A memory structure adapted simulated annealing algorithm for a green vehicle routing problem

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Abstract Currently, reduction of carbon dioxide (CO_2) emissions and fuel consumption has become a critical environmental problem and has attracted the attention of both academia and the industrial sector. Government regulations and customer demands are making environmental responsibility an increasingly important factor in overall supply chain operations. Within these operations, transportation has the most hazardous effects on the environment, i.e., CO_2 emissions, fuel consumption, noise and toxic effects on the ecosystem. This study aims to construct vehicle routes with time windows that minimize the total fuel consumption and CO_2 emissions. The green vehicle routing problem with time windows (G-VRPTW) is formulated using a mixed integer linear programming model. A memory structure adapted simulated annealing (MSA-SA) meta-heuristic algorithm is constructed due to the high complexity of the proposed problem and long solution times for practical applications. The proposed models are integrated with a fuel consumption and CO_2 emissions calculation algorithm that considers the vehicle technical specifications, vehicle load, and transportation distance in a green supply chain environment. The proposed models are validated using well-known instances with different numbers of customers. The computational results indicate that the MSA-SA heuristic is capable of obtaining good G-VRPTW solutions within a reasonable amount of time by providing reductions in fuel consumption and CO_2 emissions.

Keywords Green logistics · Fuel consumption · CO_2 emissions · Green vehicle routing problem · Mixed integer linear programming model · Simulated annealing algorithm

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Introduction

Atmospheric levels of CO_2 have increased steadily since the beginning of the industrial revolution, and these levels are projected to increase even more rapidly as the global economy grows. Significant climate changes are likely associated with increased atmospheric concentrations of certain gases, most significantly, CO_2. As concern over climate change has risen, it has become increasingly important to monitor and record CO_2 emissions into the atmosphere. Government and industry are now poised to enact broad national and international policy decisions focused on monitoring the size of their carbon footprints and reducing greenhouse gas (GHG) emissions. In highly competitive environments, industrial organizations must investigate logistics strategies and adopt green supply chain applications to consider the accurate use of natural resources as a social responsibility.

In the past, logistics systems have primarily focused on the objective of increasing the efficiency of industry activities with respect to timing and profits. However, the current atmosphere of growing concern over environmental impacts has introduced the concept of green logistics as a basis for developing methods that can reduce the environmental impacts of logistics systems.

The importance of environmental issues is continuously translated into regulations, which potentially can have a tangible impact on supply chain management. As a consequence, an increasing amount of research has been carried out on the intersection between logistics and environmental factors (Jabali et al. 2012).

The growth in emissions has resulted from an increasing demand for transport activities in passenger transport and especially in freight transport. Between 1990 and 2005, freight transport in Europe rose by 90 %. In the same period, the demand for road freight transport, which is the major polluter of all land-based transport modes, increased to an

even greater extent (by 138 %). Consequently, the contribution of road freight transport to direct CO_2 emissions is 20 % (Bühler and Jochem 2008).

A green logistics strategy includes the activities of supply of raw materials, production, packaging, transport, storage, implementation of environmental policies, energy conservation, waste recycling, and reverse logistics. Green logistics involves producing and dispatching goods in a sustainable manner by taking into account economic, social, and environmental factors. Green logistics activities also include measuring the environmental impact of different distribution strategies, reducing the energy usage in logistics activities, reducing waste and managing waste treatment. Green logistics can be viewed as an approach for planning logistics systems that incorporate sustainability goals with a primary focus on reduction of environmental externalities. The "green" vehicle routing problem, which is one of the fundamental operations in logistics, is concerned with determining routes for a fleet of vehicles to satisfy the demands from a set of customers with an emphasis on environmental concerns and CO_2 emissions in particular.

The scope of this study includes development of a new mathematical model for energy consumption optimization by reducing CO_2 emissions and fuel consumption in the green vehicle routing problem with time windows (G-VRPTW) and by solving this model using a memory structure adapted simulated annealing (MSA-SA) meta-heuristic algorithm. The remainder of this paper is organized as follows. It starts off with a "Literature review" section followed by "Problem definition", which introduces the problem. "Model formulation" explains the nonlinear fuel consumption calculation algorithm, mixed integer linear programming model, and proposed meta-heuristic algorithm. "Computational results" provides numerical examples and results. Finally, conclusions are presented in "Conclusions and future research."

Literature review

The vehicle routing problem (VRP) is a combinatorial optimization problem that seeks to serve a set of customers with a fleet of vehicles. Formulated initially by Dantzig and Ramser (1959), the VRP is an important problem in the fields of transportation, distribution, and logistics (Tan et al. 2001). In a typical VRP, the primary objective is to minimize the total transportation cost or distance, and the model consists of a depot and fleet of vehicles used to deliver goods to various customers.

The vehicle routing problem with time windows (VRPTW) is a well-known nonpolynomial-hard (NP-hard) problem that represents the generalization of the VRP with additional time constraints and is an important problem that occurs in many distribution systems. The problem involves a fleet of vehicles that set off from a depot to serve a number of customers at different geographic locations with various demands and

within specific time windows before returning to the depot. The objective of the problem is to find routes for the vehicles to serve all of the customers at a minimal total travel distance or at a minimal number of vehicles without violating the capacity and travel time constraints of the vehicles and the time window constraints set by the customers (Alvarenga et al. 2007, Tan et al. 2001). In these problems, the service to a customer must begin within a time window $[e_i, l_i]$ defined by customer i. The vehicle can arrive no earlier than time e_i and no later than time l_i. The regular VRP can be viewed as the situation $e_i = 0$ and $l_i = \infty$ for all $1 \le i \le n$. When $0 \le e_i \le l_i \le \infty$, the situation is also known as a double-sided time window, whereas a single-sided time window refers to either $e_i = 0$ or $l_i = \infty$ but not both. In the presence of time windows, the total routing and scheduling costs include not only the total travel distance and time costs considered for routing problems but also the cost of waiting time incurred if a vehicle arrives too early at a customer location or the time in which the vehicle is loaded or unloaded (Solomon 1987).

The VRP plays a vital role in logistics and has been extensively studied. By considering additional requirements and various constraints on route construction, different VRPs have been formulated, i.e., pick-up and delivery VRP, capacitated VRP (CVRP), multiple-depot VRP, and VRPTW. Although there are different forms of VRPs, most minimize cost by minimizing the total distance but without considering the fuel consumption rate. In fact, statistics show that fuel cost is a significant component of the total transportation cost (Xiao et al. 2012). Because VRPTW has been the subject of intensive research and can be used to model many real-world problems, it adds the complexity of allowable delivery times or time window constraints stemming from the fact that customers request earliest and latest service times (Amini et al. 2010).

The literature contains many studies on the topic of VRP (Bodin 1990; Bühler and Jochem 2008; Toth and Vigo 2002; Hoff et al. 2010; Huang et al. 2012 etc.) and the VRPTW (Solomon 1987; Azi et al. 2010; Macedo et al.; 2011, Banos et al. 2013; Agra et al. 2013 etc.). Solution approaches for the VRPTW can be classified into two main types: exact methods and heuristics/meta-heuristics methods. Exact methods have the ability to obtain the best solutions and guarantee their optimality. Because of the growth in the problem size and because the VRPTW is NP-hard, exact methods are not useful options because they are only suitable for application to a problem with a small size. Heuristics/meta-heuristics approaches can deliver a good-quality solution within a reasonable time but cannot guarantee the optimality of these solutions (Talbi 2009). Heuristics and meta-heuristics solution methods for the VRPTW are presented in Bräysy and Gendreau (2005a, b). Although, interest has grown in practical applications and literature on green logistics in recent years, according to our knowledge, few studies have been conducted

on the VRP under minimization of energy consumption and CO_2 emissions. Apaydın and Gönüllü (2008) studied the VRP model on reducing the CO_2 emissions of diesel vehicles used for waste collection operations in a city, and a shortest path model was used to optimize solid waste collection. They used fixed values for fuel consumption. Tavares et al. (2009) developed a model to optimize waste collection routing for vehicle fuel consumption based on a geographic information system. In the fuel consumption calculation, the speed, gradient, and load are considered but acceleration is not. Kuo (2010), proposed a model for calculating the total fuel consumption for a time-dependent vehicle routing problem (TDVRP) in which speed and travel times are assumed to depend on the time of travel when planning vehicle routes. The model calculates the fuel consumption with respect to vehicle load and vehicle speed. The SA algorithm is proposed for finding vehicle routes with the lowest total fuel consumption. Figliozzi (2010) proposed a model for the time-dependent VRP. However, the model did not consider vehicle load and acceleration. Bektaş and Laporte (2011) described an approach intended to reduce the energy requirements in vehicle routing based on a comprehensive emissions model that takes into account the load and speed. The authors present a comprehensive formulation of the problem and solve moderately sized instances. Suzuki (2011) presented a model that indicates that significant savings in fuel consumption and CO_2 emissions may be realized by delivering heavy items in the earlier segments of a tour and delivering lighter items in the later segments such that the distance that a vehicle travels while carrying heavy payloads can be minimized. The model considers the vehicle load, the average speed based on the road gradient and the average amount of fuel consumption per hour at the customer sites. Wygonik and Goodchild (2011) used a modified version of the standard ArcGIS VRP tool, extended to incorporate emissions for an urban pick-up and delivery system. They developed different scenarios and conducted a scenario analysis to check the trade-offs between cost, service quality, and emissions of an urban transportation systems. Their paper does not include a mathematical model. Xiao et al. (2012) proposed a mathematical model that minimizes the fuel consumption of the vehicles and developed a SA-based algorithm to solve the model. They formulated a linear function that depends on the vehicle load for the fuel consumption. Erdoğan and Miller-Hooks (2012) considered the fuel consumption of the vehicles and sought a set of vehicle tours that minimize the total distance traveled to serve a set of customers while incorporating stops at alternative fueling stations en route. For solution approach, modified Clarke and Wright savings heuristic, the density-based clustering algorithm and an improvement heuristic are developed. Jabali et al. (2012) proposed a framework for modeling CO_2 emissions as a function of speed in a time-dependent VRP, and the model was solved via a Tabu search procedure. Pradenas

et al. (2013) presented a mathematical model that minimizes the emission of GHG for a homogenous vehicle fleet and solved the problem by using a scatter search method. They used the problems from the literature for analysis, their results show that the traveling distance and the transportation costs increase as the required energy and fuel consumption decreases. Wu et al. (2012) studied the prediction of vehicle population, vehicle fuel consumption, and exhaust emissions in China for auto industry. For prediction of vehicle population, they used a logistic model and by using this predicted value, they formulated equations for fuel consumption and exhaust emissions prediction. They also evaluated the related industry policies. Küçükoğlu et al. (2013) presented a mixed integer linear programming model for a green-capacitated vehicle routing problem that means to minimize total fuel consumption of the route. They proposed a computation for calculating fuel consumption considering the vehicle technical specifications, vehicle load, and the distance. Their proposed model provides important reductions in fuel consumption.

Although the literature review reveals several studies for different types of vehicle routing problems, there are no adequate numbers of studies in the literature for the green vehicle routing problem intended to minimize fuel consumption and CO_2 emissions. The previous studies on green vehicle routing problem in the literature (Apaydın and Gönüllü 2008, Jabali et al. 2012, Kuo 2010, Suzuki 2011, Xiao et al. 2012, Bektaş and Laporte 2011, Tavares et al. 2009, Figliozzi 2010 etc.) ignored the acceleration rate and/or did not directly consider the vehicle load in calculating the fuel consumption. Other studies considered the effect of traffic lights and road congestion or detailed vehicle specifications; however, the size of the problem to which they can be applied is generally limited. This study aims to calculate the fuel consumption and CO_2 emissions of the vehicle routes by taking into account the vehicle technical data, vehicle speed, vehicle weight, acceleration of the vehicle, and distance traveled to obtain more realistic results in the computations. Based on this approach, a mixed integer linear programming model is presented for G-VRPTW and is solved using a new methodology that integrates the fuel consumption calculation algorithm with an SA meta-heuristic algorithm.

Problem definition

The aim of this study is to develop a new methodology for energy consumption optimization by reducing fuel consumption and CO_2 emissions in a G-VRPTW. The proposed study includes three different stages. The first stage includes the construction of a fuel consumption calculation algorithm (FCCA), the second stage includes a mixed integer linear programming model, and the third stage includes the solution methodology of the proposed G-VRPTW using the MSA-SA

meta-heuristic algorithm. The framework of the methodology is shown in Fig. 1.

The FCCA calculates the fuel consumption of a specific vehicle for a given route. The output of the FCCA is a nonlinear function of fuel consumption based on vehicle load and distance. To integrate the total fuel consumption calculation into the linear mathematical model, the nonlinear function should be linearized. The MSA-SA meta-heuristic algorithm can be integrated with either the linearized form of the fuel consumption calculation function or the nonlinear form. This study presents various numerical examples for the G-VRPTW. A mixed integer linear programming model is developed, and optimal solutions are achieved for small-sized problems. The MSA-SA meta-heuristic algorithm is included due to the complexity of the problem and the long solution times of the mathematical model. Various instances are solved with the MSA-SA for small, medium, and large-sized problems using the linearized and nonlinear forms of the fuel consumption calculation function.

Model formulation

In this section, the mixed integer linear programming model is introduced after the presentation of the fuel consumption calculation algorithm. The basic principles of the simulated annealing (SA) meta-heuristic algorithm and the proposed MSA-SA model are also covered in the "Proposed meta-heuristic algorithm" section.

Nonlinear fuel consumption calculation algorithm

Vehicles require energy to move with a constant speed or acceleration in real time. This energy is essentially equal to resistance forces. The resistance forces include rolling resistance, aerodynamic resistance, grade resistance, and acceleration resistance.

Rolling resistance is the result of deformation work on the tires and roadway and is given by:

$$F_{Ro} = fG = fmg \tag{1}$$

where f is the coefficient of rolling resistance, m is the vehicle weight (empty plus carried load), and g is the gravitational constant (Braess and Seiffert 2005). The aerodynamic resistance F_{Ae}, can be calculated from the following equation:

$$F_{Ae} = c_d A \rho \frac{v^2}{2} \tag{2}$$

where c_d is the aerodynamic coefficient, A is the frontal surface area, ρ is the air density, and v is the speed. The frontal surface area A of a passenger car ranges from 1.5 to 2.5 m^2 and from 4 to 9 m^2 for trucks (Braess and Seiffert 2005). The acceleration resistance F_{Acc} can be calculated from the following equation:

$$F_{Acc} = \lambda ma \tag{3}$$

where λ is the transmission variable ($\lambda = 1.04 + 0.0025i^2$, i is the overall gear ratio), m is the vehicle load, and a is the acceleration rate. The gear ratio i used to calculate λ is related to the gear and thus the speed, and due to the piecewise characteristics of the function, this relationship increases the complexity of the calculations. The grade resistance F_G is calculated as:

$$F_G = mg\sin\alpha \tag{4}$$

where mg is the vehicle load, and α is the coefficient related to the gradient of the road. In this study, the grade resistance is
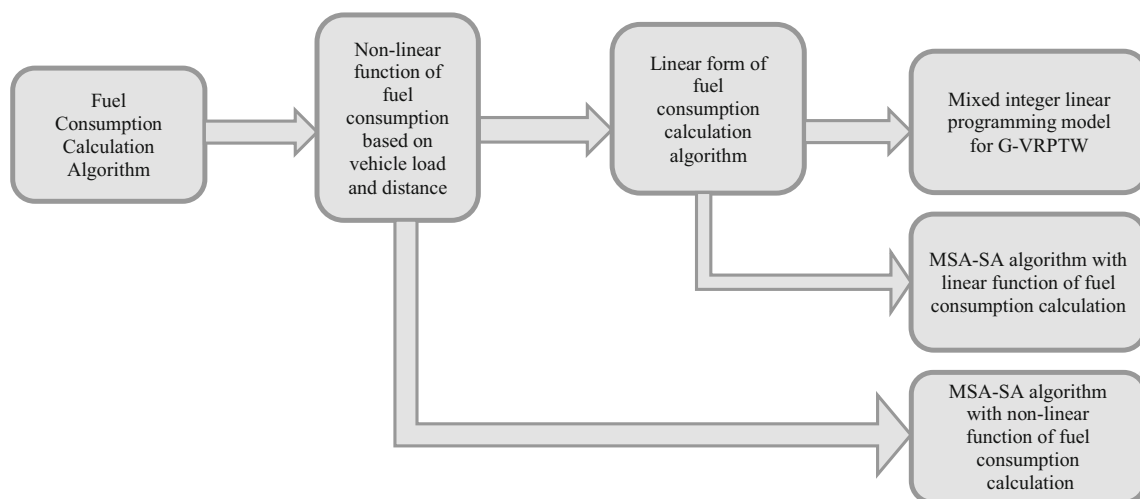

Fig. 1 Framework of the study

assumed to be constant, and $\alpha=0$. The total force (F_T) at the wheels for a given acceleration and grade is:

$$F_T = F_{Ro} + F_{Ae} + F_{Acc} \qquad (5)$$

Assume that P_T is the total power at the wheels and is calculated as:

$$P_T = F_T v \qquad (6)$$

To convert the total force to fuel consumption, the fuel consumption value per unit power (u_p) is multiplied with the P_T as shown:

$$consumption = u_p P_T \qquad (7)$$

The formulations described above present the momentary fuel consumption values. The time–speed calculations are computed to obtain the total fuel consumption value for a specified distance. To determine the time–speed graphic of a vehicle for a specified distance, it is assumed that as the vehicle accelerates from 0 to t_1, the speed is constant at speed limit (v_{max}) from t_1 to t_2 and decelerates from t_2 to t_3. Figure 2 shows the time–speed graphic with three discrete areas: A_1 represents the area for the acceleration time, A_2 represents the area for the time with constant speed, and A_3 represents the area for the deceleration time. In this study, it is accepted that the acceleration rate of the vehicle is equal to the deceleration rate of the vehicle ($a_{acc}=a_{dec}=a$), and thus the speed of the vehicle can be determined in real time.

To calculate the total fuel consumption of a vehicle over a specified distance, the total power and consumption formulations are developed as a function of time using the following equations:

$$v(t) = \begin{cases} a_{acc}\, t, & \text{if } 0 \leq t \leq t_1 \\ v_{max}, & \text{if } t_1 \leq t \leq t_2 \\ v_{max} - a_{dec}(t - t_2), & \text{if } t_2 \leq t \leq t_3 \\ 0, & \text{otherwise} \end{cases} \qquad (8)$$

$$F_T(t) = F_{Ro}(t) + F_{Ae}(t) + F_{Acc}(t) \qquad (9)$$

$$F_T(t) = (fmg) + \left(c_d A \rho \frac{v^2(t)}{2}\right) + (\lambda m a) \qquad (10)$$

$$P_T(t) = F_T(t)v(t) \qquad (11)$$

$$consumption(t) = u_p P_T(t) \qquad (12)$$

$$\text{Total consumption} = \int_0^{t_3} u_p P_T(t)\,dt \qquad (13)$$

When the traveling distance is known for a vehicle, the total traveling time t_3 and the total fuel consumption, which depend on the distance, can be calculated by using the following equations:

$$t_1 = t_3 - t_2 = \frac{v_{max}}{a} \qquad (14)$$

$$t_2 - t_1 = \frac{distance}{v_{max}} - \frac{v_{max}}{a} \qquad (15)$$

$$t_3 = \frac{v_{max}^2 + a \times distance}{a v_{max}} \qquad (16)$$

$$\text{Total consumption} = \int_0^{\left(\frac{v_{max}^2 + a \times distance}{a v_{max}}\right)} u_p P_T(t)\,dt \qquad (17)$$

The total fuel consumption of the vehicle for a defined route is computed using the nonlinear fuel consumption calculation (NFCC) formulations described above. The flowchart of the NFCC is presented in Fig. 3.

The NFCC algorithm considers the vehicle technical specifications, vehicle load, distance, and acceleration and deceleration rates. Taking into account the acceleration and

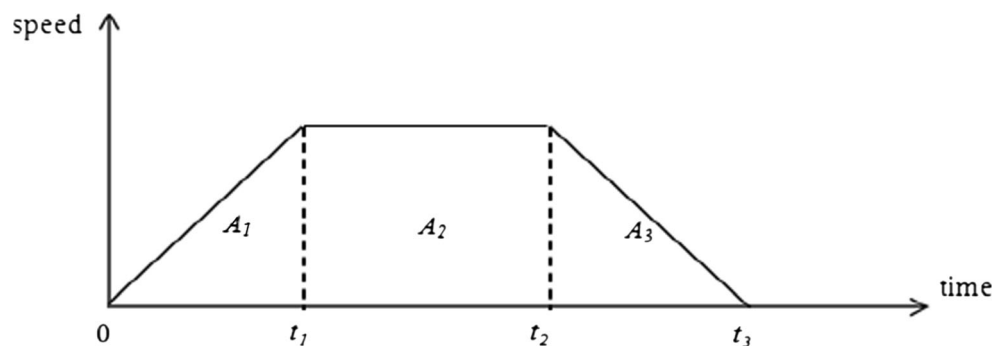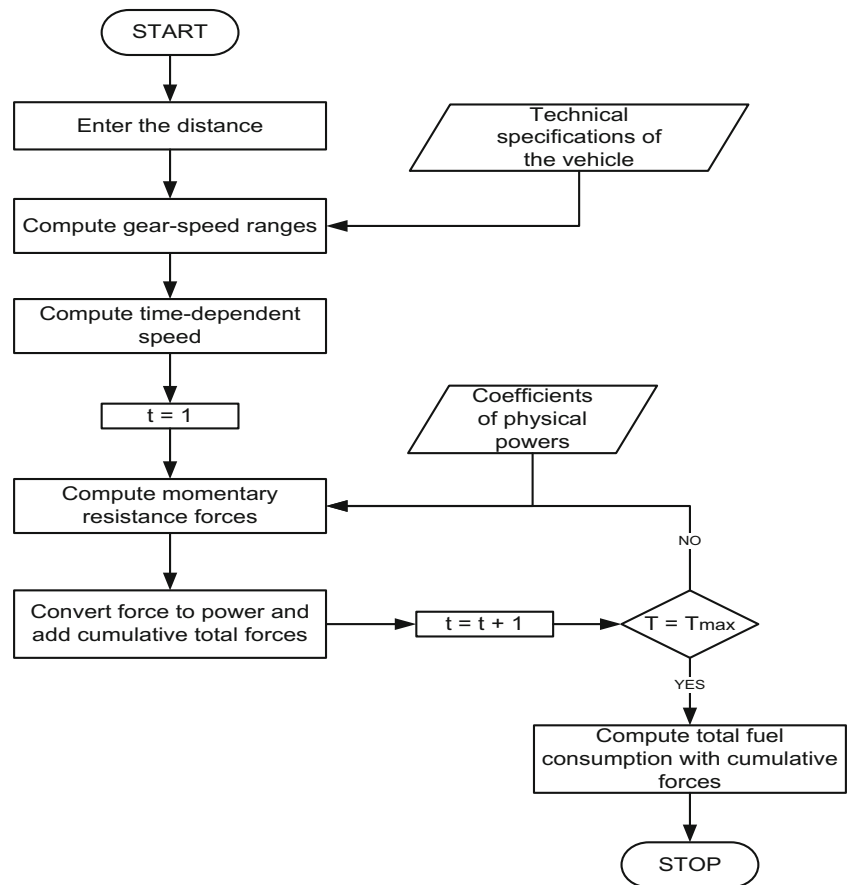**Fig. 2** Speed vs. time graphic used to calculate fuel consumption

Fig. 3 Flowchart of the NFCC algorithm

deceleration rates in the calculations is a more realistic approach, especially for the routes with small distances. The effects of the acceleration and deceleration rates on the total fuel consumption for routes of 1, 5, 10, and 25 km are shown in Fig. 4a–d, respectively. In the figures, case 1 denotes the fuel consumption value without consideration of the acceleration and deceleration rates, and case 2 denotes the fuel consumption value with consideration of the acceleration and deceleration rates. It is obvious from the figures that the fuel consumption is affected by acceleration and deceleration. For real-time applications, the NFCC will respond with realistic outcomes, especially for small distances.

The fuel consumption calculation can be converted to CO_2 emissions using the following equation (Braess and Seiffert 2005):

$$CO_2 \text{ Emissions} = \text{Total consumption} \times c_e \quad (18)$$

where c_e is the vehicle CO_2 emissions constant, which can be obtained from the technical specifications of the vehicle.

To integrate the total fuel consumption into the linear programming model, which is detailed in the next section, the complex NFCC formulations must be linearized. The parameters of the NFCC are assumed to be constant, except for load and distance, similar to the work of Huang et al.

(2012). Therefore, the fuel consumption equation is proportional to the distance and vehicle load, and regression analysis is carried out using Minitab to obtain a linear equation among load, distance, and fuel consumption, as shown (Küçükoğlu et al. 2013):

$$\text{Fuel consumption} = a_1 \text{ distance} + a_2 \text{ vehicle load} + b \quad (19)$$

The regression equation of fuel consumption is integrated into the linear programming model and also employed in the proposed meta-heuristic algorithm. Furthermore, the meta-heuristic algorithm is also solved with NFCC formulations to demonstrate the performance of the algorithm.

Mixed integer linear programming model

Bräysy and Gendreau (2005a) defined the VRPTW as follows. Let $G=(V,E)$ be a connected digraph consisting of a set of $n+1$ nodes (each of which can be serviced only within a specified time interval or time window), a set of E of arcs with nonnegative weights c_{ij}, and with associated travel times t_{ij} (including a service time at node i). A vehicle is permitted to arrive before the opening of the time window and wait at no cost until service becomes possible, but it is not permitted to arrive after
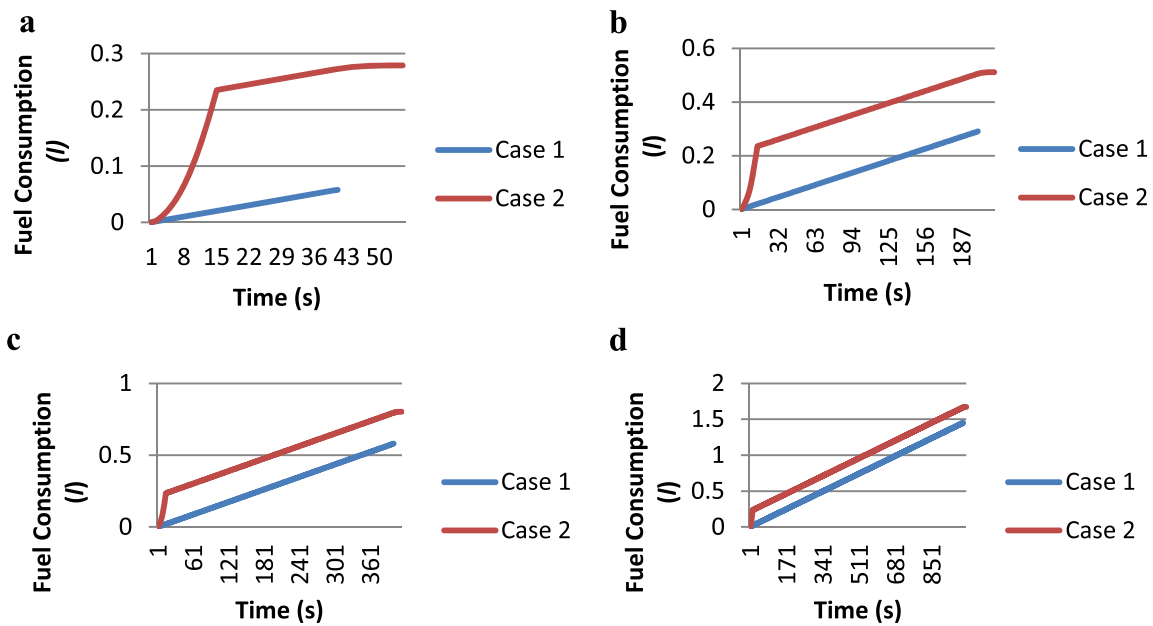
Fig. 4 **a** Cumulative fuel consumption for 1 km. **b** Cumulative fuel consumption for 5 km. **c** Cumulative fuel consumption for 10 km. **d** Cumulative fuel consumption for 25 km

the latest time window. Node *0* represents the depot. Apart from the depot, each node *i* imposes a service requirement d_i that represents a delivery to or a pick up from the depot. In most of the surveyed papers, the objective is to find the minimum number of tours K^* for a set of identical vehicles such that each node is reached within its time window, and the accumulated service up to any node does not exceed a positive number of vehicle capacity. A secondary objective is to either minimize the total distance traveled or the duration of the routes.

In this paper, a mixed integer linear programming model is considered to construct routes for a set of vehicles to meet the demand of all customers. In this model, *k* homogenous vehicles depart from and return to the depot node after serving all the customers, each customer is visited at once, vehicles cannot carry loads greater than their capacity and vehicles must respect the time windows. The overall objective is to minimize the traveled distance and the fuel consumption. In this research, the proposed optimization model of the G-VRPTW is developed as follows:

Indices

N Set of customer and the depot nodes $\forall\ i, j \in N$
K Set of vehicles $\forall\ k \in K$

Parameters

c_{ij}	Travel distance between node *i* and node *j*
t_{ij}	Travel time between node *i* and node *j*
e_i	Time window lower bound for node *i*
l_i	Time window upper bound for node *i*
p_i	Service processing time for node *i*

d_i	Demand of node *i*
Q	Capacity of vehicles
a_1	Coefficient of regression equation
a_2	Coefficient of regression equation
b	Coefficient of regression equation
M	Large scalar
Min_load	Unladen weight of the vehicle
Max_load	$Min\_load + Q$

Decision Variables

$$x_{ijk} = \begin{cases} 1 & \text{if vehicle } k \text{ travels from node } i \text{ to node } j, \quad (i \neq j) \\ 0 & \text{otherwise} \end{cases}$$

y_i Load of a vehicle at node *i*
w_i Service start time at node *i*

Model

$$Min\ z = \sum_{i=0}^{N} \sum_{j=0}^{N} \sum_{k=1}^{K} x_{ijk} \int_0^{\left(\frac{V_{max}^2 + ac_{ij}}{aV_{max}}\right)} u_p P_T(t)\,dt \quad (20)$$

Subject to

$$\sum_{i=0}^{N} \sum_{k=1}^{K} x_{ijk} = 1 \qquad \text{for } i \neq j \text{ and } j \in \{1, 2, \ldots, N\} \quad (21)$$

$$\sum_{j=0}^{N} \sum_{k=1}^{K} x_{ijk} = 1 \qquad \text{for } i \neq j \text{ and } i \in \{1, 2, ..., N\} \quad (22)$$

$$\sum_{j=1}^{N} x_{0jk} = 1 \qquad \text{for } k \in \{1, 2, ..., K\} \qquad (23)$$

$$\sum_{i=1}^{N} x_{i0k} = 1 \qquad \text{for } k \in \{1, 2, ..., K\} \qquad (24)$$

$$\sum_{i=0}^{N} x_{ijk} = \sum_{i=0}^{N} x_{jik} \text{ for } k \in \{1, 2, ..., K\} \text{ and } j \in \{1, 2, ..., N\} \qquad (25)$$

$$y_0 = Min\_load \qquad (26)$$

$$y_i - y_j \geq d_i - M\left(1 - \sum_{k=1}^{K} x_{ijk}\right) \text{ for } i \in \{1, 2, ..., N\} \text{ and } j \in \{0, 1, ..., N\} \qquad (27)$$

$$Min\_load \leq y_i \leq Max\_load \text{ for } i \in \{0, 1, ..., N\} \qquad (28)$$

$$w_i + p_i + t_{ij} \leq w_j + M\left(1 - \sum_{k=1}^{K} x_{ijk}\right) \quad \text{for } i \in \{0, 1, ..., N\} \text{ and } j \in \{1, 2, ..., N\} \qquad (29)$$

$$w_i + p_i + t_{i0} \leq l_0 + M\left(1 - \sum_{k=1}^{K} x_{i0k}\right) \quad \text{for } i \in \{1, 2, ..., N\} \qquad (30)$$

$$e_i \leq w_i \leq l_i \qquad \text{for } i \in \{0, 1, ..., N\} \qquad (31)$$

$$y_i \geq 0 \qquad (32)$$

$$w_i \geq 0 \qquad (33)$$

$$x_{ijk} \in \{0, 1\} \qquad (34)$$

The objective function (20) seeks to minimize total fuel consumption using NFCC formulation. Constraints (21) and (22) ensure that each customer is visited exactly once. Constraints (23) and (24) ensure departure from and return to the depot node. Constraint (25) ensures flow balance for each node. Constraint (22) indicates that vehicles are empty when they return to the depot node. Constraints (27) and (28) restrict the load of vehicles at the visited customer and also eliminate sub-tours. Constraint (29) states that vehicle k cannot arrive at customer j before $w_i + p_i + t_{ij}$ if it travels from customer i to customer j. Constraint (30) ensures that that vehicle k cannot arrive at the depot before $w_i + p_i + t_{io}$ if it travels from customer i to depot. Constraint (31) ensures that time windows are respected. Constraints (32) and (33) ensure that y_i and w_i are nonnegative and constraint (34) defines the binary variables x_{ijk}.

Because of the complexity of the NFCC in objective function (20), the fuel consumption calculation is stated as a linear regression equation, which is described in the "Nonlinear fuel consumption calculation algorithm" section, and integrated to the mathematical model by Eq. (35).

$$Min \ z = \sum_{i=0}^{N} \sum_{j=0}^{N} \sum_{k=1}^{K} a_1 c_{ij} x_{ijk} + \sum_{i=0}^{N} \sum_{j=0}^{N} \sum_{k=1}^{K} a_2 x_{ijk} y_j + \sum_{i=0}^{N} \sum_{j=0}^{N} \sum_{k=1}^{K} b x_{ijk} \qquad (35)$$

Although, the regression equation is linearly formed, with integration of regression equation to the mathematical model the objective function (35) became bilinear due to the multiplication of two decision variables. However, it is observed that y_i is nonnegative only if $x_{ijk}=1$. Therefore, the regression equation based bilinear objective function is converted into the equivalent linear form (36).

$$Min \ z = \sum_{i=0}^{N} \sum_{j=0}^{N} \sum_{k=1}^{K} a_1 c_{ij} x_{ijk} + Min\_load \sum_{i=1}^{N} \sum_{k=1}^{K} a_2 x_{i0k} + \sum_{i=1}^{N} a_2 y_i + \sum_{i=0}^{N} \sum_{j=0}^{N} \sum_{k=1}^{K} b x_{ijk} \qquad (36)$$

In the mixed integer linear mathematical model described above, it is assumed that a vehicle, when traveling in a given arc, accelerates to v_{max}, travels with its cruise speed, and decelerates to zero. However, vehicles would be obligated to stop more than once in an arc owing to the some unavoidable road conditions like traffic lights, junctions, and such stop-and-go conditions. Each stop-and-go condition causes the re-acceleration and re-deceleration of the vehicle, and consequently re-calculation of the regression equation. The multi-stopping condition is taken into account with objective

function (37) where p_{ij} denotes the number of acceleration–deceleration parts from node i to node j.

$$Min\ z = \sum_{i=0}^{N} \sum_{j=0}^{N} \sum_{k=1}^{K} a_1 c_{ij} x_{ijk} + \sum_{i=0}^{N} \sum_{j=0}^{N} \sum_{k=1}^{K} p_{ij} a_2 y_j x_{ijk} + \sum_{i=0}^{N} \sum_{j=0}^{N} \sum_{k=1}^{K} p_{ij} b x_{ijk}$$

(37)

The nonlinear objective function of multi-stopping condition (37) is linearized by using constraint (38) and (39) and converted to the equivalent linear form (40) where $\mu_{ij} = p_{ij} a_2 y_j$ if $x_{ijk} = 1$.

$$p_{ij} a_2 y_j - \mu_{ij} \leq M\left(1 - \sum_{k=1}^{K} x_{ijk}\right) \text{for } i \in \{0, 1, \dots, N\}$$
$$and \quad j \in \{0, 1, \dots, N\}$$

(38)

$$p_{ij} a_2 y_j - \mu_{ij} \geq -M\left(1 - \sum_{k=1}^{K} x_{ijk}\right) \quad \text{for } i \in \{0, 1, \dots, N\}$$
$$and \quad j \in \{0, 1, \dots, N\}$$

(39)

$$Min\ z = \sum_{i=0}^{N} \sum_{j=0}^{N} \sum_{k=1}^{K} a_1 c_{ij} x_{ijk} + \sum_{i=0}^{N} \sum_{j=0}^{N} \mu_{ij} + \sum_{i=0}^{N} \sum_{j=0}^{N} \sum_{k=1}^{K} p_{ij} b x_{ijk}$$

(40)

Proposed meta-heuristic algorithm

The SA is a heuristic optimization algorithm for solving combinatorial optimization problems that was introduced by Kirkpatrick et al. in 1983. The SA methodology draws its inspiration from the annealing process in metallurgy and operates by emulating the physical process in which a solid is heated to a high temperature and cooled step-by-step to allow the solid to crystallize. The algorithm has been applied to several problems, i.e., vehicle routing, scheduling, layout, cutting stock problems etc.

The SA begins with a random initial solution and uses a stochastic approach to guide the search. In each iteration, the algorithm selects a new solution X' from the neighborhood of the current solution X. The essential concept does not restrict the search moves only to better solutions. The procedure can escape from a local optimum by accepting a worse solution such that the SA allows the search to proceed to a neighboring state even if the move causes the value of the objective function to temporarily worsen (Lin et al. 2011). The SA

explores the solution space in the following manner. If a move to a neighbor X' in a neighborhood ensures an improvement in the objective value or leaves the value unchanged, then the move is always accepted. More precisely, the solution X' is accepted as the new solution if $\Delta \leq 0$, where $\Delta = f(X') - f(X)$ and $f(X)$ is the value of the objective function. Moves that cause a worse result for the objective function (i.e., $\Delta > 0$) are accepted according to a probability function $e^{\left(\frac{-\Delta}{T}\right)} > \theta$, where T is the temperature parameter and θ is a random number between [0,1]. Additionally, if the move improves the best solution, then the new solution is accepted as the new best solution. The value of T varies from a relatively large number to a value close to zero and is often controlled by the cooling rate (c) to linearly reduce the temperature (Otten and Ginneken 1988).

Presentation

The presentation of the solution used in this study is an integer string of length ($N+K+1$), where N is the number of customer nodes, and K is the number of vehicles, as applied by Tan et al. (2001). For example, consider three tour routes as shown in Fig. 5.

The definitions of the routes are as follows:

Route No.1 : $0 \rightarrow 1 \rightarrow 4 \rightarrow 5 \rightarrow 0$
Route No.2 : $0 \rightarrow 3 \rightarrow 2 \rightarrow 6 \rightarrow 8 \rightarrow 0$
Route No.3 : $0 \rightarrow 9 \rightarrow 7 \rightarrow 11 \rightarrow 10 \rightarrow 12 \rightarrow 0$

The coded integer string is defined as follows:

$0 \rightarrow 1 \rightarrow 4 \rightarrow 5 \rightarrow 0 \rightarrow 3 \rightarrow 2 \rightarrow 6 \rightarrow 8 \rightarrow 0 \rightarrow 9 \rightarrow 7 \rightarrow 11 \rightarrow 10 \rightarrow 12 \rightarrow 0$

The zeros represent the bounds of the routes, and this representation provides an incentive to return to the original routes after decoding.
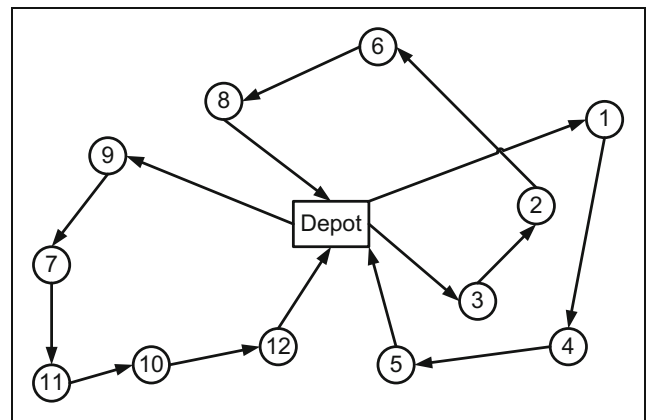

Fig. 5 Solution to a vehicle routing problem

Initial solution

Most of the meta-heuristic algorithms involve finding an initial feasible solution and subsequently improving that solution using local search methods. In this study, Solomon's time-oriented nearest neighbor algorithm (first presented in 1987) is used for initial solution generation as modified using the nearest neighbor algorithm by considering the time windows and capacity constraints.

The nearest neighbor (NN) method is easy and straightforward. The method begins every route by finding the customer closest to the depot that has not yet been scheduled on a route. In each iteration, the heuristic searches for the customer closest to the last customer and inserts new customers into the route one by one until all of the customers are serviced. This search is performed among all the customers that can be feasibly added to the end of the route. A new route is initialized at any time if the search fails according to the capacity constraints or if there are no additional customers that can be scheduled. In addition to the NN, Solomon's time-oriented NN considers not only capacity constraints to decide on the closest customer but also considers the time windows' constraints and uses a weighted metric that measures the direct distance between the customers, the time difference between the completion of service at node i and the beginning of service at node j, and the urgency of delivery to customer j. As a result of this measurement, a new customer is selected (Solomon 1987).

Fitness function

The fuel consumption of a route is computed separately using both the linear regression equation and the NFCC. This procedure calculates the cumulative fuel consumption amount by starting from the first customer on the route to the depot. For each move of the vehicle from node i to node j, the distance between two nodes c_{ij} and the weight of the vehicle at node i y_i are used for the calculations. Moreover, the capacity and time window constraints are taken into account for computing the fitness function, and a penalty cost is applied if the vehicle load at the depot exceeds the vehicle capacity or if the vehicle arrives at a customer after its time window upper bound. Table 1 shows an illustrative example for the linear regression fuel consumption calculation and the NFCC that includes 10 demand nodes.

For the linear regression, the cost function can be easily computed using the coefficients of the regression Eq. (15), and this approach has a structure similar to the distance-related cost functions. However, this situation is not same for the NFCC because of the complicated formulations described in the "Nonlinear fuel consumption calculation algorithm" section. If a vehicle moves from node i to node j, the fuel consumption value of the vehicle is cumulatively computed by considering the instantaneous resistance forces. Therefore, the NFCC increases the number of calculations for each fitness function computation and directly affects the performance of the algorithm with respect to CPU times. To speed up the fitness function computation process, a memory structure is used to avoid duplicate examinations for the SA. This structure is employed to store the route information, and when a route is examined, the fitness value can be easily obtained if the route has been generated previously. Otherwise, the NFCC is applied for the route, and the new information is recorded to memory in a string form. The string form of the route includes the customer nodes excepting the depot node and the fitness function value of the route which are split with the use of comma: e.g., the route $0 \rightarrow 1 \rightarrow 2 \rightarrow 3 \rightarrow 4 \rightarrow 5 \rightarrow 6 \rightarrow 7 \rightarrow 8 \rightarrow 9 \rightarrow 10 \rightarrow 0$ described in Table 1 and its fuel consumption amount of 29.56 is coded as "1, 2, 3, 4, 5, 6, 7, 8, 9, 10, and 29.56." To reduce the searching process time for retrieving the stored information of a route, this memory structure is designed in the form of a three-

Table 1 Illustrative example for the linear regression fuel consumption calculation and the NFCC

Nodes		0	1	2	3	4	5	6	7	8	9	10	0
Distance from node i to node $i+1$ (km)		20.2	2.8	6.3	6.1	10.8	13.0	12.0	8.1	14.1	7.8	9.0	–
Demand amount of node i (kg)		0	110	205	95	90	80	175	10	80	60	75	0
Vehicle weight at node i (kg)		2,280	2,170	1,965	1,870	1,780	1,700	1,525	1,515	1,435	1,375	1,300*	–
Fuel consumption (l) from node i to node $i+1$	Regression	1.8858	0.6609	0.7429	0.9503	0.8658	0.9549	0.9219	0.6544	1.0402	0.6223	0.6695	–
	NFCC	1.9217	0.6792	0.6840	0.6480	0.9024	1.0011	0.8850	0.6679	0.9642	0.6128	0.6545	–
Cumulative fuel consumption (l) from node i to node $i+1$	Regression	1.8858	2.5467	3.2896	4.2399	5.1057	6.0606	6.9828	7.6369	8.6771	9.2994	9.9689	–
	NFCC	1.9217	2.6009	3.2849	3.9329	4.8353	5.8364	6.7214	7.3893	8.3535	8.9663	9.6208	–
Cumulative CO_2 emissions (kg) value from node i to node $i+1$	Regression	5.79	7.82	10.11	13.03	15.69	18.62	21.45	23.46	26.66	28.57	30.63	–
	NFCC	5.90	7.99	10.09	12.08	14.86	17.93	20.65	22.70	25.66	27.55	29.56	–

*The unladen vehicle weight at the last node before the depot node is 1,300 kg

dimensional array. Each dimension of the array indicates a key factor of a route: the first customer node of the route, the last customer node of the route and the number of customer in the route. Instead of a single array, the proposed key factors prevent the unnecessary searches. In Fig. 6, an illustrative memory structure example is presented for new route information. In addition to the classical SA operations, this memory structure provides efficiency in the CPU times. Figure 7 shows the pseudo-code of the memory structure-adapted fitness function calculation.

Local search

As introduced by Osman and Christofides in 1994, the λ-interchange local search method is based on the interchange of customers between sets of routes. This method was first applied for capacitated clustering problems and was also shown to be an effective neighborhood-searching algorithm for most VRP problems.

The local search method is based on the λ number, where λ is the maximum number of customers that can be interchanged between routes. The customers that are interchanged can be chosen systematically or randomly. For example, consider $\lambda=2$, which means that one or more customers may be interchanged between routes in the following orders: (0, 1), (1, 0), (1, 1), (0, 2), (2, 0), (21,), (1, 2), and (2, 2). These operators include both shift and exchange procedures between routes. A neighboring solution is selected using first-improvement or best-improvement strategies. Figure 8a, b illustrate an example for the (0, 1) and (1, 1) operators, respectively. For the first illustration, node 3 is shifted from one route to another route, and for the second illustration, nodes 3 and 4 are exchanged. In this paper, the λ-interchange method is improved by considering the time windows and capacity constraints to obtain a feasible

```
Start
   Nₖ        : Number of customer at route k
   0, Nₖ + 1: Denote the depot node
   costₖ     : Fitness function value of route k
   y₀ = Min_load
   Route_memory = ∅
   For i = 1 to Nₖ + 1
       y₀ = y₀ + dᵢ
       Compute the service starting time wᵢ
       If wᵢ ≥ lᵢ Then
           costₖ = costₖ + penalty_cost
       End If
   Next
   If y₀ > Max_load Then
       costₖ = costₖ + penalty_cost
   End If
   If Regression equation fuel consumption calculation is selected Then
       For i = 1 to Nₖ + 1
           costₖ = costₖ + a₁cᵢ₋₁,ᵢ + a₂yᵢ₋₁ + b
           yᵢ = yᵢ − dᵢ₋₁
       Next
   Else If NFCC is selected Then
       If Route_memory contains the existing route Then
           Determine the costₖ from Route_memory
       Else
           For i = 1 to Nₖ + 1
               Compute the time-dependent speed according to cᵢ₋₁,ᵢ
               Determine the t₁, t₂, t₃, A₁, A₂, A₃
               For t=1 to t₃
                   Determine the speed of vehicle at time t
                   Determine the F_Ro and F_Ae
                   If t ≤ t₁ Then
                       Determine the F_Acc
                   End If
                   Determine the F_T
                   P_T = F_T v
                   consumption = u_p P_T
                   costₖ = costₖ + consumption
               Next
               yᵢ = yᵢ − dᵢ₋₁
           Next
       End If
   End If
End
```

Fig. 7 Pseudo-code of the memory structure-adapted fitness function calculation

solution for each move of the nodes and avoid redundant computations. These constraints are carried out by

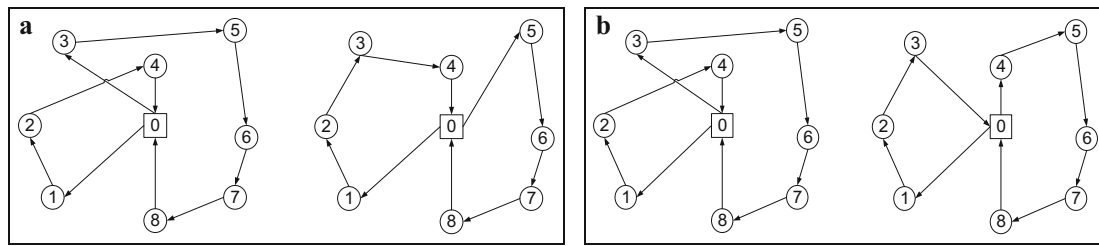**Fig. 6** An illustrative example for the memory structure
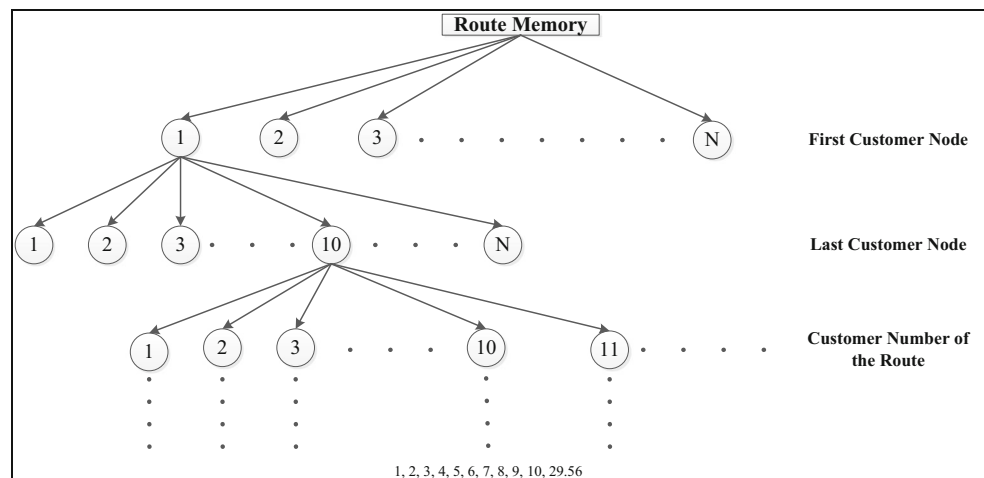
Fig. 8 a. An illustrative example for the (0, 1) operator. **b** An illustrative example for the (1, 1) operator

checking the feasibility status of any move such that node j can be inserted between node i and node $i+1$ into route k if the following conditions are met:

- $e_i + p_i + t_{i,j} \leq l_j$
- $e_j + p_j + t_{j,i+1} \leq l_{i+1}$
- $y_\varepsilon + d_j \leq Max\_load$, where ε is the first node of the route k.

For the computational studies, the improved λ-interchange method is applied for $\lambda=1$ and $\lambda=2$, which means that a maximum of two customer nodes may be interchanged between routes and that both first-improvement and best-improvement strategies are implemented together in selecting a new neighbor.

Acceptance of a solution and stopping criteria

The maximum iteration number is used as termination criteria such that the solution search procedures proceed until the iteration number reaches the maximum iteration number. At the end of each iteration, the SA temperature is progressively reduced by a constant cooling parameter. Based on the described statements above, the pseudo-code of the proposed MSA-SA algorithm is shown in Fig. 9.

Computational results

The proposed MSA-SA algorithm is developed in the Visual Basic programming language, and numerical experiments are performed on a 2.20-GHz Intel Core i7 processor with 8-GB memory. To evaluate the performance of the proposed algorithm, Solomon's benchmark problem data set which was developed to test the VRPTW (Solomon 1987) and Gehring and Homberger's benchmark problem data set which is the extension of the Solomon's problems and ranges from 200 up to 1,000 customers (Gehring and Homberger 2001) are used.

Fig. 9 Pseudo-code of the proposed MSA-SA algorithm

```
Start
    Select initial temperature T
    Select cooling rate c
    Select stopping criteria
    Select the fuel consumption calculation type (NFCC or Linear Regression Equation)
    X^best = ∅
    Generate X
    Cumpute the f(X) using the fuel consumption formulations
    X^best = X
    Repeat
        Generate all possible neighbor solutions from X using the λ-interchange local search
        Compute the memory structure adapted fitness function of each neighbor using the selected fuel
        consumption calculation type
        Select the best neighbor solution X′ from the neighborhood and specify as the new solution
        If f X′ ≤ f(X) Then
            X = X′
        Else
            Δ= f X′ − f(X)
            If exp −Δ/T > random 0,1 Then
                X = X′
            End If
        End If
        T = T × c
        If f X ≤ f(X^best) Then
            X^best = X
        End If
    Until stopping criteria is met
End
```

For the small and medium-sized computations, Solomon's R1 type problems which consist of 12 basic instances are utilized. In each problem, there are 100 randomly located demand nodes, and the depot is located in the center. For this study, we used the first five R1 type instances, and in addition to the existing problems, we considered the first 5, 10, and 15 customer nodes to obtain small-sized problems to demonstrate the efficiency of the G-VRPTW model. For the large-sized problems, the MSA-SA is performed on a number of Gehring and Homberger's C1, R1, and RC1 type instances (for 200 and 400 customers). It should be noted that each customer node location is defined with coordinates in the data set, and the distances between nodes are determined in terms of Euclidean distance for the VRP and its extensions. However, to be able to use in fuel consumption calculations with real-life vehicle technical specifications and physical coefficients, the distances, and demand amounts are assumed as metric units (1 distance unit=1 km, 1 demand unit=1 kg). Moreover, the maximum iteration number, which is the stopping criteria of the proposed algorithm, is set to 1,000 iterations for all instances to get proper comparisons. Within these assumptions, the G-VRPTW and VRPTW models are first compared with small-sized problems to emphasize the energy savings (reduction of fuel consumption and CO_2 emissions) of the solutions. Afterward, MSA-SA algorithm is tested on medium and large-sized problems for the VRPTW, G-VRPTW with linear function of fuel consumption calculation, and G-VRPTW with NFCC.

For the first part of the computational studies, Table 2 shows the optimum solutions of the considered small-sized problems and compares the VRPTW and G-VRPTW results for both total route distances and fuel consumptions. It can be noted from the table that the G-VRPTW obtains eight different solutions from the VRPTW and that G-VRPTW provides considerable fuel consumption and CO_2 emissions reductions for these problems though the instances contains maximum 15 customer nodes. According to the overall instances, the percentage gap (GAP%) between the VRPTW and G-VRPTW solutions based on the total route distance-related calculations is 0.69 % on average, and the G-VRPTW exposes slight increases in the total distances. However, it can be clearly observed that the G-VRPTW obtains noticeable energy savings for these problems such that the average GAP% between the VRPTW and G-VRPTW solutions is −1.70 % based on the fuel consumption-related calculations that consider the linear regression equation. Consequently, the proposed G-VRPTW reduces the fuel consumption with respect to that of the VRPTW by taking into account the distance, vehicle load, and vehicle specifications.

In order to demonstrate the effects of multi-stopping condition on total fuel consumption and CO_2 emissions, small-sized instances are re-solved by considering objective function (40). For each instance, the data p_{ij} is randomly generated. Table 3 shows the result of the computations and compares the VRPTW and G-VRPTW results. It can be concluded from the results

Table 2 Optimum solutions of the proposed G-VRPTW and VRPTW models

Problem type	Customer number	VRPTW				G-VRPTW				GAP%	
		Fuel consumption (l)	CO_2 emissions (kg)	Total distance (km)	Number of vehicles	Fuel consumption (l)	CO_2 emissions (kg)	Total distance (km)	Number of vehicles	Distance	Fuel consumption
R101	5	8.3535	25.67	156.3	2	8.0455	24.72	156.6	3	0.19	−3.69
R102		7.5420	23.17	130.6	1	7.1509	21.97	134.2	2	2.76	−5.19
R103		7.5420	23.17	130.6	1	7.1509	21.97	134.2	2	2.76	−5.19
R104		7.5420	23.17	130.6	1	7.1509	21.97	134.2	2	2.76	−5.19
R105		7.5202	23.11	141.9	2	7.5202	23.10	141.9	2	0.00	0.00
R101	10	14.0088	43.04	269.4	4	14.0088	43.29	269.4	4	0.00	0.00
R102		12.2925	37.77	229.6	3	12.2925	37.77	229.6	3	0.00	0.00
R103		12.2925	37.77	229.6	3	12.2925	37.77	229.6	3	0.00	0.00
R104		10.9809	33.74	198.1	2	10.9809	33.74	198.1	2	0.00	−0.45
R105		13.5816	41.73	253.0	3	13.5816	41.73	253.0	3	0.00	0.00
R101	15	20.3529	62.53	383.7	5	20.3529	62.53	383.7	5	0.00	0.00
R102		18.2825	56.17	326.7	4	17.9819	55.25	327.4	4	0.21	−1.64
R103		18.2825	56.17	326.7	4	17.9819	55.25	327.4	4	0.21	−1.64
R104		16.5896	50.97	288.8	3	16.5889	50.97	288.8	3	0.00	0.00
R105		19.7607	60.71	349.8	3	19.2795	59.23	354.9	4	1.46	−2.44
Total		194.9242	598.89	3545.4	41	192.4398	591.26	3563.0	46	10.35	−25.43

Table 3 Optimum solutions of the proposed G-VRPTW and VRPTW models for multi-stopping condition

Problem type	Customer number	VRPTW				G-VRPTW				GAP%	
		Fuel consumption (l)	CO$_2$ emissions (kg)	Total distance (km)	Number of vehicles	Fuel consumption (l)	CO$_2$ emissions (kg)	Total distance (km)	Number of vehicle	Distance	Fuel consumption
R101	5	8.9140	27.3883	156.3	2	8.6142	26.4671	156.6	3	0.19	−3.36
R102		9.0149	27.6983	130.6	1	7.8478	24.1124	134.2	2	2.76	−12.95
R103		9.0149	27.6983	130.6	1	7.8478	24.1124	134.2	2	2.76	−12.95
R104		9.0149	27.6983	130.6	1	7.8478	24.1124	134.2	2	2.76	−12.95
R105		8.7371	26.8447	141.9	2	8.7371	26.8447	141.9	2	0.00	0.00
R101	10	15.2109	46.7355	269.4	4	15.2109	46.7355	269.4	4	0.00	0.00
R102		13.5878	41.7485	229.6	3	13.5878	41.7485	229.6	3	0.00	0.00
R103		13.5878	41.7485	229.6	3	13.5878	41.7485	229.6	3	0.00	0.00
R104		12.3666	37.9964	198.1	2	12.3305	37.8855	198.1	2	0.00	−0.29
R105		14.8299	45.5649	253.0	3	14.8299	45.5649	253.0	3	0.00	0.00
R101	15	22.5711	69.3497	383.7	5	22.5711	69.3497	383.7	5	0.00	0.00
R102		21.9149	67.3335	326.7	4	20.3375	62.4870	327.4	4	0.21	−7.20
R103		21.9149	67.3335	326.7	4	20.3375	62.4870	327.4	4	0.21	−7.20
R104		18.3291	56.3162	288.8	3	18.3291	56.3162	288.8	3	0.00	0.00
R105		22.5962	69.4268	349.8	3	21.8990	67.2847	354.9	4	1.46	−3.08
Total		224.6050	690.0989	3545.4	41	213.9158	657.2563	3563.0	46	10.35	−59.98

that considering the multi-stopping condition for a given arc highlights, the performance of the proposed G-VRPTW. The average percentage gaps for the fuel consumption between the VPRTW and G-VRPTW, which are −1.70 for single-stopping condition and −4.00 % for multi-stopping condition, point out the energy savings and bring out the

effectiveness of the G-VRPTW model explicitly. In this study, the following computational studies are performed for single stopping condition, and also, it is expected that the fuel consumption and CO$_2$ emissions savings will also increase for multi-stopping conditions regarding the results summarized in Tables 2 and 3.

Table 4 The MSA-SA solutions for VRPTW problems of Solomon

Problem type	Customer number	Optimal solution		SA solution			GAP%
		Number of vehicles	Total distance	Number of vehicles	Total distance	CPU time	Optimal solution
R101	25	8	617.1	8	617.1	0.02	0.00
R102		7	547.1	7	547.1	0.23	0.00
R103		4	454.6	4	454.6	0.62	0.00
R104		5	416.9	5	416.9	0.74	0.00
R105		6	530.5	6	530.5	0.19	0.00
R101	50	12	1044.0	12	1044.0	1.02	0.00
R102		11	909.0	11	909.0	4.21	0.00
R103		9	772.9	9	773.5	18.70	0.08
R104		6	625.4	6	629.0	26.43	0.58
R105		9	899.3	9	899.3	2.76	0.00
R101	100	20	1637.7	20	1640.2	17.48	0.15
R102		18	1466.6	18	1467.1	75.56	0.03
R103		14	1208.7	14	1223.1	115.26	1.19
R104		11	971.5	11	1004.1	145.47	3.36
R105		15	1355.3	15	1366.0	23.15	0.79
Total		155	13456.6	155	13521.5	431.84	6.18

Table 5 The comparisons with the best known VRPTW results of Solomon's problems

Problem type	Customer number	Best known solution			SA solution		GAP%
		Number of vehicles	Total distance	Reference	Number of vehicles	Total distance	Best known solution
R101	100	19	1,645.79	Homberger	20	1,640.2	−0.34
R102		17	1,486.12	Rochart and Taillard	18	1,467.1	−1.28
R103		13	1,292.68	Li et al.	14	1,223.1	−5.38
R104		9	1,007.24	Mester	11	1,004.1	−0.31
R105		14	1,377.11	Rochart and Taillard	15	1,366.0	−0.81
Total		72	6,808.94	–	78	6,700.5	−8.12

The second component of the computational studies is based on the performance testing of the MSA-SA. To identify the efficiency of the proposed algorithm, the MSA-SA is performed for VRPTW problems and compared with the optimal and best known solutions. In this paper, the objective function of the VRPTW is considered as the minimization of the total travel distance. For the Solomon's benchmark problems, Table 4 shows the optimal and the MSA-SA solutions.

The results of these experiments, which are reported in Table 4, show that the MSA-SA exhibits superior performance for 25, 50, and 100 customer nodes based on the optimal solutions. According to the GAP% between the optimal and MSA-SA solutions, proposed algorithm finds the optimum solutions with 0.41 % deviation on average. Besides, the total traveling distance is reduced according to the best known solution for each problem consists of 100 customers despite the increment in the total number of vehicles. Table 5 compares the MSA-SA solutions with the best known solutions obtained by Solomon's web page and results show that the proposed algorithm provides improvement on total distances with the range of [0.31, 5.38] and 1.62 % on average. It can be noted that our solutions improve to the best known solutions, in terms of total distance traveled by the vehicles, not in terms of the number of vehicles used. On the other hand, indicated results are found in a very short time such that the average CPU times for R1 type problems are 0.36 s for 25 customers, 10.62 s for 50 customers, and 75.38 s for 100 customers.

For the large-sized problems (200 and 400 customer nodes), the MSA-SA and best known solutions (seen in http://www.sintef.no) are presented in Table 6. The average GAP% between the MSA-SA and best known solutions is 2.40 % on the basis of the total distance traveled by the vehicles. Furthermore, the MSA-SA produces these results in satisfactory running times. Consequently, the results of the VRPTW benchmark problems of Solomon and Gehring and Homberger demonstrate the efficiency of the proposed algorithm.

For the G-VRPTW, the MSA-SA is first performed for small-sized problems (5, 10, 15 customer nodes) and the optimum solution is found in less than 0.05 s for each instance. Table 7 includes the MSA-SA results for the G-VRPTW with linear fuel consumption calculation and G-VRPTW with the NFCC. However, these solutions are not directly comparable with the optimum solutions of the G-VRPTW because of the differences between the fuel consumption calculations, but, the results show that the G-VRPTW with the NFCC solutions are notably close to the G-VRPTW solutions with respect to the fuel consumption amount such that the difference between the total fuel consumption amount of all instances is 0.07 l. When considering the difference of calculations between the G-VRPTW with linear fuel consumption calculation and G-VRPTW with the NFCC, the obtained solutions are nearly equal and the average CPU time of the G-VRPTW with NFCC is nearly 0.05 s.

Table 6 The comparisons with the best known VRPTW results of Gehring and Homberger problems

Problem name	Best known solution		SA solution			GAP%
	Distance	Number of vehicle	Distance	Number of vehicle	CPU time	distance
c1_2_1	2,704.57	20	2,704.57	20	117.11	0.00
c1_2_2	2,917.89	18	2,809.66	20	276.53	−3.71
r1_2_1	4,784.11	20	4,864.76	22	88.31	1.69
r1_2_2	4,039.86	18	4,137.59	20	309.40	2.42
rc1_2_1	3,602.80	18	3,713.50	22	92.32	3.07
rc1_2_2	3,249.05	18	3,380.70	20	308.42	4.05
c1_4_1	7,152.02	40	7,152.02	40	258.28	0.00
c1_4_2	7,686.38	36	7,519.53	42	914.42	−2.17
r1_4_1	10,372.31	40	10,824.42	44	419.33	4.36
r1_4_2	8,926.70	36	9,689.74	38	716.10	8.55
rc1_4_1	8,576.97	36	8,924.73	39	403.07	4.05
rc1_4_2	7,905.66	36	8,415.45	36	551.13	6.45
Total	71,918.32	336	74,136.67	363	4,454.42	28.76

Table 7 The MSA-SA solutions for the G-VRPTW and G-VRPTW with the NFCC (small-sized)

| Problem type	Customer number	G-VRPTW				G-VRPTW with NFCC			
		Fuel consumption (l)	CO_2 emissions (kg)	Number of vehicles	CPU time	Fuel consumption (l)	CO_2 emissions (kg)	Number of vehicles	CPU time
R101	5	8.0455	24.72	3	~0.00	8.0921	24.86	3	~0.00
R102		7.1509	21.97	2	~0.00	7.1145	21.86	2	~0.00
R103		7.1509	21.97	2	~0.00	7.1145	21.86	2	~0.00
R104		7.1509	21.97	2	~0.00	7.1145	21.86	2	~0.00
R105		7.5202	23.10	2	~0.00	7.4886	23.01	2	~0.00
R101	10	14.0088	43.29	4	~0.00	14.5643	44.75	4	~0.00
R102		12.2925	37.77	3	~0.00	12.1367	32.29	3	~0.00
R103		12.2925	37.77	3	~0.00	12.1367	32.29	3	~0.00
R104		10.9809	33.74	2	~0.00	11.2864	34.68	2	~0.00
R105		13.5816	41.73	3	~0.00	12.9504	39.79	3	~0.00
R101	15	20.3529	62.53	5	~0.00	20.1130	61.80	5	0.01
R102		17.9819	55.25	4	~0.00	18.0549	55.47	4	0.18
R103		17.9819	55.25	4	0.02	18.0549	55.47	4	0.16
R104		16.5889	50.97	3	0.03	16.1398	49.59	3	0.33
R105		19.2795	59.23	4	~0.00	20.0065	61.47	4	0.05
Total		192.4398	591.26	46	~0.06	192.3678	581.05	46	~0.75

Table 8 presents the SA solutions for medium-sized problems (25, 50, 100 customer nodes) and compares them according to the distance and fuel consumption values. It can be clearly seen from Table 8 that, the G-VRPTW and G-VRPTW with NFCC solutions are very close, especially for the 25 and 50 customer nodes. Nevertheless, the GAP% between the solutions for 100 customer nodes show a small increase based on the fuel consumption values in the range of [−6.09, 4.52]

Table 8 The MSA-SA solutions for the G-VRPTW and G-VRPTW with the NFCC (medium-sized)

Problem type	Customer number	G-VRPTW					G-VRPTW with NFCC					GAP%	
		Fuel consumption (l)	CO_2 emissions (kg)	Total distance (km)	Number of vehicles	CPU time	Fuel consumption (l)	CO_2 emissions (kg)	Total distance (km)	Number of vehicles	CPU time	Distance	Fuel consumption
R101	25	37.1076	114.01	618.1	8	0.02	37.9429	116.58	618.1	8	0.12	0.00	2.25
R102		34.6636	106.50	565.8	7	0.83	34.1664	104.97	564.6	7	4.57	−0.21	−1.43
R103		29.1911	89.69	464.4	5	1.08	29.4295	90.42	463.6	5	4.99	−0.17	0.82
R104		27.0308	83.05	424.5	5	0.67	27.2506	83.72	428.3	5	13.41	0.90	0.81
R105		33.7990	103.84	531.5	6	0.78	33.1333	101.80	531.5	6	0.25	0.00	−1.97
R101	50	63.5691	195.31	1,052.7	12	1.14	65.2552	200.49	1,053.6	12	2.05	0.09	2.65
R102		59.4365	182.61	943.9	12	5.03	60.0466	184.49	933.7	11	12.28	−1.08	1.03
R103		50.5505	155.31	791.5	9	25.42	49.0912	150.83	791.5	9	33.41	0.00	−2.89
R104		45.0687	138.47	653.2	6	23.53	44.3871	136.38	648.3	7	73.03	−0.75	−1.51
R105		54.3898	167.11	907.5	9	2.48	54.4537	167.30	905.5	10	12.34	−0.22	0.12
R101	100	97.1913	298.61	1,658.2	21	23.12	100.5422	308.91	1,655.5	21	38.14	−0.18	3.45
R102		98.1468	301.55	1,493.1	20	110.08	93.7787	288.13	1,493.1	20	175.63	0.00	−4.45
R103		92.4638	284.09	1,271.0	14	123.68	88.7266	272.60	1,274.5	14	249.55	0.27	−4.04
R104		82.8964	254.69	1,018.3	11	205.10	77.8454	239.17	1,020.4	11	389.46	0.21	−6.09
R105		88.0960	270.67	1,398.6	15	32.97	92.0812	282.91	1,393.9	15	76.58	−0.34	4.52
Total		893.6010	2,745.51	13,792.3	160	555.93	888.1306	2728.7	13,766.1	161	1,085.81	−1.48	−6.73

Fig. 10 An iteration-time graphic for R105 type problem
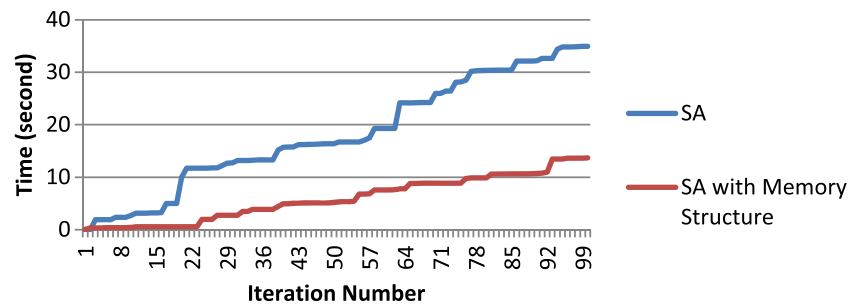
due to the deviation of the regression equation. Therefore, although both two approaches are satisfactory for solving the G-VRPTW with the MSA-SA, it should be stated that the G-VRPTW with NFCC approach is better than the G-VRPTW with linear fuel consumption calculation, to obtain more realistic results. Based on the CPU times, the problems are solved in a short time for both G-VRPTW and G-VRPTW with NFCC.

In order to show the contribution of the memory structure in NFCC, an iteration-time graphic for R105 type problem is presented in Fig. 10 for the G-VRPTW with NFCC. The SA with memory structure terminates the run of 100 iterations in about 15 s while the SA without memory structure does the same number of iterations in about 35 s. Especially for the large-sized problems, although the memory structure aggravates the processing times because of the memory usage and searching procedure for VRPTW or G-VRPTW, this operator provides significant CPU time savings on fitness function calculation for G-VRPTW with NFCC.

The final results for the computational studies are shown in Table 9 for the large-sized problems of Gehring and Homberger. The G-VRPTW with NFCC presents average −0.80 % fuel consumption difference from the G-VRPTW with average 0.24 % total distance increment and an acceptable rise in solution times. Consequently, although the MSA-SA displays a remarkable performance for both linear regression and nonlinear fuel consumption calculations, the G-VRPTW with NFCC presents more exact solutions with respect to the G-VRPTW.

Eventually, the fuel consumption calculation system for the G-VRPTW, which is integrated with the SA, produces a route plan with lower fuel consumption and CO_2 emissions and requires smaller operation times to obtain the efficient solutions indicated above. According to the general structure of the proposed system shown in Fig. 11, the inputs are the technical specifications of the vehicles, customer information (e.g., demand, location, coordinates, etc.) and time restrictions of the current situation. The system operates with the integrated SA and NFCC algorithms, and the outputs are

Table 9 The MSA-SA solutions for large-sized problems

| Problem name	G-VRPTW					G-VRPTW with NFCC					GAP%	
	Fuel consumption (l)	CO_2 emissions (kg)	Total distance (km)	Number of vehicles	CPU time	Fuel consumption (l)	CO_2 emissions (kg)	Total distance (km)	Number of vehicles	CPU time	Distance	Fuel consumption
c1_2_1	208.2874	639.94	2,913.68	22	113.03	201.4810	619.03	2,842.64	21	195.58	−2.44	−3.27
c1_2_2	224.9228	691.05	2,986.01	23	272.46	233.9273	718.72	3,077.11	24	668.42	3.05	4.00
r1_2_1	312.8767	961.28	4,875.78	23	99.50	302.8373	930.44	4,872.34	23	248.47	−0.07	−3.21
r1_2_2	295.5302	907.99	4,305.56	22	479.34	309.7615	951.71	4,482.98	24	879.30	4.12	4.82
rc1_2_1	253.7279	779.55	3,910.35	22	166.36	246.4736	757.27	3,915.50	22	346.21	0.13	−2.86
rc1_2_2	261.2828	802.77	3,664.54	21	520.51	261.0058	801.91	3,696.81	22	533.56	0.88	−0.11
c1_4_1	532.5286	1,636.14	7,856.09	43	270.43	506.9528	1,557.56	7,772.63	43	681.19	−1.06	−4.80
c1_4_2	576.6878	1,771.82	8,251.46	45	1,261.54	579.0113	1,778.95	8,116.79	44	1,508.08	−1.63	0.40
r1_4_1	678.4595	2,084.50	11,171.86	45	551.16	677.3424	2,081.07	11,148.40	45	558.25	−0.21	−0.16
r1_4_2	668.2586	2,053.16	10,102.19	42	1,368.59	660.1942	2,028.38	9,913.80	42	1,645.49	−1.86	−1.21
rc1_4_1	648.1533	1,991.39	9,476.69	43	324.48	644.4037	1,979.87	9,668.89	44	659.56	2.03	−0.58
rc1_4_2	614.2980	1,887.37	8,852.48	43	761.33	598.5369	1,838.94	8,849.13	43	1,579.43	−0.04	−2.57
Total	5275.0136	16,206.95	78,366.69	394	6,188.73	5,221.9278	78,357.02	78,357.02	397	9,503.54	2.90	−9.54
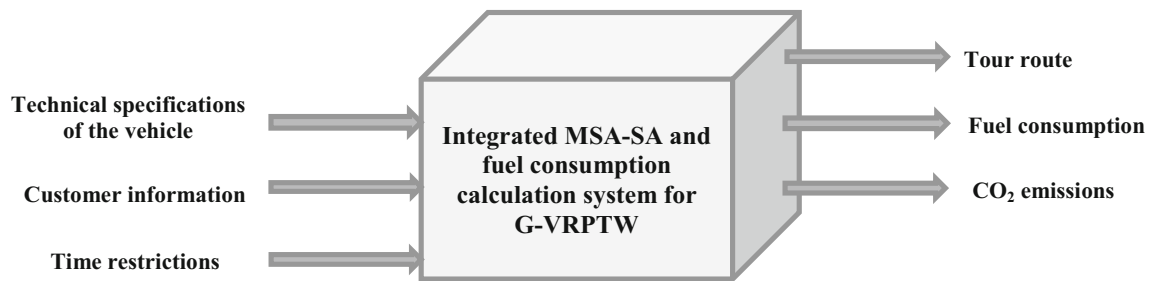
Fig. 11 General structure of the proposed system

the optimum vehicle routes, fuel consumption quantity and CO_2 emissions.

Conclusions and future research

In recent years, as concern over environmental issues has grown, researchers and companies have focused additional attention on green logistics subjects associated primarily with climate change and air pollution. The aim of this study is to introduce a methodology for an energy consumption model that minimizes fuel consumption and CO_2 emissions. The methodology includes three stages. In the first stage, the fuel consumption is calculated by considering the vehicle technical specifications, loads, and distance. The output of the first stage is the nonlinear fuel consumption calculation function. The second stage includes the mixed integer linear programming model which considers single-stopping and multi-stopping conditions. This model is integrated with the fuel consumption calculation by linearization of the fuel consumption calculation function. In the third stage, the MSA-SA meta-heuristic algorithm is included due to the complexity of the proposed problem and the requirements of the long solution times.

The proposed mixed integer linear programming model of the G-VRPTW is tested on small-sized problems due to the long operation times of the model. The results of the mathematical model show that the proposed G-VRPTW provides considerable fuel consumption and CO_2 emission reductions for small-sized problems. The SA meta-heuristic model is presented to determine the operation times and complexity of the mathematical model. The performance of the proposed MSA-SA model is evaluated for the VRPTW, G-VRPTW with linear fuel consumption calculation and G-VRPTW with NFCC on Solomon's (small and medium-sized) and Gehring and Homberger's (large-sized) benchmark problems, which are the most popular data sets to test VRPTW problems in literature. The MSA-SA exhibits superior performance for the VRPTW and finds near optimum solution in a very short time for medium-sized and large-sized problems. Moreover, the total traveling distance is reduced according to the best known solution for the first five R1 type problems of Solomon's data set and c1_2_2 and c1_4_2 problems of Gehring and

Homberger's benchmark problems despite the increment in the total number of vehicles. Following the tests on the VRPTW, the MSA-SA is applied to the G-VRPTW with the linear fuel consumption calculation and G-VRPTW with NFCC. The MSA-SA with the linear fuel consumption calculation function produces optimal solutions for each instance in small-sized problem sets. Additionally, the efficiency of the MSA-SA with both fuel consumption calculation types is demonstrated for the G-VRPTW in the pre-mentioned problems and the computational results show that the MSA-SA displays remarkable performance for each instance in a practical time.

In future research, the nonlinear fuel consumption calculations should be studied with integration of the time-dependent vehicle routing problem to incorporate the road congestions, especially for urban logistics. Additionally, the performance of the NFCC algorithm should be investigated with different heuristic algorithms.

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