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Published online: 24 Oct 2018.

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The (a, b)-Zagreb index of some derived networks

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ABSTRACT

There is a very wide application of mathematics in communication theory, signal processing and networking. A network is formed by nodes communicating with each other. Graph theory plays a vital role in the area of networking research. A topological index is a numeric quantity obtained from a graph structure which is invariant under graph isomorphism. In this study, we obtained a generalized degree-based topological index, called (a, b)-Zagreb index of some derived networks such as Sierpinski network, butterfly network, Benes network and mesh-derived network and hence consider some particular cases.

1. Introduction

An interconnection network can be considered as a graph $G = (V(G), E(G))$, where $V(G)$ denotes a set of nodes or vertex and $E(G)$ is the set of interconnections between the nodes of G. The degree of a vertex $v \in V(G)$ is defined as the number of adjacent vertices of v in G and is denoted as $d_G(v)$. A topological index is a function $f : \sum \rightarrow \mathbb{R}$ with the property that $f(G) = f(H)$ for all $G, H \in \sum$ are isomorphic, where $\sum$ denotes the set of all finite simple graphs and $\mathbb{R}$ is the set of all real numbers. In the last decade, a lot of studies on different topological indices for different graph families and network structures were conducted and successfully applied in mathematical chemistry. Recently, H. Alih et al. in [1] studied on the topological properties of hierarchical interconnection networks. The Sierpinski network was developed from the studies of Lipscomb's space and was first introduced by S. Klavžar et al. in [2]. The vertex set of $S[m, n]$ is the Cartesian product of m sets $\{1, 2, 3, ..., n\}$, while the edges are defined according to a certain relation. The vertex set of $S[m, n]$ is defined as $V(S[m, n]) = \{1, 2, 3, ..., n\}^m$ and $|V(S[m, n])| = n^m$. Recently, M. Imran et al. studied topological properties of Sierpinski networks in [3]. Butterfly network $BF[n]$ is a well-known and important topological structure of interconnection networks. It is extensively used for parallel architectures and for achieving a technique to interpret Fast Fourier Transform (FFT), which is comprehensively used for signal processing. Butterfly network and some other closely multistage interconnection networks are used for several proposed designs for the switching fabric of sealable high-speed ATM networks. A Benes network is derived from the

butterfly network by overlapping the low-order cycles of two butterfly networks. An n dimensional Benes network is denoted as $B[n]$ with $(2n + 1)2^n$ nodes. If P_r is a path of r vertices, then $P_r \times P_t$ is defined as the two-dimensional mesh network with r rows and t columns. This network was first introduced by I. Rajasingh et al. in [4], and is denoted as $M_{r,t}$. The bounded medial of $r \times t$ mesh is obtained by applying medial operations on $r \times t$ mesh and then by deleting those vertices that are placed on unbounded faces. The mesh-derived network was first introduced by M. Imran et al. [5] in 2014. This network was obtained by taking union of $r \times t$ mesh and its bounded medial in a way that the vertices of bounded medial are placed in the middle of each edge of $r \times t$ mesh, the resulting structure is called as a mesh-derived network of first type and is denoted as $MDN1[r, t]$. The vertex and edge cardinalities of $MDN1[r, t]$ are $3rt - r - t$ and $8rt - 6(r + t) + 4$, respectively. The second type of mesh-derived network is obtained from the union of $r \times t$ mesh and its bounded dual $r - 1 \times t - 1$ mesh by joining every vertex of $r - 1 \times t - 1$ mesh to every vertex of the corresponding face of $r \times t$ mesh. The resulting structure is called second type mesh-derived network and is denoted as $MDN2[r, t]$. The vertex and edge cardinalities of $MDN2[r, t]$ are $2rt - r - t + 1$ and $8(rt - r - t + 1)$, respectively. The parameters r and t are defined as the number of vertices in any row and column, respectively. A new branch cheminformatics is a combination of mathematics, information sciences and chemistry. This branch studies QSAR/QSPR study, physicochemical properties and topological indices such as Zagreb indices, "forgotten topological index", redefined Zagreb

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index, general first Zagreb index, general Randić index, symmetric division deg index, generalized Zagreb index and so on to predict physicochemical properties and biological activities of the chemical compounds theoretically. The most oldest and extremely studied vertex degree-based topological indices are Zagreb indices, which were introduced by Gutman and Trinajestić in 1972 [6], to study the total π-electron energy(ϵ) of carbon atoms and are defined as

$$M_1(G) = \sum_{v \in V(G)} d_G(v)^2 = \sum_{uv \in E(G)} [d_G(u) + d_G(v)]$$

and

$$M_2(G) = \sum_{uv \in E(G)} d_G(u)d_G(v).$$

We refer our reader to [7–9], for further study about this index. The "forgotten topological index" or F-index of a graph was introduced in the same paper [6], where Zagreb indices were introduced. This index is defined as

$$F(G) = \sum_{v \in V(G)} d_G(v)^3 = \sum_{uv \in E(G)} [d_G(u)^2 + d_G(v)^2].$$

Some recent study about this index we refer our reader to [10–13]. One of the redefined versions of the Zagreb index is defined as

$$ReZM(G) = \sum_{uv \in E(G)} d_G(u)d_G(v)[d_G(u) + d_G(v)]$$

and was first introduced by Ranjini et al. in 2013 [14]. Li and Zheng in [15] generalized the first Zagreb index and F-index as follows

$$M^\alpha(G) = \sum_{u \in V(G)} d_G(u)^\alpha,$$

where $\alpha \neq 0, 1$ and $\alpha \in \mathbb{R}$. Clearly, when $\alpha = 2$, we obtain the first Zagreb index and when $\alpha = 3$ it gives the F-index. Gutman and Lepović generalized the Randić index in [16] and is defined as

$$R_a = \sum_{uv \in E(G)} \{d_G(u).d_G(v)\}^a,$$

where $a \neq 0, a \in \mathbb{R}$. The symmetric division deg index of a graph is defined as

$$SDD(G) = \sum_{uv \in E(G)} \left[\frac{d_G(u)}{d_G(v)} + \frac{d_G(v)}{d_G(u)} \right].$$

We refer our reader to [17, 18], for further study about this index. Based on Zagreb indices, Azari et al.[19] introduced an another type of topological index called generalized Zagreb index or the (a, b)-Zagreb index in 2011 and is defined as

$$Z_{a,b}(G) = \sum_{uv \in E(G)} (d_G(u)^a d_G(v)^b + d_G(u)^b d_G(v)^a).$$

For further study about this index, we refer our reader to [20, 21]. It is clear that all the topological indices discussed previously in this paper are derived from this

Table 1. Relations between (a,b)-Zagreb index with some other topological indices

Topological index	Corresponding (a,b)-Zagreb index
First Zagreb index $M_1(G)$	$Z_{1,0}(G)$
Second Zagreb index $M_2(G)$	$\frac{1}{2}Z_{1,1}(G)$
Forgotten topological index $F(G)$	$Z_{2,0}(G)$
Redefined Zagreb index $ReZM(G)$	$Z_{2,1}(G)$
General first Zagreb index $M^a(G)$	$Z_{a-1,0}(G)$
General Randić index R_a	$\frac{1}{2}Z_{a,a}$
Symmetric division deg index $SDD(G)$	$Z_{1,-1}(G)$

(a, b)-Zagreb index for some particular values of a and b. Table 1 shows the relation between (a, b)-Zagreb index to some other topological indices for some particular values of a and b. In this work, we study the mathematical property of (a, b)-Zagreb index for some derived networks such as Sierpinski network $S[m, n]$, butterfly network $BF[n]$, Benes network $B[n]$ and mesh-derived network $MDN1[r, t]$.

2. Main results

In this section, we derived (a,b)-Zagreb index of some well-defined networks. First, we consider $S[m, n]$. The edge sets of $S[m, n]$ are divided as follows:

$$E_1(S[m, n]) = \{e = uv \in E(S[m, n]) :$$
$$d_{S[m,n]}(u) = n - 1 \quad \text{and} \quad d_{S[m,n]}(v) = n\},$$
$$E_2(S[m, n]) = \{e = uv \in E(S[m, n]) :$$
$$d_{S[m,n]}(u) = n \quad \text{and} \quad d_{S[m,n]}(v) = n\},$$

note that,

$$|E_1(S[m, n])| = 2n, |E_2(S[m, n])| = \frac{n^{m+1} - 5n}{2}.$$

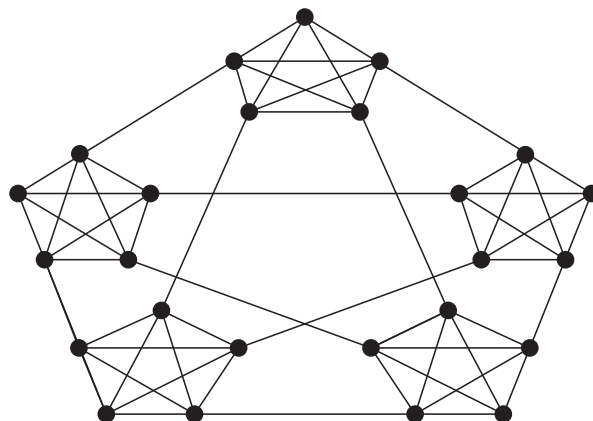The Sierpinski network $S[4, 4]$ is depicted in Figure 1.

Figure 1. Sierpinski network $S[3, 4]$.

Theorem 2.1: *The (a, b)-Zagreb index of Sierpinski network $(S[m, n])$ is given by*

$$Z_{a,b}(S[m, n]) = 2.n^{b+1}(n - 1)^a + 2n^{a+1}(n - 1)^b$$
$$+ n^{m+a+b+1} - 5n^{a+b+1}. \qquad (1)$$

Proof: Applying the definition of (a, b)-Zagreb index, we obtain

$$Z_{a,b}(S[m, n]) = \sum_{uv \in E(S[m,n])} (d_{S[m,n]}(u)^a d_{S[m,n]}(v)^b$$
$$+ d_{S[m,n]}(u)^b d_{S[m,n]}(v)^a)$$
$$= \sum_{uv \in E_1(S[m,n])} [(n - 1)^a n^b + (n - 1)^b n^a]$$
$$+ \sum_{uv \in E_2(S[m,n])} (n^a n^b + n^b n^a)$$
$$= |E_1(S[m, n])|[(n - 1)^a n^b + (n - 1)^b n^a]$$
$$+ |E_2(S[m, n])|2.n^{a+b}$$
$$= 2n[(n - 1)^a n^b + (n - 1)^b n^a]$$
$$+ \frac{n^{m+1} - 5n}{2}.2.n^{a+b}.$$

Hence, the theorem. ∎

Corollary 2.2: *From Equation (1), we derived the following results.*

$$(i)\ M_1(S[m, n]) = Z_{1,0}(S[m, n])$$
$$= n(n^{m+1} - n - 2),$$
$$(ii)\ M_2(S[m, n]) = \frac{1}{2}Z_{1,1}(S[m, n])$$
$$= \frac{1}{2}n(n^{m+2} - n^2 - 4n),$$
$$(iii)\ F(S[m, n]) = Z_{2,0}(S[m, n])$$
$$= n(n^{m+2} - n^2 - 2n + 2),$$
$$(iv)\ ReZM(S[m, n]) = Z_{2,1}(S[m, n])$$
$$= n(n^{m+2} - n^2 - 4n + 2),$$
$$(v)\ M^a(S[m, n]) = Z_{a-1,0}(S[m, n])$$
$$= 2n(n - 1)^{a-1} + n^{m+a} - 3n^a,$$
$$(vi)\ R_a(S[m, n]) = \frac{1}{2}Z_{a,a}(S[m, n])$$
$$= 4n^{a+1}(n - 1)^a$$
$$+ n^{m+2a+1} - 5n^{2a+1},$$
$$(vii)\ SDD(S[m, n]) = Z_{1,-1}(S[m, n])$$
$$= 2n^2(n - 1)^{-1} + n^{m+1} - 3n - 2.$$

Now, we consider the butterfly network $BF[n]$ and obtained the (a,b)-Zagreb index of this network. The figure of two-dimensional butterfly network $BF[2]$ is
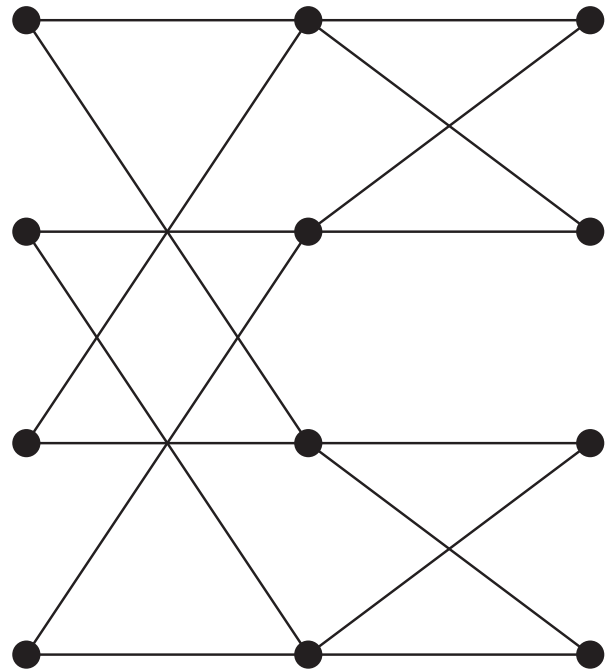

Figure 2. An example of two-dimensional butterfly network $BF[2]$.

shown in Figure 2. The edge set of this network is partitioned as follows:

$$E_1(BF[n]) = \{e = uv \in E(BF[n]) : d_{BF[n]}(u)$$
$$= 2 \quad \text{and} \quad d_{BF[n]}(v) = 4\},$$
$$E_2(BF[n]) = \{e = uv \in E(BF[n]) : d_{BF[n]}(u)$$
$$= 4 \quad \text{and} \quad d_{BF[n]}(v) = 4\},$$

such that,

$$|E_1(BF[n])| = 2^{n+2}, |E_2(BF[n])| = 2^{n+1}(n - 2).$$

Theorem 2.3: *The (a, b)-Zagreb index of $BF[n]$ is given by*

$$Z_{a,b}(BF[n]) = 2^{n+2}(2^{a+2b} + 2^{2a+b})$$
$$+ 2^{n+1}(n - 2).2^{2(a+b)+1}. \qquad (2)$$

Proof: Using the definition of (a, b)-Zagreb index, we obtain

$$Z_{a,b}(BF[n]) = \sum_{uv \in E(NF[n])} (d_{BF[n]}(u)^a d_{BF[n]}(v)^b$$
$$+ d_{BF[n]}(u)^b d_{BF[n]}(v)^a)$$
$$= \sum_{uv \in E_1(BF[n])} (2^a 4^b + 2^b 4^a)$$
$$+ \sum_{uv \in E_2(BF[n])} (4^a 4^b + 4^b 4^a)$$
$$= |E_1(BF[n])|(2^a 2^{2b} + 2^b 2^{2a})$$
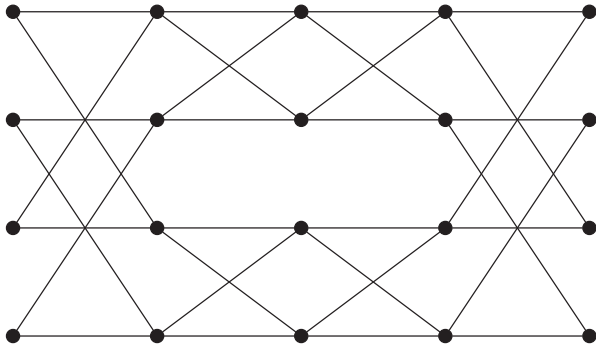$$+ |E_2(BF[n])|2.4^{a+b}$$

Figure 3. An example of two-dimensional Benes network $B[2]$.

$$= 2^{n+2}(2^{a+2b} + 2^{2a+b})$$
$$+ 2^{n+1}(n-2).2.2^{2(a+b)},$$

which is the required theorem. ∎

Corollary 2.4: *We derived the following results by using Equation (2),*

$$(i)\ M_1(BF[n]) = Z_{1,0}(BF[n]) = 2^{n+3}(2n-1),$$

$$(ii)\ M_2(BF[n]) = \frac{1}{2}Z_{1,1}(BF[n]) = 2^{n+5}(n-1),$$

$$(iii)\ F(BF[n]) = Z_{2,0}(BF[n]) = 2^{n+4}(4n-3),$$

$$(iv)\ ReZM(BF[n]) = Z_{2,1}(BF[n]) = 2^{n+5}(8n-11),$$

$$(v)\ M^a(BF[n]) = Z_{a-1,0}(BF[n])$$
$$= 2^{n+2}(2^{(a-1)} + 2^{2(a-1)})$$
$$+ 2^{n+2a}(n-2),$$

$$(vi)\ R_a(BF[n]) = \frac{1}{2}Z_{a,a}(BF[n])$$
$$= 2^{n+2}.2^{3a} + 2^{n+1}(n-2).2^{4a},$$

$$(vii)\ SDD(BF[n]) = Z_{1,-1}(BF[n]) = 2^{n+1}(2n-1).$$

Here, we consider the Benes network $B[n]$ and derived the (a,b)-Zagreb index of this network. An example of two-dimensional Benes network $B[2]$ is shown in Figure 3. The edge sets of $B[n]$ are shown as follows:

$$E_1(B[n]) = \{e = uv \in E(B[n]) : d_{B[n]}(u)$$
$$= 2 \quad \text{and} \quad d_{B[n]}(v) = 4\},$$
$$E_2(B[n]) = \{e = uv \in E(B[n]) : d_{B[n]}(u)$$
$$= 4 \quad \text{and} \quad d_{B[n]}(v) = 4\},$$

such that,

$$|E_1(B[n])| = 2^{n+2}, |E_2(B[n])| = 2^{n+2}(n-1).$$

Theorem 2.5: *The (a,b)-Zagreb index of $B[n]$ is given by*

$$Z_{a,b}(B[n]) = 2^{n+2}(2^{a+2b} + 2^{2a+b})$$
$$+ 2^{n+2}(n-1).2^{2(a+b)+1}. \tag{3}$$

Proof: From the definition of (a,b)-Zagreb index, we obtain

$$Z_{a,b}(B[n]) = \sum_{uv \in E(B[n])} (d_{B[n]}(u)^a d_{B[n]}(v)^b$$
$$+ d_{B[n]}(u)^b d_{B[n]}(v)^a)$$
$$= \sum_{uv \in E_1(B[n])} (2^a 4^b + 2^b 4^a)$$
$$+ \sum_{uv \in E_2(B[n])} (4^a 4^b + 4^b 4^a)$$
$$= |E_1(B[n])|(2^a 2^{2b} + 2^b 2^{2a})$$
$$+ |E_2(B[n])|.2.4^{a+b}$$
$$= 2^{n+2}(2^{a+2b} + 2^{2a+b})$$
$$+ 2^{n+2}(n-1).2.2^{2(a+b)}.$$

Hence the theorem. ∎

Corollary 2.6: *From Equation (3), we derived the following results.*

$$(i)\ M_1(B[n]) = Z_{1,0}(B[n]) = 2^{n+3}(4n-1),$$

$$(ii)\ M_2(B[n]) = \frac{1}{2}Z_{1,1}(B[n]) = 2^{n+5}(4n-3),$$

$$(iii)\ F(B[n]) = Z_{2,0}(B[n]) = 2^{n+2}(32n-12),$$

$$(iv)\ ReZM(B[n]) = Z_{2,1}(B[n]) = 2^{n+4}(32n-29),$$

$$(v)\ M^a(B[n]) = Z_{a-1,0}(B[n])$$
$$= 2^{n+2}(2^{(a-1)} + 2^{2(a-1)})$$
$$+ 2^{n+2a+1}.(n-1),$$

$$(vi)\ R_a(B[n]) = \frac{1}{2}Z_{a,a}(B[n])$$
$$= 2^{n+2}\{2^{3a} + (n-1).2^{4a}\},$$

$$(vii)\ SDD(B[n]) = Z_{1,-1}(B[n]) = 2^{n+1}(4n+1).$$

Now, we obtained (a,b)-Zagreb index of mesh-derived network $MDN1[r,t]$. The edge sets of $MDN1[r,t]$ are shown as follows:

$$E_1(MDN1[r,t]) = \{e = uv \in E(MDN1[r,t]) : d_{MDN1[r,t]}(u)$$
$$= 2 \quad \text{and} \quad d_{MDN1[r,t]}(v) = 4\},$$
$$E_2(MDN1[r,t]) = \{e = uv \in E(MDN1[r,t]) : d_{MDN1[r,t]}(u)$$
$$= 3 \quad \text{and} \quad d_{MDN1[r,t]}(v) = 4\},$$
$$E_3(MDN1[r,t]) = \{e = uv \in E(MDN1[r,t]) : d_{MDN1[r,t]}(u)$$
$$= 3 \quad \text{and} \quad d_{MDN1[r,t]}(v) = 6\},$$
$$E_4(MDN1[r,t]) = \{e = uv \in E(MDN1[r,t]) : d_{MDN1[r,t]}(u)$$
$$= 4 \quad \text{and} \quad d_{MDN1[r,t]}(v) = 6\},$$
$$E_5(MDN1[r,t]) = \{e = uv \in E(MDN1[r,t]) : d_{MDN1[r,t]}(u)$$
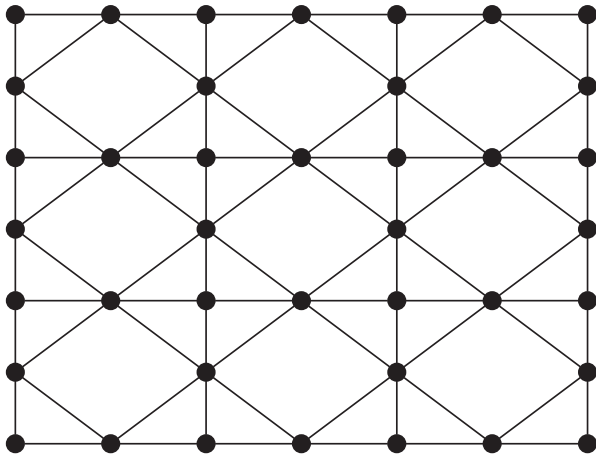$$= 4 \quad \text{and} \quad d_{MDN1[r,t]}(v) = 4\},$$

Figure 4. An example of $MDN1[4, 4]$ network.

$E_6(MDN1[r, t]) = \{e = uv \in E(MDN1[r, t]) : d_{MDN1[r,t]}(u)$

$$= 6 \quad \text{and} \quad d_{MDN1[r,t]}(v) = 6\},$$

note that,

$$|E_1(MDN1[r, t])| = 8, |E_2(MDN1[r, t])| = 4(r + t - 4),$$

$$|E_3(MDN1[r, t])| = 2(r + t - 4), |E_4(MDN1[r, t])|$$

$$= 4(rt - r - t),$$

$$|E_5(MDN1[r, t])| = 4, |E_6(MDN1[r, t])|$$

$$= 4(rt - 2r - 2t + 4).$$

So that, $|V(MDN1[r, t])| = (3rt - r - t)$ and $|E(MDN1[r, t])| = \{8rt - 6(r + t) + 4\}$.

An example $MDN1[4, 4]$ is shown in Figure 4.

Theorem 2.7: *For $MDN1[r, t]$, the (a, b)-Zagreb index is given by*

$$Z_{a,b}(MDN1[r, t]) = 8(2^{a+2b} + 2^{2a+b})$$

$$+ 4(r + t - 4)(3^a.4^b + 3^b.4^a)$$

$$+ 2(r + t - 4).3^{a+b}$$

$$.(2^a + 2^b) + 4.2^{2(a+b)+1}$$

$$+ 4(rt - r - t)(2^{2a+b}.3^b + 2^{a+2b}.3^a)$$

$$+ 8(rt - 2r - 2t + 4)6^{a+b}. \tag{4}$$

Proof: Using the definition of (a, b)-Zagreb index, we obtain

$$Z_{a,b}(MDN1[r, t]) = \sum_{uv \in E(MDN1[r,t])} (d(u)^a d(v)^b$$

$$+ d(u)^b d(v)^a)$$

$$= \sum_{uv \in E_1(MDN1[r,t])} (2^a 4^b + 2^b 4^a)$$

$$+ \sum_{uv \in E_2(MDN1[r,t])} (3^a 4^b + 3^b 4^a)$$

$$+ \sum_{uv \in E_3(MDN1[r,t])} (3^a 6^b + 3^b 6^a)$$

$$+ \sum_{uv \in E_4(MDN1[r,t])} (4^a 6^b + 4^b 6^a)$$

$$+ \sum_{uv \in E_5(MDN1[r,t])} (4^a 4^b + 4^b 4^a)$$

$$+ \sum_{uv \in E_6(MDN1[r,t])} (6^a 6^b + 6^b 6^a)$$

$$= |E_1(MDN1[r, t])|(2^a 2^{2b} + 2^b 2^{2a})$$

$$+ |E_2(MDN1[r, t])|(3^a 4^b + 3^b 4^a)$$

$$+ |E_3(MDN1[r, t])|$$

$$\times (3^a.3^b.2^b + 3^b.3^a.2^a)$$

$$+ |E_4(MDN1[r, t])|$$

$$\times (2^{2a}.2^b.3^b + 2^{2b}.2^a.3^a)$$

$$+ |E_5(MDN1[r, t])|.2.2^{2(a+b)}$$

$$+ |E_6(MDN1[r, t])|.2.6^{a+b}$$

$$= 8(2^{a+2b} + 2^{2a+b})$$

$$+ 4(r + t - 4)(3^a 4^b + 3^b 4^a)$$

$$+ 2(r + t - 4).3^{a+b}.(2^b + 2^a)$$

$$+ 4(rt - r - t)(2^{2a+b}.3^b + 2^{a+2b}.3^a)$$

$$+ 4.2^{2(a+b)+1}$$

$$+ 8(rt - 2r - 2t + 4).6^{a+b},$$

which is the desired result. ∎

Corollary 2.8: *Using Equation (4), we derived the following results,*

(i) $M_1(MDN1[r, t]) = Z_{1,0}(MDN1[r, t])$

$$= 46(r + t - 4) + 40(rt - r - t)$$

$$+ 48(rt - 2r - 2t + 4) + 80,$$

(ii) $M_2(MDN1[r, t]) = \frac{1}{2}Z_{1,1}(MDN1[r, t])$

$$= 84(r + t - 4) + 96(rt - r - t)$$

$$+ 144(rt - 2r - 2t + 4) + 128,$$

(iii) $F(MDN1[r, t]) = Z_{2,0}(MDN1[r, t])$

$$= 190(r + t - 4) + 160(rt - r - t)$$

$$+ 288(rt - 2r - 2t + 4) + 288,$$

(iv) $ReZM(MDN1[r, t]) = Z_{2,1}(MDN1[r, t])$

$$= 660(r + t - 4) + 960(rt - r - t)$$

$$+ 1728(rt - 2r - 2t + 4) + 896,$$

$(v)\ M^a(MDN1[r,t]) = Z_{a-1,0}(MDN1[r,t])$

$$= 8(2^{a-1} + 2^{2(a-1)})$$

$$+ 4(r+t-4)(3^{a-1} + 4^{a-1})$$

$$+ 2(r+t-4)3^{a-1}(2^{a-1} + 1)$$

$$+ 4.2^{2(a-1)+1} + 4(rt - r - t)$$

$$\times (2^{2(a-1)} + 2^{a-1}.3^{a-1})$$

$$+ 8(rt - 2r - 2t + 4).6^{a-1},$$

$(vi)\ R_a(MDN1[r,t]) = \dfrac{1}{2}Z_{a,a}(MDN1[r,t])$

$$= 8.2^{3a} + 4(r+t-4).3^a.4^a$$

$$+ 2(r+t-4).3^{2a}.2^a + 2^{2(2a+1)}$$

$$+ 4(rt - r - t).2^{3a}.3^a$$

$$+ 4(rt - 2r - 2t + 4).6^{2a},$$

$(vii)\ SDD(MDN1[r,t]) = Z_{1,-1}(MDN1[r,t])$

$$= \frac{40}{3}(r+t-4) + \frac{26}{3}(rt - r - t)$$

$$+ 8(rt - 2r - 2t + 4) + 28.$$

Finally, we consider a type 2 mesh-derived network $MDN2[r,t]$ and obtain the (a,b)-Zagreb index of this network. An example of the type 2 mesh-derived network $MDN2[4,4]$ is shown in Figure 5. The edge set of this network partitioned as follows:

$E_1(MDN2[r,t]) = \{e = uv \in E(MDN2[r,t]) :$

$\quad d_{MDN2[r,t]}(u) = 3 \quad \text{and} \quad d_{MDN2[r,t]}(v) = 6\},$

$E_2(MDN2[r,t]) = \{e = uv \in E(MDN2[r,t]) :$

$\quad d_{MDN2[r,t]}(u) = 3 \quad \text{and} \quad d_{MDN2[r,t]}(v) = 5\},$

$E_3(MDN2[r,t]) = \{e = uv \in E(MDN2[r,t]) :$

$\quad d_{MDN2[r,t]}(u) = 5 \quad \text{and} \quad d_{MDN2[r,t]}(v) = 6\},$

$E_4(MDN2[r,t]) = \{e = uv \in E(MDN2[r,t]) :$

$\quad d_{MDN2[r,t]}(u) = 5 \quad \text{and} \quad d_{MDN2[r,t]}(v) = 5\},$

$E_5(MDN2[r,t]) = \{e = uv \in E(MDN2[r,t]) :$

$\quad d_{MDN2[r,t]}(u) = 6 \quad \text{and} \quad d_{MDN2[r,t]}(v) = 8\},$

$E_6(MDN2[r,t]) = \{e = uv \in E(MDN2[r,t]) :$

$\quad d_{MDN2[r,t]}(u) = 5 \quad \text{and} \quad d_{MDN2[r,t]}(v) = 8\},$

$E_7(MDN2[r,t]) = \{e = uv \in E(MDN2[r,t]) :$

$\quad d_{MDN2[r,t]}(u) = 5 \quad \text{and} \quad d_{MDN2[r,t]}(v) = 7\},$

$E_8(MDN2[r,t]) = \{e = uv \in E(MDN2[r,t]) :$

$\quad d_{MDN2[r,t]}(u) = 7 \quad \text{and} \quad d_{MDN2[r,t]}(v) = 7\},$

$E_9(MDN2[r,t]) = \{e = uv \in E(MDN2[r,t]) :$

$\quad d_{MDN2[r,t]}(u) = 6 \quad \text{and} \quad d_{MDN2[r,t]}(v) = 7\},$

$E_{10}(MDN2[r,t]) = \{e = uv \in E(MDN2[r,t]) :$

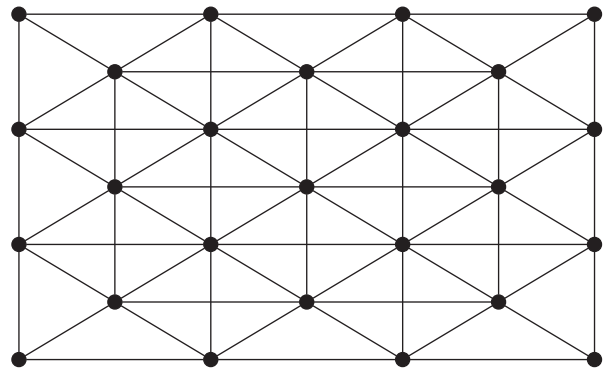$\quad d_{MDN2[r,t]}(u) = 7 \quad \text{and} \quad d_{MDN2[r,t]}(v) = 8\},$

Figure 5. An example of $MDN2[4,4]$ network.

$E_{11}(MDN2[r,t]) = \{e = uv \in E(MDN2[r,t]) :$

$\quad d_{MDN2[r,t]}(u) = 8 \quad \text{and} \quad d_{MDN2[r,t]}(v) = 8\},$

note that,

$|E_1(MDN2[r,t])| = 4, |E_2(MDN2[r,t])|$

$$= 8, |E_3(MDN2[r,t])| = 8,$$

$|E_4(MDN2[r,t])| = 2(r+t-6), |E_5(MDN2[r,t])|$

$$= 4, |E_6(MDN2[r,t])| = 2(r+t-4),$$

$|E_7(MDN2[r,t])| = 4(r+t-6), |E_8(MDN2[r,t])|$

$$= 2(r+t-8), |E_9(MDN2[r,t])| = 8,$$

$|E_{10}(MDN2[r,t])| = 6(r+t-6), |E_{11}(MDN2[r,t])|$

$$= 8rt - 24(r+t) + 72.$$

Theorem 2.9: *The (a,b)-Zagreb index of $MDN2[r,t]$, is given by*

$$Z_{a,b}(MDN2[r,t]) = 4(3^a.6^b + 3^b.6^a) + 8(3^a.5^b + 3^b.5^a)$$

$$+ 8(5^a.6^b + 5^b.6^a) + 4(r+t-6).5^{a+b}$$

$$+ 4(6^a.8^b + 6^b.8^a) + 2(r+t-4).(5^a.8^b + 5^b.8^a)$$

$$+ 4(r+t-6)(5^a.7^b + 5^b.7^a) + 4(r+t-8).7^{a+b}$$

$$+ 8(7^a.6^b + 7^b.6^a) + 6(r+t-6)(7^a.8^b + 7^b.8^a)$$

$$+ 2\{8rt - 24(r+t) + 72\}.8^{a+b}. \tag{5}$$

Proof: From the definition of (a,b)-Zagreb index, we obtain

$$Z_{a,b}(MDN2[r,t]) = \sum_{uv \in E(MDN2[r,t])} (d(u)^a d(v)^b$$

$$+ d(u)^b d(v)^a) = \sum_{uv \in E_1(MDN2[r,t])} (3^a 6^b + 3^b 6^a)$$

$$+ \sum_{uv \in E_2(MDN2[r,t])} (3^a 5^b + 3^b 5^a)$$

$$+ \sum_{uv \in E_3(MDN2[r,t])} (5^a 6^b + 5^b 6^a)$$

$$+ \sum_{uv \in E_4(MDN2[r,t])} (5^a 5^b + 5^b 5^a)$$

$$+ \sum_{uv \in E_5(MDN2[r,t])} (6^a 8^b + 6^b 8^a)$$

$$+ \sum_{uv \in E_6(MDN2[r,t])} (5^a 8^b + 5^b 8^a)$$

$$+ \sum_{uv \in E_7(MDN2[r,t])} (5^a 7^b + 5^b 7^a)$$

$$+ \sum_{uv \in E_8(MDN2[r,t])} (7^a 7^b + 7^b 7^a)$$

$$+ \sum_{uv \in E_9(MDN2[r,t])} (6^a 7^b + 6^b 7^a)$$

$$+ \sum_{uv \in E_{10}(MDN2[r,t])} (7^a 8^b + 7^b 8^a)$$

$$+ \sum_{uv \in E_{11}(MDN2[r,t])} (8^a 8^b + 8^b 8^a)$$

$$= |E_1(MDN2[r,t])|(3^a 6^b + 3^b 6^a)$$

$$+ |E_2(MDN2[r,t])|(3^a 5^b + 3^b 5^a)$$

$$+ |E_3(MDN2[r,t])|(5^a 6^b + 5^b 6^a)$$

$$+ |E_4(MDN2[r,t])|(5^a 5^b + 5^b 5^a)$$

$$+ |E_5(MDN2[r,t])|(6^a 8^b + 6^b 8^a)$$

$$+ |E_6(MDN2[r,t])|(5^a 8^b + 5^b 8^a)$$

$$+ |E_7(MDN2[r,t])|(5^a 7^b + 5^b 7^a)$$

$$+ |E_8(MDN2[r,t])|(7^a 7^b + 7^b 7^a)$$

$$+ |E_9(MDN2[r,t])|(6^a 7^b + 6^b 7^a)$$

$$+ |E_{10}(MDN2[r,t])|(7^a 8^b + 7^b 8^a)$$

$$+ |E_{11}(MDN2[r,t])|(8^a 8^b + 8^b 8^a)$$

$$= 4(3^a 6^b + 3^b 6^a) + 8(3^a 5^b + 3^b 5^a)$$

$$+ 8(5^a 6^b + 5^b 6^a) + 4(r + t - 6).5^{a+b}$$

$$+ 4(6^a 8^b + 6^b 8^a) + 2(r + t - 4)(5^a 8^b + 5^b 8^a)$$

$$+ 4(r + t - 6)(5^a 7^b + 5^b 7^a)$$

$$+ 4(r + t - 8).7^{a+b} + 8(6^a 7^b + 6^b 7^a)$$

$$+ 6(r + t - 6)(7^a 8^b + 7^b 8^a)$$

$$+ 2\{8rt - 24(r + t) + 72\}.8^{a+b}.$$

Hence, the desired result. ∎

Corollary 2.10: *From Equation (5), we obtain the following results.*

(i) $M_1(MDN2[r,t]) = Z_{1,0}(MDN2[r,t])$

$$= 158(r + t - 6) + 26(r + t - 4) + 28(r + t - 8)$$

$$+ 16\{8rt - 24(r + t) + 72\} + 348,$$

(ii) $M_2(MDN2[r,t]) = \frac{1}{2}Z_{1,1}(MDN2[r,t])$

$$= 526(r + t - 6) + 80(r + t - 4) + 98(r + t - 8)$$

$$+ 64\{8rt - 24(r + t) + 72\} + 960,$$

(iii) $F(MDN2[r,t]) = Z_{2,0}(MDN2[r,t])$

$$= 1074(r + t - 6) + 178(r + t - 4)$$

$$+ 196(r + t - 8)$$

$$+ 128\{8rt - 24(r + t) + 72\} + 2020,$$

(iv) $ReZM(MDN2[r,t]) = Z_{2,1}(MDN2[r,t])$

$$= 7220(r + t - 6) + 1040(r + t - 4)$$

$$+ 1372(r + t - 8)$$

$$+ 1024\{8rt - 24(r + t) + 72\} + 11304,$$

(v) $M^a(MDN2[r,t]) = Z_{a-1,0}(MDN2[r,t])$

$$= 4(3^{a-1} + 6^{a-1}) + 8(3^{a-1} + 5^{a-1})$$

$$+ 8(5^{a-1} + 6^{a-1}) + 4(r + t - 6).5^{a-1}$$

$$+ 4(6^{a-1} + 8^{a-1}) + 2(r + t - 4)(5^{a-1} + 8^{a-1})$$

$$+ 4(r + t - 6)(5^{a-1} + 7^{a-1}) + 4(r + t - 8).7^{a-1}$$

$$+ 8(7^{a-1} + 6^{a-1}) + 6(r + t - 6)(7^{a-1} + 8^{a-1})$$

$$+ 2\{8rt - 24(r + t) + 72\}.8^{a-1},$$

(vi) $R_a(MDN2[r,t]) = \frac{1}{2}Z_{a,a}(MDN2[r,t])$

$$= 4.3^a.6^a + 8.3^a.5^a + 8.5^a.6^a$$

$$+ 2(r + t - 6).5^{2a} + 4.6^a.8^a + 2(r + t - 4).5^a.8^a$$

$$+ 4(r + t - 6).5^a.7^a + 2(r + t - 8).7^{2a}$$

$$+ 8.7^a.6^a + 6(r + t - 6).7^a.8^a$$

$$+ \{8rt - 24(r + t) + 72\}.8^{2a},$$

(vii) $SDD(MDN2[r,t]) = Z_{1,-1}(MDN2[r,t])$

$$= \frac{3439}{140}(r + t - 6) + \frac{89}{20}(r + t - 4)$$

$$+ 4(r + t - 8) + 2\{8rt - 24(r + t) + 72\} + \frac{7237}{105}.$$

3. Conclusions

In this work, we study the (a, b)-Zagreb index of some derived networks and hence obtain some other topological indices such as Zagreb indices, "forgotten topological index", redefined Zagreb index, general first Zagreb index, general Randić index, and symmetric division deg index for some particular values of a and b. For further study, some other network structures can be considered for studying this (a, b)-Zagreb index.

Disclosure statement

No potential conflict of interest was reported by the authors.

Funding

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