

Yukawa Unification and Sparticle Spectroscopy in Gauge Mediation Models

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Abstract

We explore the implications of t - b - τ (and b - τ) Yukawa coupling unification condition on the fundamental parameter space and sparticle spectroscopy in the minimal gauge mediated supersymmetry breaking (mGMSB) model. We find that this scenario prefers values of the CP-odd Higgs mass $m_A \gtrsim 1$ TeV, with all colored sparticle masses above 3 TeV. These predictions will be hard to test at LHC13 but they may be testable at HE-LHC 33 TeV or a 100 TeV collider. Both t - b - τ and b - τ Yukawa coupling unifications prefer a relatively light gravitino with mass $\lesssim 30$ eV, which makes it a candidate hot dark matter particle. However, it cannot account for more than 15% of the observed dark matter density.

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1 Introduction

The apparent unification of the Standard Model (SM) gauge couplings at the scale $M_{\text{GUT}} \simeq 2 \times 10^{16}$ GeV in the presence of low scale supersymmetry (SUSY) [1] is nicely consistent with supersymmetric grand unified theories (GUTs), but it does not significantly constrain the sparticle spectrum. On the other hand, imposing t - b - τ Yukawa coupling unification (YU) condition at M_{GUT} [2] can place significant constraints on the supersymmetric spectrum in order to fit the top, bottom and tau masses [3, 4, 5]. Most work on t - b - τ YU condition has been performed in the framework of gravity mediation SUSY breaking [6]. Some well known choices for GUT-scale boundary conditions on the soft supersymmetry breaking (SSB) terms yield, with t - b - τ YU condition, quite severe constraints on the sparticle spectrum [3, 4, 5, 7, 8], which is further exacerbated after the discovery of the SM-like Higgs boson with mass $m_h \simeq 125 - 126$ GeV [9, 10].

Models with gauge mediated SUSY breaking (GMSB) provide a compelling resolution of the SUSY flavor problem, since the SSB terms are generated by the flavor blind gauge interactions [11, 12, 13]. In both the minimal [11] and general [13] GMSB scenarios, the trilinear SSB A-terms are relatively small at the messenger scale, even if an additional sector is added to generate the $\mu/B\mu$ terms [14]. Because of the small A-terms, accommodating the light CP-even Higgs boson mass around 125 GeV requires a stop mass in the multi-TeV range. This, in turn, pushes the remaining sparticle mass spectrum to values that are out of reach of the 14 TeV LHC [15, 16]. There is hope, however, that the predictions can be tested at 33 TeV High-Energy LHC (HE-LHC). This tension between the Higgs mass and the sparticle spectrum can be relaxed if we assume the existence of low scale vector-like particles that provide significant contribution to the CP-even Higgs boson mass [17, 18]. We will not consider this possibility here.

In this paper we investigate YU in the framework of minimal GMSB (mGMSB) scenario. Here, the decoupled messenger fields generate SSB terms of particular non-universal pattern. We know from previous studies in gravity mediation SUSY breaking scenarios that non-universal SSB terms are necessary to achieve successful YU [5]. Moreover, below M_{GUT} , the mGMSB model contains messenger fields that modify the evolution of the gauge and Yukawa couplings, thereby affecting YU. We consider mGMSB in which the messenger fields reside in $5 + \bar{5}$ of $SU(5)$, which is the simplest scenario that preserves unification of the MSSM gauge couplings. We note that previous studies of mGMSB only considered evolution below the messenger scale, which is usually much lower than M_{GUT} , and therefore could not address the question of YU.

The remainder of this paper is organized as follows. In Section 2 we briefly outline the mGMSB framework that we use for our analysis. We describe our scanning procedure along with the experimental constraints we applied in Section 3. In Section 4 we present our results, focusing in particular on the sparticle mass spectrum. The table in this section presents some benchmark points which summarize the prospects for testing these predictions at the LHC. Our conclusions are presented in Section 5. In Appendix A, we briefly discuss the renormalization group equations (RGEs) from the messenger scale to M_{GUT} .

2 Minimal Gauge Mediation

In GMSB models supersymmetry breaking is communicated from a hidden sector to the superpartners of SM particles via some messenger fields [12]. In the minimal version, the messengers interact with the visible sector only via SM gauge interactions and to the hidden sector through an arbitrary Yukawa coupling. The theory is described by the superpotential

$$W = W_{MSSM} + \lambda \hat{S} \Phi \bar{\Phi}, \quad (1)$$

where W_{MSSM} is the usual superpotential of MSSM, \hat{S} is the hidden sector gauge singlet chiral superfield, and $\Phi, \bar{\Phi}$ are the messenger fields. In order to preserve perturbative gauge coupling unification, the messenger fields are usually taken to form complete vector-like multiplets of $SU(5)$. Thus one could have N_5 (1 to 4) pairs of $5 + \bar{5}$, or a single pair of $10 + \bar{10}$, or the combination $5 + \bar{5} + 10 + \bar{10}$. For simplicity, we only consider the case with N_5 (one to four) pairs of $5 + \bar{5}$ vector-like multiplets.

The singlet field \hat{S} develops non-zero VEVs for both its scalar and auxiliary components, $\langle \hat{S} \rangle = \langle S \rangle + \theta^2 \langle F \rangle$, from the hidden sector dynamics. This results in masses for the bosonic and fermionic components of the messenger superfields,

$$m_b = M_{\text{mess}} \sqrt{1 \pm \frac{\Lambda}{M_{\text{mess}}}}, \quad m_f = M_{\text{mess}}, \quad (2)$$

where $M_{\text{mess}} = \lambda \langle S \rangle$ is the messenger scale, and $\Lambda = \langle F \rangle / \langle S \rangle$ sets the scale of the SSB terms. Note that $\langle F \rangle < \lambda \langle S \rangle^2$ to avoid tachyonic messengers, and in many realistic cases $\langle F \rangle \ll \lambda \langle S \rangle^2$.

Below the messenger scale M_{mess} , the fields Φ and $\bar{\Phi}$ decouple generating SSB masses for MSSM fields. The masses of the MSSM gauginos are generated at M_{mess} from 1-loop diagrams with messenger fields. In the approximation $\langle F \rangle \ll \lambda \langle S \rangle^2$, the gaugino masses are given by

$$M_i(M_{\text{mess}}) = \frac{\alpha_i}{4\pi} N_5 \Lambda, \quad (3)$$

where $i = 1, 2, 3$ stand for $U(1)_Y$, $SU(2)_L$ and $SU(3)_c$ sectors, respectively. Since the MSSM scalars do not couple directly to the messengers, their SSB masses are induced at two loop level:

$$m_\phi^2(M_{\text{mess}}) = 2N_5 \Lambda^2 \sum_{i=1}^3 C_i(\phi) \left(\frac{\alpha_i}{4\pi} \right)^2, \quad (4)$$

where $C_i(\phi)$ is the appropriate quadratic Casimir associated with the MSSM scalar field ϕ .

The trilinear A-terms only arise at two-loop level, and thus are vanishing at the messenger scale. However, they are generated below the messenger scale from the RGE evolution. The bilinear SSB term also vanishes at M_{mess} , although it is often ignored. We do not impose the relation $B = 0$, anticipating that the value needed to achieve the electroweak symmetry breaking (EWSB) can be explained by some other mechanism operating at the messenger scale [14].

The mGMSB spectrum is therefore completely specified by the following set of parameters:

$$M_{\text{mess}}, \Lambda, N_5, \tan \beta, \text{sign}(\mu), c_{grav}, \quad (5)$$

where $\tan\beta$ is the ratio of the two MSSM Higgs doublet VEVs. The magnitude of μ is determined by EWSB condition at the weak scale. The parameter $c_{grav}(\geq 1)$ affects the decay rate of the sparticles into gravitino, but it does not affect the MSSM spectrum, and we set it equal to unity.

It is worth noting that the gaugino masses in mGMSB are non-universal at the messenger scale (if $M_{\text{mess}} < M_{\text{GUT}}$). However, identical relations at the same scale among the gauginos can be obtained through RGE evolution in the gravity mediated SUSY breaking framework starting with universal gaugino masses at M_{GUT} . On the other hand, Eq. (4) implies a universal SSB mass terms for the up (H_u) and down (H_d) Higgs doublets at the messenger scale. Within the gravity mediated framework this relation can be obtained at the same scale if one imposes non-universal Higgs masses $m_{H_u}^2 < m_{H_d}^2$ at M_{GUT} . Indeed, this non-universality is a necessary condition for compatibility with t - b - τ YU in the gravity mediation case with universal gaugino masses at M_{GUT} [7, 8]. However, the relation $m_{H_u}^2 < m_{H_d}^2$ is introduced by hand in gravity mediated SO(10) GUT, while t - b - τ YU in mGMSB can be achieved without such *ad hoc* conditions, as we will see later. From Eq. (4) the sfermion masses in mGMSB are non-universal at the messenger scale, but it is not easy to find the same pattern at M_{mess} within the gravity mediation scenario. Hence, the sfermion sector could be a good place to see differences between gravity and gauge mediation SUSY breaking scenarios that are compatible with YU condition at the GUT scale.

3 Scanning Procedure and Experimental Constraints

For our scan over the fundamental parameter space of mGMSB, we employed ISAJET 7.84 package [19] that we modified to include the RGE evolution above M_{mess} in MSSM with N_5 pairs of $5 + \bar{5}$ messengers. In this package, the weak-scale values of gauge and third generation Yukawa couplings are evolved from M_Z to M_{mess} via the MSSM RGEs in the \overline{DR} regularization scheme. Above M_{mess} the messenger fields are present, which changes the beta-functions for the SM gauge and Yukawa couplings. The modified RGEs between M_{mess} and M_{GUT} are given in Appendix A. M_{GUT} is defined as the scale at which $g_1 = g_2$ and is approximately equal to 2×10^{16} GeV, the same as in MSSM. We do not strictly enforce the unification condition $g_3 = g_1 = g_2$ at M_{GUT} , since a few percent deviation from unification can be accounted by unknown GUT-scale threshold corrections [20]. For simplicity, we do not include the Dirac neutrino Yukawa coupling in the RGEs, whose contribution is expected to be small.

The SSB terms are induced at the messenger scale and we set them according to expressions (3) and (4). From M_{mess} the SSB parameters, along with the gauge and Yukawa couplings, are evolved down to the weak scale M_Z . In the evolution of Yukawa couplings the SUSY threshold corrections [21] are taken into account at the common scale $M_{\text{SUSY}} = \sqrt{m_{\tilde{t}_L} m_{\tilde{t}_R}}$, where $m_{\tilde{t}_L}$ and $m_{\tilde{t}_R}$ denote the soft masses of the left and right-handed top squarks. The entire parameter set is iteratively run between M_Z and M_{GUT} using the full 2-loop RGEs until a stable solution is obtained. To better account for leading-log corrections, one-loop step-beta functions are adopted for the gauge and Yukawa couplings, and the SSB parameters m_i are extracted from RGEs at multiple scales: at the scale of its own mass, $m_i = m_i(m_i)$, for the unmixed sparticles, and at the common scale M_{SUSY} for the mixed ones. The RGE-improved 1-loop effective potential is minimized at M_{SUSY} , which

effectively accounts for the leading 2-loop corrections. Full 1-loop radiative corrections [21] are incorporated for all sparticle masses.

We have performed random scans over the model parameters (5) in the following range:

$$\begin{aligned}
10^3 \text{ GeV} &\leq \Lambda \leq 10^7 \text{ GeV}, \\
10^5 \text{ GeV} &\leq M_{\text{mess}} \leq 10^{16} \text{ GeV}, \\
40 &\leq \tan\beta \leq 60, \\
N_5 &= 1, \quad \mu < 0.
\end{aligned}
\tag{6}$$

Varying N_5 from 1 to 4, we find that the low scale sparticle spectrum does not change significantly [15]. Thus we present results only for $N_5 = 1$ case. Regarding the MSSM parameter μ , its magnitude but not its sign is determined by the radiative electroweak symmetry breaking. In our model we set $\text{sign}(\mu) = -1$. A negative μ -term together with a positive gaugino mass M_2 gives negative contribution to the muon anomalous magnetic moment calculation [22] which would disagree with the measured value for muon $g-2$ [23]. This, however, is not a problem, since the sparticle spectrum is quite heavy in our scenario. As we will see, the smuons are often heavier than 1 TeV. Hence, the SUSY contribution to muon $g-2$ is not significant, which also is the case in t - b - τ YU scenarios [24, 5]. On the other hand, a negative sign of μ with positive signs for all gauginos helps to realize YU. The reason for this is that in order to implement YU, we require significant SUSY threshold corrections to the bottom quark Yukawa coupling. One of the dominant finite corrections is proportional to μM_3 [25] and it helps to realize Yukawa unification if this combination has negative sign [5]. Finally, we employ the current central value for the top mass, $m_t = 173.3$ GeV. Our results are not too sensitive to one or two sigma variation of m_t [27].

In scanning the parameter space, we employ the Metropolis-Hastings algorithm as described in Ref. [28]. The data points collected all satisfy the requirement of radiative electroweak symmetry breaking (REWSB). After collecting the data, we impose the mass bounds on the particles [29] and use the IsaTools package [30, 24] to implement the various phenomenological constraints. We successively apply mass bounds including the Higgs [9, 10] and gluino masses [31], and the constraints from the rare decay processes $B_s \rightarrow \mu^+ \mu^-$ [32], $b \rightarrow s\gamma$ [33] and $B_u \rightarrow \tau\nu_\tau$ [34]. The constraints are summarized below in Table 1. Notice that we used wider range for the Higgs boson mass, since there is an approximate error of around 2 GeV in the estimate of the mass that largely arises from theoretical uncertainties in the calculation of the minimum of the scalar potential, and to a lesser extent from experimental uncertainties in the values of m_t and α_s .

$$\begin{aligned}
123 \text{ GeV} &\leq m_h \leq 127 \text{ GeV} \\
& m_{\tilde{g}} \geq 1.5 \text{ TeV} \\
0.8 \times 10^{-9} &\leq \text{BR}(B_s \rightarrow \mu^+ \mu^-) \leq 6.2 \times 10^{-9} \quad (2\sigma) \\
2.99 \times 10^{-4} &\leq \text{BR}(b \rightarrow s\gamma) \leq 3.87 \times 10^{-4} \quad (2\sigma) \\
0.15 &\leq \frac{\text{BR}(B_u \rightarrow \tau\nu_\tau)_{\text{MSSM}}}{\text{BR}(B_u \rightarrow \tau\nu_\tau)_{\text{SM}}} \leq 2.41 \quad (3\sigma)
\end{aligned}$$

Table 1: Phenomenological constraints implemented in our study.

Following ref. [28] we define the parameters $R_{tb\tau}$ and $R_{b\tau}$ which quantify t - b - τ YU and

b - τ YU respectively:

$$R_{tbr} = \frac{\text{Max}(y_t, y_b, y_\tau)}{\text{Min}(y_t, y_b, y_\tau)}, \quad R_{b\tau} = \frac{\text{Max}(y_b, y_\tau)}{\text{Min}(y_b, y_\tau)} \quad (7)$$

Thus, R is a useful indicator for Yukawa unification with $R \lesssim 1.1$, for instance, corresponding to Yukawa unification to within 10%, while $R = 1.0$ denotes ‘perfect’ Yukawa unification.

4 Results

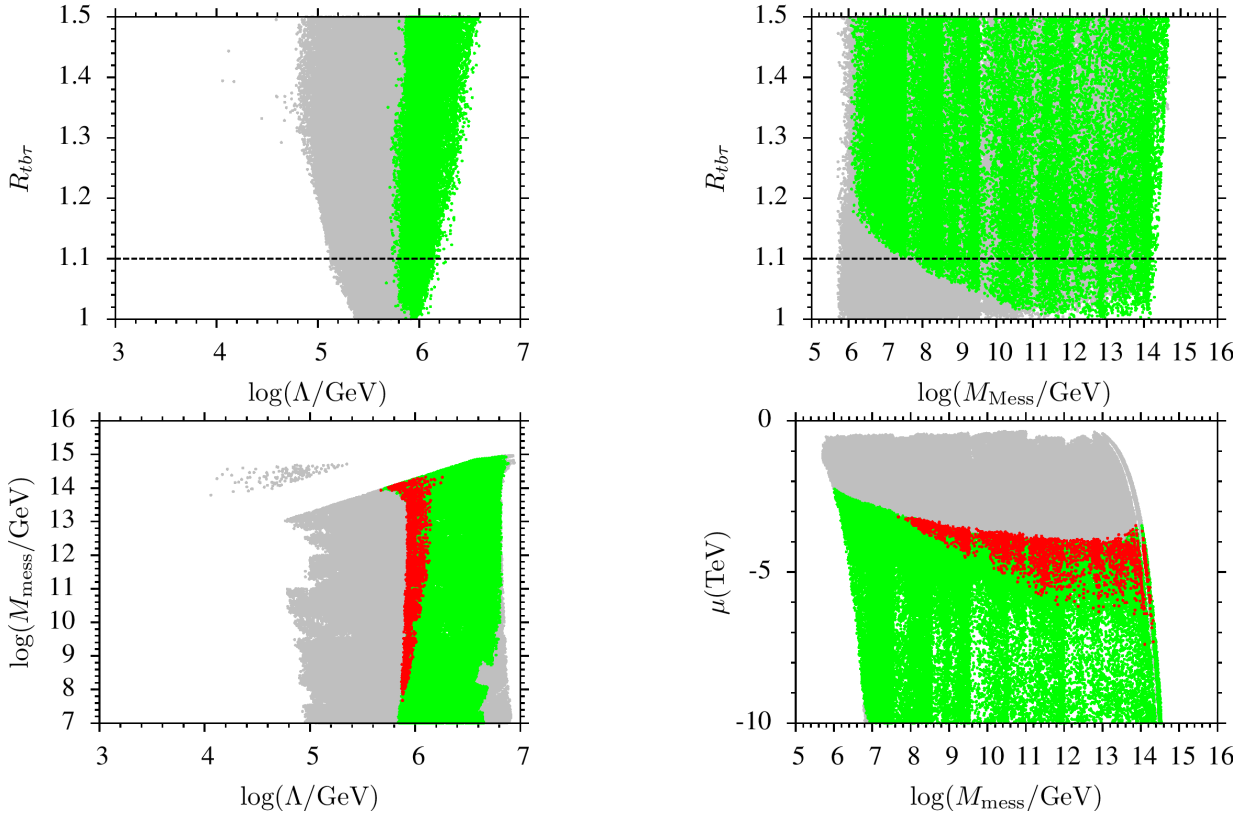


Figure 1: Plots in $R_{tbr} - \Lambda$, $R_{tbr} - M_{\text{mess}}$, $M_{\text{mess}} - \Lambda$ and $\mu - M_{\text{mess}}$ planes. All points are consistent with REWSB. Green points also satisfy mass bounds and B-physics constraints. Red points are a subset of green and they are compatible with t - b - τ YU. In the top panels regions below the horizontal line are compatible with YU such that $R_{tbr} \leq 1.1$

Figure 1 displays the regions in the parameter space that are compatible with t - b - τ YU with plots in $R_{tbr} - \Lambda$, $R_{tbr} - M_{\text{mess}}$, $M_{\text{mess}} - \Lambda$ and $\mu - M_{\text{mess}}$ planes. All points are consistent with REWSB. Green points satisfy the mass bounds and B-physics constraints. Red points are a subset of green ones and they are compatible with t - b - τ YU. In the top panels the dashed line indicates the region consistent with YU with $R_{tbr} \leq 1.1$. The $R_{tbr} - \Lambda$ plane shows that the parameter Λ in mGMSB is constrained by $10^5 \text{ GeV} \lesssim \Lambda \lesssim 10^6 \text{ GeV}$ for t - b - τ YU with $R_{tbr} \leq 1.1$. As seen from the $R_{tbr} - M_{\text{mess}}$ plane, YU imposes a far less stringent constraint on M_{mess} , namely $10^8 \text{ GeV} \lesssim M_{\text{mess}} \lesssim 10^{14} \text{ GeV}$. We also present,

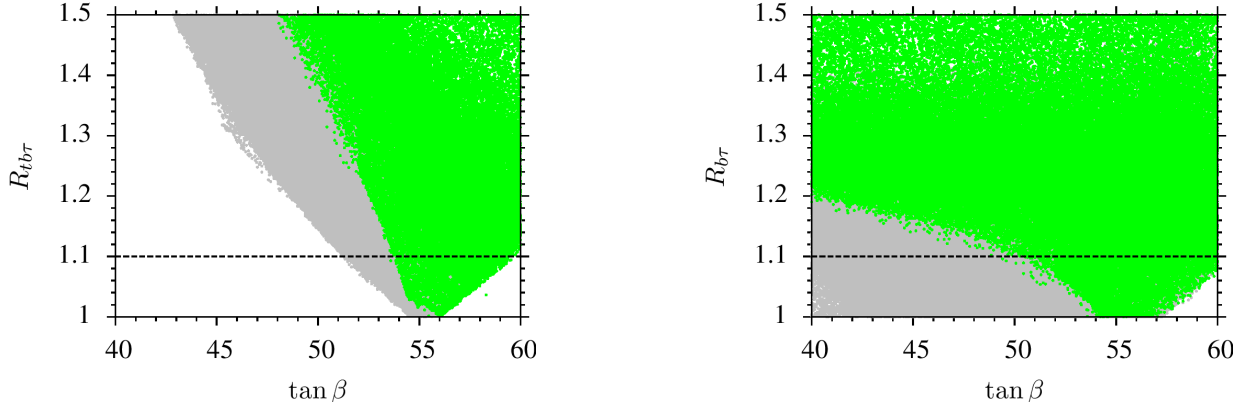


Figure 2: Plots in $R_{tb\tau} - \tan\beta$ and $R_{b\tau} - \tan\beta$ planes. The color coding is the same as in Figure 1.

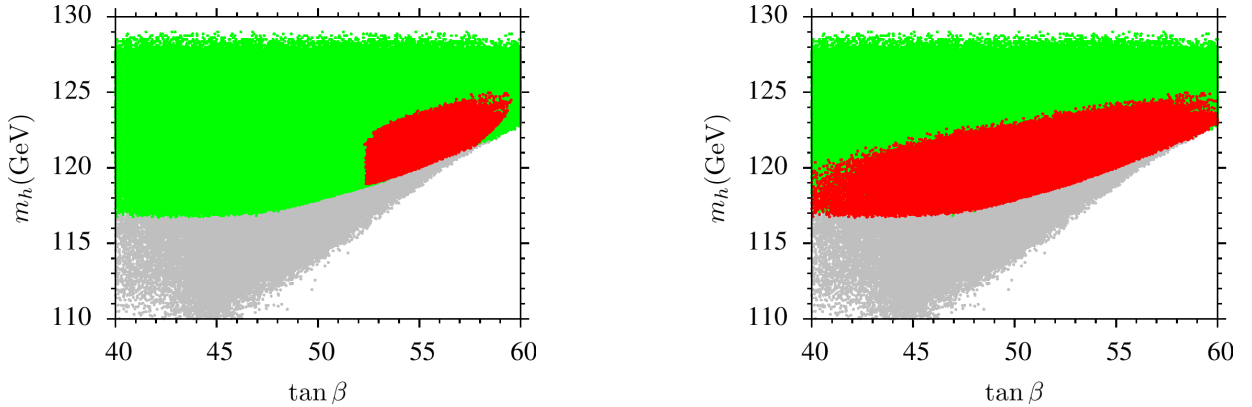


Figure 3: Plots in $m_h - \tan\beta$ plane for t - b - τ YU (left panel) and b - τ YU (right panel). The color coding is the same as in Figure 1 except the Higgs mass bound is not applied in these panels.

for additional clarity, the results in $M_{\text{mess}} - \Lambda$ plane. The MSSM parameter μ lies in the range $-8 \text{ TeV} \lesssim \mu \lesssim -3 \text{ TeV}$ for compatibility with YU. We see that there are plenty of solutions consistent with the mass bounds and constraints from rare B-decays. However, only the thin red stripe is compatible with YU.

The strong impact of t - b - τ YU on the fundamental parameter space is somewhat relaxed if only b - τ YU is imposed. Figure 2 shows the differences between t - b - τ and b - τ YU cases with plots in $R_{tb\tau} - \tan\beta$ and $R_{b\tau} - \tan\beta$ planes. The color coding is the same as in Figure 1. One sees that, t - b - τ YU only allows a very narrow range for $\tan\beta$, namely $54 \lesssim \tan\beta \lesssim 60$, and perfect t - b - τ YU consistent with the experimental constraints can be realized when $\tan\beta \approx 56$. On the other hand, the $R_{b\tau} - \tan\beta$ plane shows that the range for $\tan\beta$ is slightly enlarged, $50 \lesssim \tan\beta \lesssim 60$, with b - τ YU, and perfect b - τ YU is obtained for $54 \lesssim \tan\beta \lesssim 57$.

A similar comparison based on the range for the SM-like Higgs boson mass is given in Figure 3 with plots in $m_h - \tan\beta$ planes for t - b - τ (left panel) and b - τ (right panel) YU cases. The color coding is the same as in Figure 1, except that the Higgs mass bound is not applied in these panels. The left panel shows that demanding 10% or better YU, the lightest CP-even Higgs boson mass can be $m_h \approx 119 - 125 \text{ GeV}$ for t - b - τ case, while it can

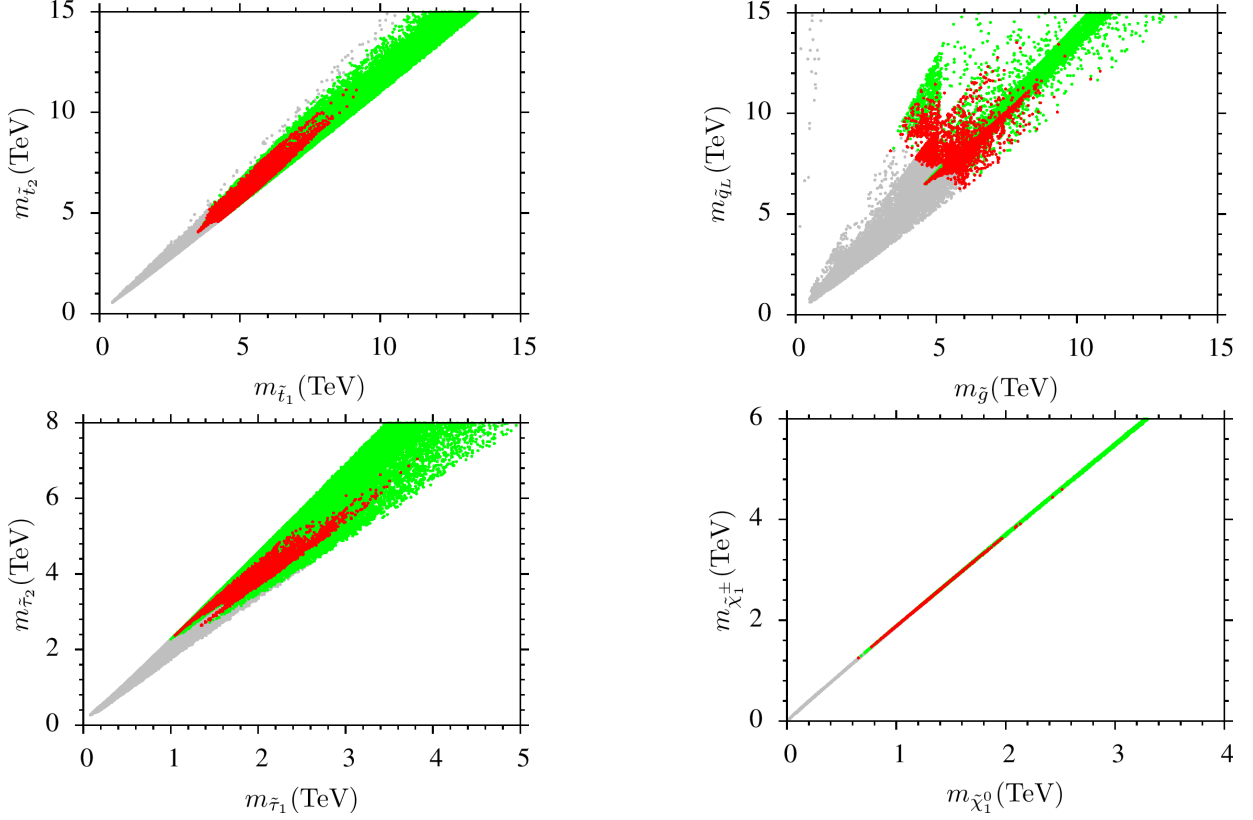


Figure 4: Plots in $m_{\tilde{t}_1} - m_{\tilde{t}_2}$, $m_{\tilde{u}_L} - m_{\tilde{g}}$, $m_{\tilde{\tau}_1} - m_{\tilde{\tau}_2}$ and $m_{\tilde{\chi}_1^\pm} - m_{\tilde{\chi}_1^0}$ planes. The color coding is the same as in Figure 1.

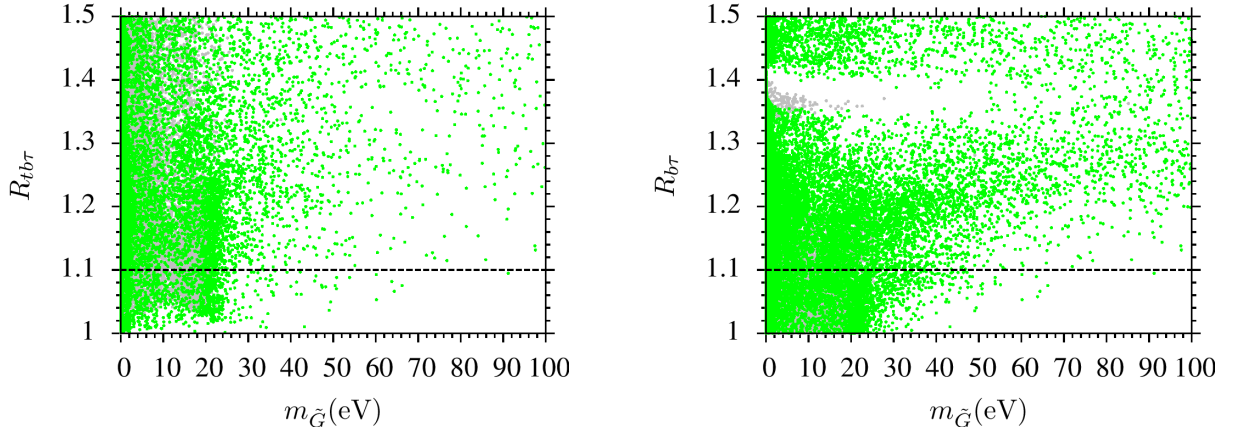


Figure 5: Plots in $M_{\text{mess}} - m_{\tilde{G}}$, $\Lambda - m_{\tilde{G}}$, $R_{tbr} - m_{\tilde{G}}$ and $R_{b\tau} - m_{\tilde{G}}$ planes. The color coding is the same as in Figure 1.

be as light as 117 GeV in the case of b - τ YU, as shown in the right panel. The panels of Figure 3 highlight how stringent the Higgs boson mass constraint is on the fundamental parameter space of mGMSB compatible with YU. The left panel shows that the 125 GeV Higgs boson mass and t - b - τ YU more or less equivalently constrain the parameter space. On the other hand, as seen from the right panel, b - τ YU can be realized for $\tan \beta \gtrsim 40$, while the mass bound on the Higgs boson excludes the region with $\tan \beta \lesssim 50$.

Figure 4 displays the mass spectrum of supersymmetric particles in $m_{\tilde{t}_2} - m_{\tilde{t}_1}$, $m_{\tilde{u}_L} - m_{\tilde{g}}$,

	Point 1	Point 2	Point 3	Point 4	Point 5
Λ	0.94×10^6	1.51×10^6	7.6×10^5	4.7×10^5	1.16×10^6
M_{mess}	1.5×10^{14}	1.3×10^{14}	1.2×10^8	8.6×10^{13}	1.3×10^{15}
$\tan \beta$	56.1	59.3	58.7	54.8	52.7
μ	-4906	-6727	-2995	-4268	-3984
A_t	-3564	-5479	-1933	-1834	-4513
A_b	-3600	-5540	-1946	-1891	-4868
A_τ	-268.1	-462.9	-123.2	-140.4	-535.1
m_h	124	125	123	123	124
m_H	1148	1640	929	1665	2378
m_A	1141	1629	923	1654	2363
m_{H^\pm}	1153	1643	934.2	1668	2380
$m_{\tilde{\chi}_{1,2}^0}$	1305, 2438	2098, 3868	1045, 1971	654, 1249	1602, 2960
$m_{\tilde{\chi}_{3,4}^0}$	4902, 4903	6725, 6725	3001, 3002	4190, 4190	3993, 3995
$m_{\tilde{\chi}_{1,2}^\pm}$	2440, 4902	3871, 6724	1972, 3002	1251, 4154	2961, 3996
$m_{\tilde{g}}$	6113	9350	4995	3372	7178
$m_{\tilde{u}_{L,R}}$	9545, 8590	13434, 12152	6903, 6469	8140, 7206	7818, 7255
$m_{\tilde{t}_{1,2}}$	6325, 7784	9152, 11112	5569, 6186	5011, 6515	5752, 6745
$m_{\tilde{d}_{L,R}}$	9545, 8317	13434, 11796	6903, 6412	8140, 6937	7819, 7124
$m_{\tilde{b}_{1,2}}$	6280, 7743	9134, 11053	5617, 6148	5054, 6482	6022, 6709
$m_{\tilde{\nu}_{e,\mu}}$	5181	7122	2727	4659	3560
$m_{\tilde{\nu}_\tau}$	4843	6625	2659	4358	3372
$m_{\tilde{e}_{L,R}}$	5186, 3698	7129, 5044	2735, 1478	4663, 3324	3567, 2378
$m_{\tilde{\tau}_{1,2}}$	2600, 4845	3397, 6628	1193, 2667	2340, 4357	1748, 3381
$R_{tb\tau}$	1.00	1.09	1.09	1.06	1.23
$R_{b\tau}$	1.00	1.08	1.06	1.02	1.07

Table 2: Benchmark points for exemplifying our results. All masses and scales are given in GeV units. Point 1 depicts a solution with perfect YU. Point 2 shows a solution with 125 GeV mass for h consistent with YU. Point 3 displays a solution with essentially the lowest stau mass consistent with YU. Similarly, point 4 shows essentially the lightest mass for the lightest neutralino. This point also demonstrates the lightest gluino and stop masses obtained from our scans. Point 5 displays a solution which is compatible with b - τ YU, but not with t - b - τ YU.

$m_{\tilde{\tau}_2} - m_{\tilde{\tau}_1}$ and $m_{\tilde{\chi}_1^\pm} - m_{\tilde{\chi}_1^0}$ planes, with the color coding the same as in Figure 1. As seen from the top panels, the colored sparticles are quite heavy. Thus t - b - τ YU requires $m_{\tilde{t}_1} \gtrsim 4$ TeV. Since the SSB trilinear scalar interaction term A_t is relatively small [15], the heavier stop is expected to be about the same mass ($\gtrsim 4$ TeV) as the lighter stop. Similarly, the squarks of the first two families and the gluino have masses in the few TeV range. The $m_{\tilde{q}_L} - m_{\tilde{g}}$ plane shows that $m_{\tilde{g}} \gtrsim 3$ TeV and $m_{\tilde{q}_L} \gtrsim 6$ TeV for consistency with t - b - τ YU as well as the experimental constraints.

The bottom panels of Figure 4 display the mass spectra for the lightest sparticles which are found to be stau, chargino and neutralino. According to the $m_{\tilde{\tau}_2} - m_{\tilde{\tau}_1}$ panel, $\tilde{\tau}_1$ can be as light as 1 TeV, while $m_{\tilde{\tau}_2} \gtrsim 2$ TeV. Similarly, the $m_{\tilde{\chi}_1^\pm} - m_{\tilde{\chi}_1^0}$ plane displays the masses of the lightest chargino and neutralino with the latter $\gtrsim 600$ GeV, and the former $\gtrsim 1200$ GeV.

Finally, Figure 5 displays the gravitino mass, $m_{\tilde{G}}$, in $R_{tb\tau} - m_{\tilde{G}}$ and $R_{b\tau} - m_{\tilde{G}}$ planes with the color coding as in Figure 1. As is well-known, the gravitino is usually the LSP

in the mGMSB framework, and its mass can be as heavy as 10 TeV. A light gravitino is a plausible dark matter candidate and it can also manifest itself through missing energy in colliders [36].

In the standard scenarios, the relic density bound ($\Omega h^2 \simeq 0.11$ [35]) is satisfied with a gravitino mass ~ 200 eV [36], which makes it a hot dark matter candidate. The hot component of dark matter, however, cannot be more than 15% which, in turn, implies that the gravitino mass should be less than about 30 eV [36]. This indeed is predicted from Figure 5. The panels of Figure 5 represent the impact of YU on the gravitino mass. Both t - b - τ and b - τ YU do not allow a gravitino that is too heavy, preferring instead a gravitino lighter than 100 eV or so. The $R_{tb\tau} - m_{\tilde{G}}$ plane shows that t - b - τ YU is mostly realized with $m_{\tilde{G}} \lesssim 30$ eV, with essentially perfect YU possible for nearly massless gravitinos. The bound on the gravitino mass is not significantly relaxed for b - τ YU. However, essentially perfect YU is found for $m_{\tilde{G}} \lesssim 50$ eV. In order to have a complete dark matter scenario one could invoke axions as cold dark matter.

In Table 2 we present five benchmark points that exemplify our results. The points are chosen to be consistent with the experimental constraints and YU. The masses are given in GeV. Point 1 depicts a solution with perfect YU. Point 2 shows the heaviest mass in our scan for the lightest CP-even Higgs boson, while point 3 displays the lightest stau mass that we found, consistent with YU. Similarly, point 4 represents the lightest neutralino mass found in our scan. This point also shows the lightest gluino and stop masses that we obtained. Point 5 displays a solution which is compatible with b - τ YU, but not with t - b - τ YU.

5 Conclusion

Models based on the gauge mediated SUSY breaking are highly motivated, as they provide an attractive solution to the SUSY flavor problem. We have explored the implications of t - b - τ and b - τ YU condition on the sparticle spectroscopy in the framework of mGMSB where the messengers reside in $5+\bar{5}$ multiplets of $SU(5)$ and only interact with visible sector via gauge interactions. We find that YU leads to a heavy sparticle spectrum with the lowest mass sparticles being the lightest neutralino ($m_{\tilde{\chi}_1^0} \gtrsim 600$ GeV), chargino ($m_{\tilde{\chi}_1^\pm} \gtrsim 1200$ GeV) and stau ($m_{\tilde{\tau}} \gtrsim 1000$ GeV). In addition, YU prefers values of the CP-odd Higgs boson mass m_A to be greater than 1 TeV and all colored sparticle masses above 3 TeV. Such heavy sparticles can escape detection at the 13-14 TeV LHC, but should be accessible at HE-LHC 33 TeV or the proposed 100 TeV collider. We find that YU requirement in the mGMSB framework leads to a spectrum of gauginos, Higgses and staus, that is very similar to the one in gravity mediated $SO(10)$ GUT, although for different reasons. The details of the rest of sfermion spectrum are different, but they are at about the same heavy scale. However, YU in mGMSB framework does not require any *ad hoc* assumptions, such as as SSB non-universalities that are necessary in gravity mediated GUTs. It is also interesting to note that t - b - τ and b - τ YU favor a gravitino mass less than 30 eV, which makes it a hot dark matter candidate. In this case, the gravitino cannot comprise more than 15% of the dark matter density, and one may invoke axions in order to have a complete dark matter scenario.

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A RGEs for MSSM and $\mathbf{5} + \bar{\mathbf{5}}$

Above the messenger scale M_{mess} the MSSM is supplemented with vector-like messenger fields and is described by the superpotential in Eq. (1). Since the theory is supersymmetric above M_{mess} , there are no SSB terms in the lagrangian. The N_5 messenger pairs $\mathbf{5} + \bar{\mathbf{5}}$ of $SU(5)$ decompose under the SM gauge group as

$$\Phi + \bar{\Phi} = L_M \left(1, 2, \frac{1}{2}\right) + \bar{L}_M \left(1, 2, -\frac{1}{2}\right) + D_M \left(\bar{3}, 1, \frac{1}{3}\right) + \bar{D}_M \left(3, 1, -\frac{1}{3}\right), \quad (8)$$

where numbers in parentheses represent transformation properties under $SU(3)_c$, $SU(2)_L$ and $U(1)_Y$ respectively.

At the 1-loop level the MSSM gauge couplings contain additional term $g_a^3 N_5$, where $a = 1, 2, 3$ correspond to the gauge groups $U(1)_Y$, $SU(2)_L$ and $SU(3)_c$, respectively. At the 2-loop level additional contributions appear from terms where the Dynkin index is summed over all chiral multiplets. Hence the gauge coupling RGEs have the form

$$\frac{dg_a}{dt} = \frac{1}{16\pi^2} (\beta_a^{(1)} + g_a^3 N_5) + \frac{1}{(16\pi^2)^2} \left(\beta_a^{(2)} + g_a^3 \sum_{b=1}^3 2N_5 B_{ab}^{(2)}(5) g_b^2 \right). \quad (9)$$

Here, $\beta_a^{(1,2)}$ are one- and two-loop MSSM beta functions that can be found in Ref. [37], and

$$B_{ab}^{(2)}(5) = \begin{pmatrix} 7/30 & 9/10 & 16/15 \\ 3/10 & 7/2 & 0 \\ 2/15 & 0 & 17/3 \end{pmatrix} \quad (10)$$

is the contribution from a single 5-plet. Note that $\mathbf{5}$ and $\bar{\mathbf{5}}$ give the same contribution to RGEs.

The anomalous dimensions for MSSM fields are modified only at the 2-loop level, since messengers only couple to MSSM fields via gauge interactions. The RGEs for the MSSM

Yukawa couplings are

$$\frac{dy_t}{dt} = \frac{1}{16\pi^2}\beta_t^{(1)} + \frac{1}{(16\pi^2)^2} \left(\beta_t^{(2)} + y_t N_5 \left(\frac{8}{3}g_3^4 + \frac{3}{2}g_2^4 + \frac{13}{30}g_1^4 \right) \right), \quad (11)$$

$$\frac{dy_b}{dt} = \frac{1}{16\pi^2}\beta_b^{(1)} + \frac{1}{(16\pi^2)^2} \left(\beta_b^{(2)} + y_b N_5 \left(\frac{8}{3}g_3^4 + \frac{3}{2}g_2^4 + \frac{7}{30}g_1^4 \right) \right), \quad (12)$$

$$\frac{dy_\tau}{dt} = \frac{1}{16\pi^2}\beta_\tau^{(1)} + \frac{1}{(16\pi^2)^2} \left(\beta_\tau^{(2)} + y_\tau N_5 \left(\frac{3}{2}g_2^4 + \frac{9}{10}g_1^4 \right) \right), \quad (13)$$

where $\beta_i^{(1,2)}$ are one- and two-loop MSSM beta functions [37].

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