

THE INVESTIGATION OF THE EFFECTS OF FATIGUE ON THE RESIDUAL STRENGTH OF COMPOSITE MATERIALS*

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ABSTRACT

This paper presents the results of an investigation into the effects of fatigue on the residual strength of a $0^\circ/90^\circ$ CFRP composite. Cycling was carried out at five different stress levels over a range of 60 % to 90 % of the ultimate tensile strength of the composite.

For a given stress level a relationship has been derived between the residual strength and the fatigue life of the composite. This relationship the fatigue characteristics for any given stress level without the need for time consuming and expensive testing.

It has been shown that stress concentrations affect a crossplied laminate to a lesser extent than unidirectional laminates and homogeneous materials.

NOTATION

	DESCRIPTION	UNITS
d	Damage ratio	No units
GPa	Gigapascals	$\times 10^9$ N/m ²
Hz	Hertz	Cycles/second
k	Stress intensity factor	No units
N	Number of cycles	Cycles
Nf	Fatigue life	Cycles
P	Load	KN
r	Stress ratio	No units
t	Cycle ratio	No units
Vf	Fibre volume fraction	%

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η	Efficiency factor	%
σ	Cyclic stress	N/m^2
σ_{app}	Applied stress	N/m^2
σ_{av}	Average stress	N/m^2
σ_c	Composite strength	N/m^2
σ_F	Ultimate tensile strength	N/m^2
σ_{Fi}	Strength of fibre	N/m^2
σ_m	Strength of matrix	N/m^2
σ_{max}	Maximum stress	N/m^2
σ_{mean}	Mean stress	N/m^2
σ_{min}	Minimum stress	N/m^2
σ_r	Residual strength	N/m^2

1. INTRODUCTION

1.1. General Introduction

This report presents the results of a program of research to investigate the effects of fatiguing on the residual strength of a carbon fibre reinforced plastic.

The purpose of this experimental program has been to study the effect of stress concentrations (in this study holes) on the fatigue behaviour of the reinforced plastic laminates, at several different stress levels ranging from 60 % to 90 % of the ultimate tensile strength of the material. From previous research¹ the axial load fatigue properties of carbon fibre composites in both unidirectional and crossplied forms have been found to be excellent. The data, however, was based mainly around unnotched specimens and there is a need at present to collect further data on the fatigue behaviour of the material containing stress concentrations and holes. The fatigue loading was tension-tension to ensure no compressive loads being applied to the specimens. The compressive strength of the material is substantially lower than the tensile strength² and so any compressive loading would affect results.

1.2. Material

A composite consists of at least two distinct intimately mixed components known as the reinforcement (fibre) and the matrix (epoxy resin). Figure 1 shows the individual characteristics of both the fibre and the resin and also their combined characteristic for two different fibre volume fractions.

Practical advantages of using high performance composite materials are many and varied. For instance, composites can have very high specific strengths and stiffnesses [TABLE 1(a)] because of their low densities and good mechanical properties.

The term 'high performance composite' describes orientated fibrous composites based on high strength or high modulus fibre or both which contain 50 % or more by volume of reinforcement.

The produce high performance composites, fibres are usually aligned to give a unidirectional laminate. The laminae may then be stacked with the fibre axis at different angles to a reference direction to make a laminated plate. This gives improved transverse properties with fibres helping to take the transverse load as well

as the matrix although, of course, strength and stiffness in other directions are somewhat reduced for the same weight of the components [TABLE 1(a)].

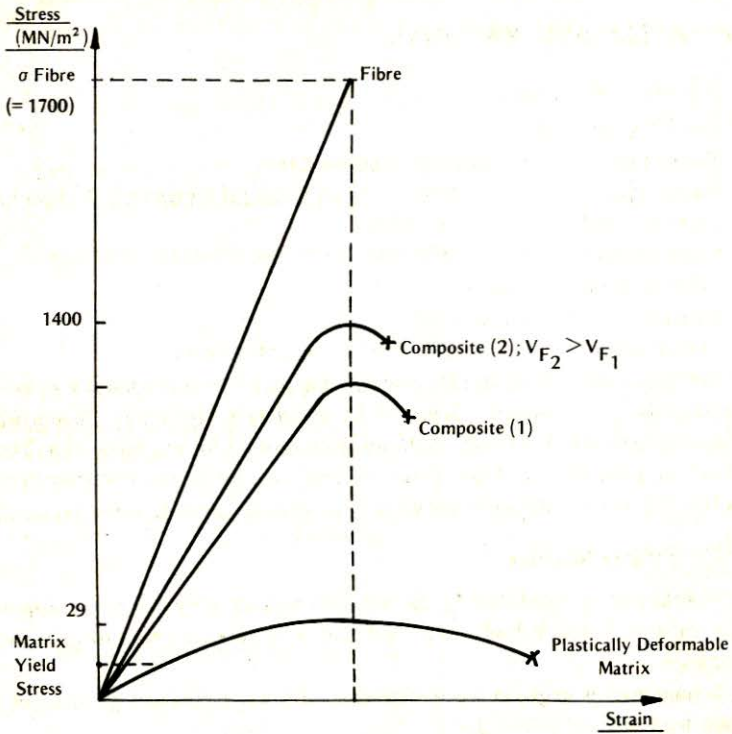


Figure: 1 – Graph of Stress v's strain for CFRP and its constituents

Table: 1 (a)

Material	Density ρ (kN /M)	Tensile Strength σ (GN/M ²)	Specific Strength σ/ρ (Mx10 ³)	Tensile Stiffness E (GN/M ²)	Specific Stiffness E/ρ (Mx10 ⁶)
CFRP 0°/90° Vf = 53.9 %	14.8	0.65	43.9	62.9	4.25
CFRP 0° Vf = 60 %	15.0	1.40	93.3	130.0	8.7
Epoxy Resin	11.9	0.029	2.4	3.38	0.28
High Tensile Steel	78.0	1.55	19.8	210	2.7

The strength and stiffness of components made from high performance fibre reinforced plastics are not directly related to the cross-sectional area of the material so much as to the fibre content³, and so the volume fraction becomes a main parameter in the definition of a composite.

2. APPARATUS AND MATERIAL

2.1. Material

Specification

Fibre type	—	T 300 Carbon fibre
Resin type	—	Epoxy resin, Fothergill Rotorway Composites Ltd.
Fibre strength	—	1.62 GPa
Fibre modulus	—	140 GPa
Fibre volume fraction, V_f		$= 53.9\%$
20 laminates crossplied		$0^\circ-90^\circ$
Curing process	—	120°C for 1 hour at 50 p.s.i.

The test pieces used in this research program were cut from a sheet of carbon fibre-epoxy resin composite, supplied by Fothergill Rotorway Composites Ltd to the above specification. The sheet dimensions were 300 mm square by 3 mm (nominal) thick and the different test piece dimensions are shown in Figure 2(a). The test pieces had 2.5 mm diameter holes drilled in them to provide stress concentrations.

2.2. Fatigue Machine

The fatigue rig used during the research was an Avery/Shenck fatigue machine (60 KN capacity) which had been modified over several years to give the required performance.

A specimen is gripped by wedge-shaped jaws, tightened by a tapered pin and an Allen head set bolt [FIG. 2(c)].

A static load is applied, first manually, and then by tensioning a core spring using a motor driven chain. An alternating load is applied through an annular spring excited by means of a force which is produced by an out of balance rotating mass. The alternating force is controlled by adjusting the demand potentiometer on the control panel.

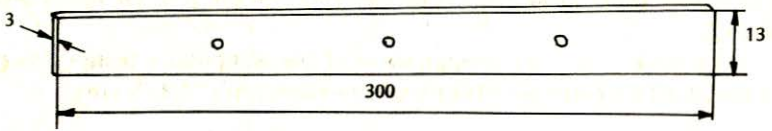
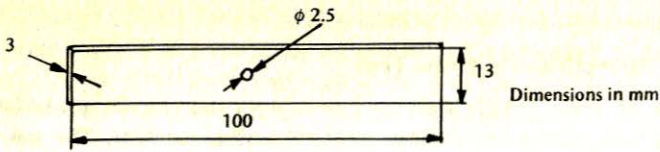
The alternating load cycles at 36 Hz.

A circuit-breaker acts as a safety device switching off the alternating load, when triggered by the test specimen breaking or slipping in the jaws.

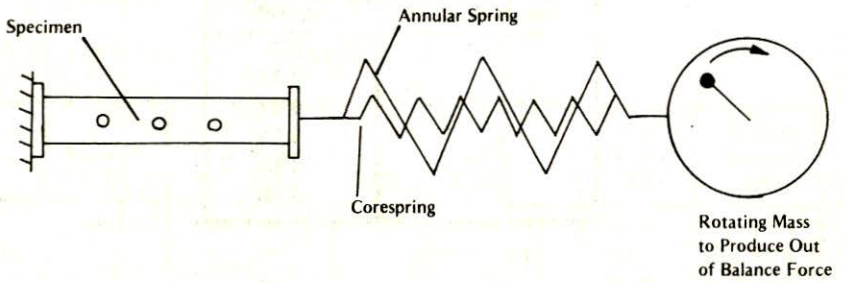
2.3. Static Test Machine

An Avery-Dennison static tensile test machine was used to test the residual strength of the specimens having been fatigued for certain proportions of their fatigue life. This machine can produce a static load of up to 50 KN through a hydraulically-operated crosshead and a fixed platten.

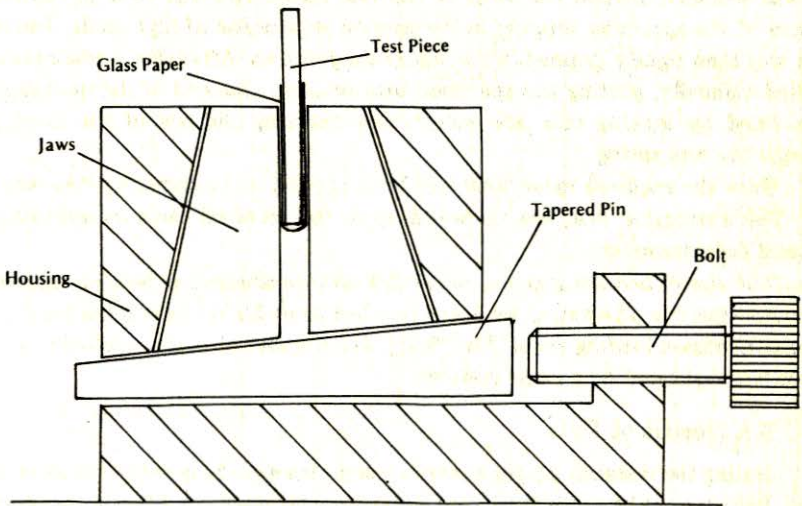
For static tensile stiffness tests, two extensometers were used; a Mercer extensometer (Model G165), and an Avery-Dennison extensometer (Model 7609C1).



(a) – Dimensions of specimens used in testing



(b) – Representation of rig



(c) – Method of gripping of specimen

Figure: 2

3. METHOD

3.1. Static Strength and Stiffness Tests

A number of static tensile tests to failure were carried out on unfatigued holed specimens using the Avery Dennison tensile testing machine. The specimens were gripped vertically in wedge jaws and a steadily increasing load was applied hydraulically until failure. The failure load was recorded and the cross-sectional area at failure point measured. From the load and the area the failure stress was calculated.

From these values an average value of the composite's tensile strength, σ_F , was calculated for specimens with stress concentrations [TABLE 1(b)].

Table: 1 (b)

Tensile Test	σ_F (GPa)
1	0.5
2	0.43
3	0.51
4	0.52
5	0.48
6	0.46

3.2. Fatigue Tests

Before being gripped in the jaws of the fatigue machine, glass paper (facing in) was wrapped around the ends of the test piece. This was done to reduce the chance of the specimen slipping in the jaws on application of high loads. The specimen was then tightly gripped in the wedge-shaped jaws. After this, a small load was applied manually, putting the specimen into tension. One end of the specimen was then fixed by locking two side nuts before applying the rest of the mean load through the core spring.

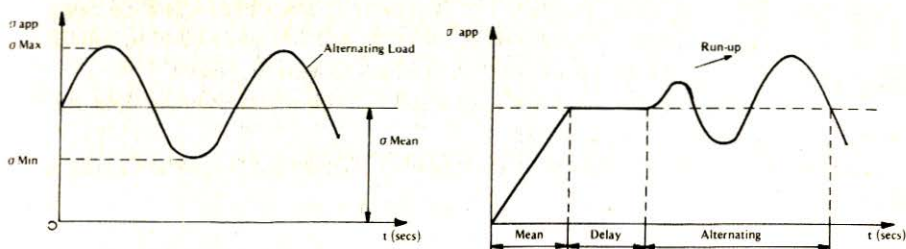
Once the required mean load had been applied, the alternating load was applied. The alternating load was increased up to the required value by adjusting the demand potentiometer.

The circuit-breaker gap was set at 2-3 mm. In the fatigue tests on carbon fibre composites the alternating load was selected to be 50 % of the mean load, giving a tension-tension loading cycle [FIG. 3(a)]. The number of cycles applied to the specimen was registered by a circle counter.

3.3. Program of Tests

During the research program several test pieces were fatigued at the same stress level. For a particular stress level, test pieces were fatigued for different proportions of their fatigue life before being loaded to failure in the Avery-Dennison tensile testing machine [TABLE 2].

The stress at which these fatigued specimens failed was their residual strength, σ_r , after being fatigued at a particular stress level, σ for a number of load cycles, N . Five different stress levels were used during the program of research, ranging from 60 % to 90 % of the composite's ultimate tensile strength, σ_F .



(a)- Illustration of tension-tension loading cycle

(b)- Actual run-up to loading cycle

Figure: 3

Table: 2

Test	Maxload KN	Mean KN	Alt KN	DVM	No. OF Cycles	Failure Load KN	Residual Strength GPa
FT2 No hole	13.5	8.5	± 5	3.1	5.021×10^6	20.0	0.605
					0.81×10^7	20.7	0.590
					1.182×10^7	20.9	0.569
					2.063×10^7	18.1	0.515
FT3 * Fatigue Failure	12.1	8.1	± 4	2.5	1.056×10^7	16.5	0.505
					1.30×10^7	16.0	0.472
					1.432×10^7	14.6	0.459
					2.063×10^7 *	12.1	0.37
FT4	10.5	7.0	± 3.5	2.2	8.16×10^6	13.8	0.458
					1.323×10^7	14.1	0.409
					1.924×10^7	14.0	0.437
					1.2×10^5	15.1	0.468
FT5	9.0	6.0	± 3.0	1.9	1.1×10^4	15.7	0.480
					8.22×10^6	14.5	0.440
					1.314×10^7	13.8	0.396
					1.962×10^7	14.0	0.424
FT6 * Fatigue Failure	14.0	9.0	± 5	3.0	7.973×10^6	14.9	0.493
					1.301×10^7	16.5	0.471
					2.05×10^7 *	14.0	0.438

The maximum applied stress, σ , is calculated by dividing the maximum applied load by the cross-sectional area of the specimen at the hole.

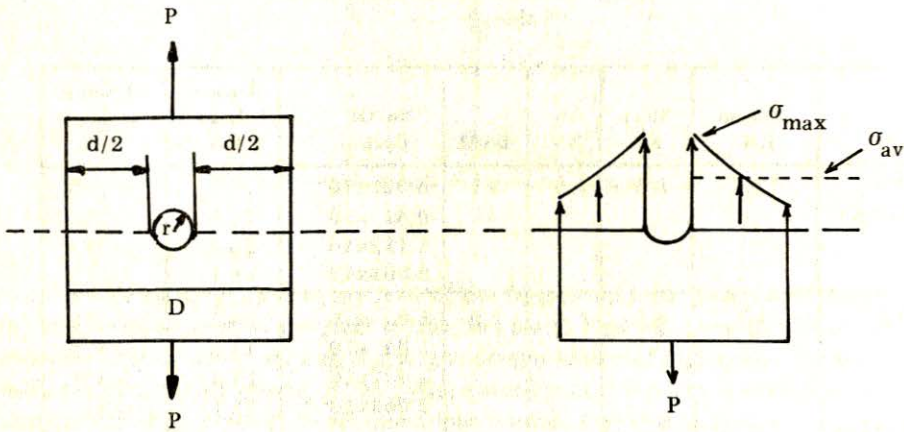
Early on in the research, single specimens were used but, to speed up the results, long strips of the composite, which contained three holes, were used [FIG. 2(a)]. Fatiguing one strip was equivalent to fatiguing three specimens in series at the same time. By temporarily stopping the fatigue test, specimens could be sawn off from the strip at suitable proportions of their fatigue life and loaded to failure in the tensile testing machine to discover their residual strength, as before.

By replacing the single test piece with a strip, three times as much data can be collected in the same amount of time.

Loading a specimen to failure is equivalent to failure by fatigue after half a loading cycle.

4. THEORY

4.1. Stresses at a circular hole — or perfectly homogeneous material⁴



When a specimen containing a hole is loaded, high localised stresses occur. The above diagrams show the distribution of stresses in a flat test piece with a circular hole.

The stresses are independent of the size of the specimen and of the material used; they only depend upon the ratios of the geometric parameters involved, i.e. upon the ratio r/d . The maximum stress occurs at the edges of the hole and using the ratio defined as

$$k = \frac{\sigma_{\max.}}{\sigma_{av}}$$

where k is the stress concentration factor of the hole, and assuming a linear relationship between stress and strain, it can be shown that k can have a maximum value of 3 at the edge of the hole.

$$\text{i.e. } \sigma_{\max.} = 3 \times \sigma_{av}$$

4.2. Strength of Composites⁵

A composite consists of two or more materials combined on a macroscopic scale to utilise the best properties of each and often qualities that neither possess alone.

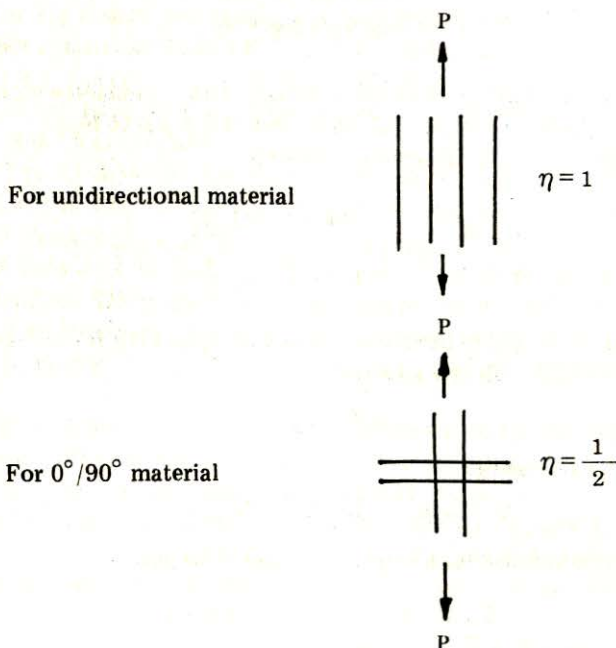
A stress and strain plot (Fig. 1) demonstrates the different properties of the fibres and the matrix before and after being combined. The composite combines the high strength of the carbon fibres with the bonding properties of the matrix which binds the load-carrying fibres. An increase in the fibre volume fraction leads to an increase in the tensile strength of the composite and also its brittleness.

The tensile strength of the composite is given by the following equation:

$$\sigma_c = \eta \sigma_{Fi} V_F + \sigma_M (1 - V_F)$$

where η is the efficiency factor which takes account of the fibre orientation. However, this equation does not account for any reduction in strength of the matrix which may occur during fatigue.

E.g.



(These values of η ignore the strength of the resin).

4.3. Normalisation of fatigue results and curve fit⁶

The wear out model used to estimate values of N_f can be seen in Fig (9). This uses the reasoning that failure will eventually occur when the value of the residual strength falls to the value of the maximum applied cyclic stress.

Since there is a similarity in the shapes of the fatigue curves using this model (and with actual curves where failure occurred through fatigue) for different stress levels, it is possible that some normalising factor can take account of the stress dependence of the rate damage accumulation.

The residual strength σ_r , has limits of σ_F , the ultimate tensile strength, and σ , the maximum applied value of the cyclic stress at each particular stress level. The abscissa represents N , the number of cycles at the given stress level, and the limits are $\frac{1}{2}$ (ie N_F is $\frac{1}{2}$ cycle for static tensile test) and N_f the number of cycles at which failure will eventually occur at any given stress level.

The stress levels are normalised using the factor,

$$\frac{(\sigma_r - \sigma)}{(\sigma_F - \sigma)}$$

The fatigue life values are normalised using the factor,

$$\frac{(\log N - \log \frac{1}{2})}{(\log N_F - \log \frac{1}{2})}$$

This enables all data to be recorder on axes with a common datum. The limit of both factors in unity (i.e. σ_r is equal to σ_F and N is equal to N_F).

The general form of the interaction curve is

$$\left(\frac{\sigma_r - \sigma}{\sigma_f - \sigma}\right)^y + \left(\frac{\log N - \log \frac{1}{2}}{\log N_F - \log \frac{1}{2}}\right)^x = 1$$

For correct x, y , the experimental values of σ_r and N_F for each point on the normalised curve (A, B) will give an error,

$$e_i = A_i^x + B_i^y - 1,$$

the sum of the squares being,

$$S = \sum e_i^2 = \sum (A_i^x + B_i^y - 1)^2$$

This sum is minimised with respect to x and y , so that

$$\sum (A_i^x + B_i^y - 1) A_i^x \ln A_i = 0$$

or

$$\sum A_i^{2x} \ln A_i + \sum A_i^x B_i^y \ln A_i - \sum A_i^x \ln A_i = 0$$

and

$$\sum B_i^{2y} \ln B_i + \sum A_i^x B_i^y \ln B_i - \sum B_i^y \ln B_i = 0$$

Solving these equations the best form of the interaction curve can be obtained.

Another way of representing the normalised results is in terms of damage, d , where $d = 1 - r$. This is useful to see when the damage is being done to the material as it undergoes fatigue.

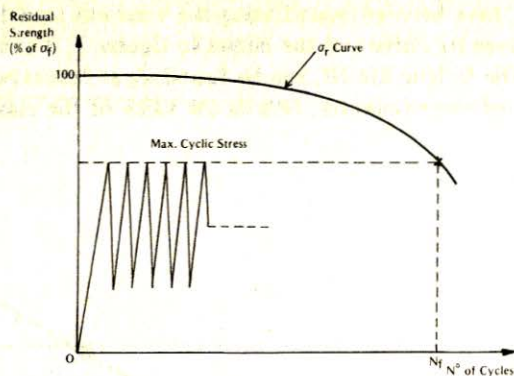


Figure: 4 – Schematic illustration of wear out model for composites

5. PRESENTATION OF RESULTS

Figures 5 and 6 show the variation of load with extension for specimens of the carbon fibre composite. They are both linear plots passing through the origin. From the slopes of the lines the values of the tensile stiffness, E , were calculated to be $62.9 \times 10^9 \text{ N/m}^2$ and $63.5 \times 10^9 \text{ N/m}^2$ respectively. Therefore, an average value of $63.2 \times 10^9 \text{ N/m}^2$ was obtained.

Figures 7 to 11 show the reduction in strength of the carbon fibre composite during fatigue at five different stress levels over a range from 60 % to 90 % of its ultimate tensile strength [TABLE 2].

The first fatigue curve [FIG. 7] shows data points from specimens without stress concentrations. Three specimens were fatigued at a stress of 0.37 GPa and their residual strengths found. No fatigue failure was obtained so the wear out model was used to predict the fatigue life. The value of N_f obtained was 5.3×10^7 cycles.

Figures 8 to 11 are curves for tests on specimens with stress concentrations. Figure 8 was obtained from six data points. The maximum applied stress was 0.37 GPa. Failure due to fatigue occurred after 2.06×10^7 cycles.

Figure 9 shows a similar curve obtained from five data points. The maximum applied stress was 0.35 GPa, which was lower than that in the previous test. A fatigue failure was not obtained and again the wear out model was used to give an estimate of the fatigue life N_f of 2.1×10^7 cycles.

Figure 10 is the fourth curve, this time obtained from five residual strength data points. The maximum applied stress was 0.32 GPa. The wear out model was used to predict an estimate for N_f of 2.9×10^7 cycles.

Figure 11, the final curve, was obtained from a maximum applied stress of 0.438 GPa. Four data points were used and a fatigue failure occurred at 2.05×10^7 cycles.

Figures 8 to 11 give a range of values of N_f of $2.05 \times 10^7 - 2.9 \times 10^7$ cycles. Two of these, [FIGS. 9 and 11] are actual fatigue failures. The other values of N_f

(FIGS. 7, 8 and 10), have been estimated using the wear-out model. The wear-out model can be used since its curve and the curves in figures 7, 8 and 10 are similar and so estimates of the fatigue life N_f , can be found since failure occurs when the residual strength, σ_r , of the composite, falls to the value of the maximum applied cyclic stress.

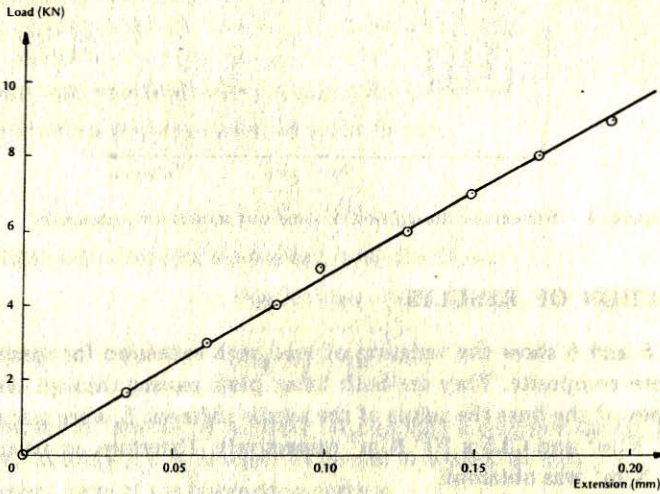


Figure: 5 – Graph of load vs extension for calculation of modulus, E

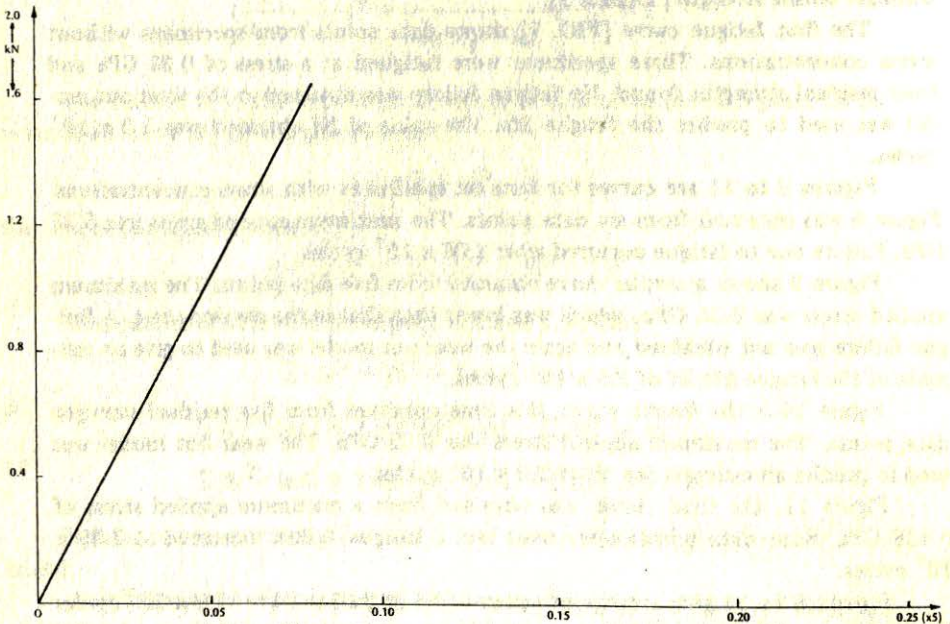


Figure: 6 – Graph of load vs extension up to failure

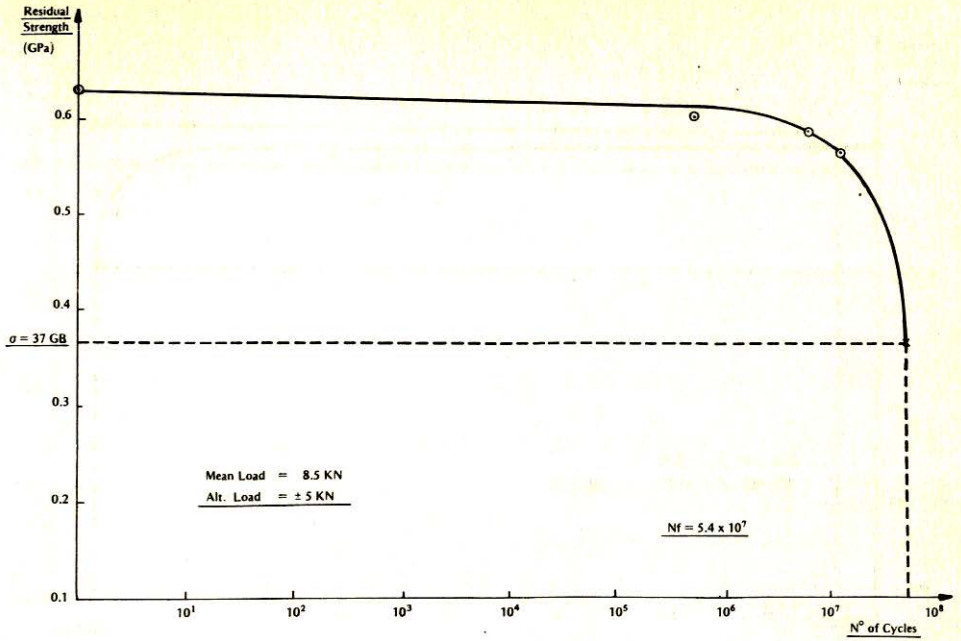


Figure: 7 – Graph of residual strength v's number of Cycles for FT2 (without hole)

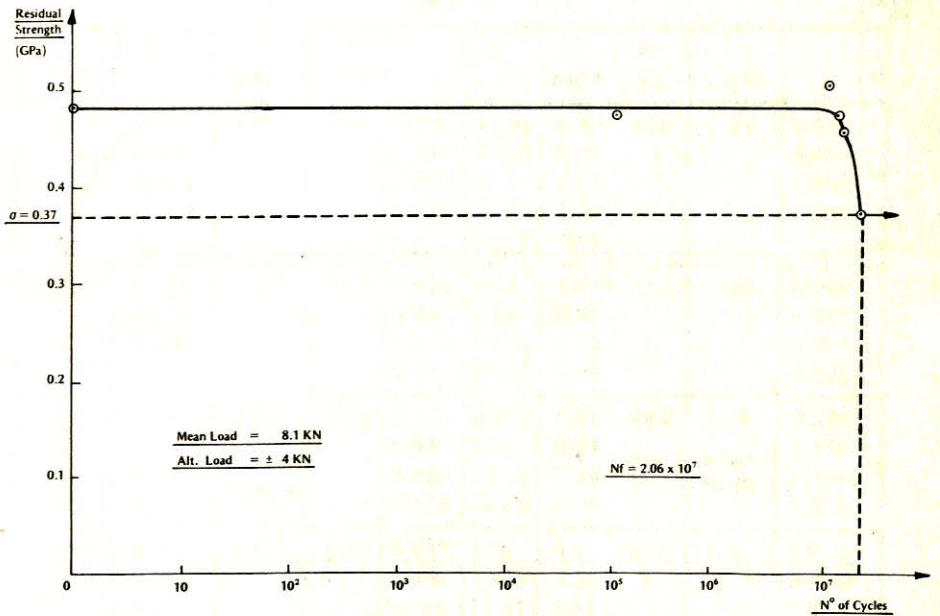


Figure: 8 – Graph of residual strength v's N⁰, of cycles for FT3

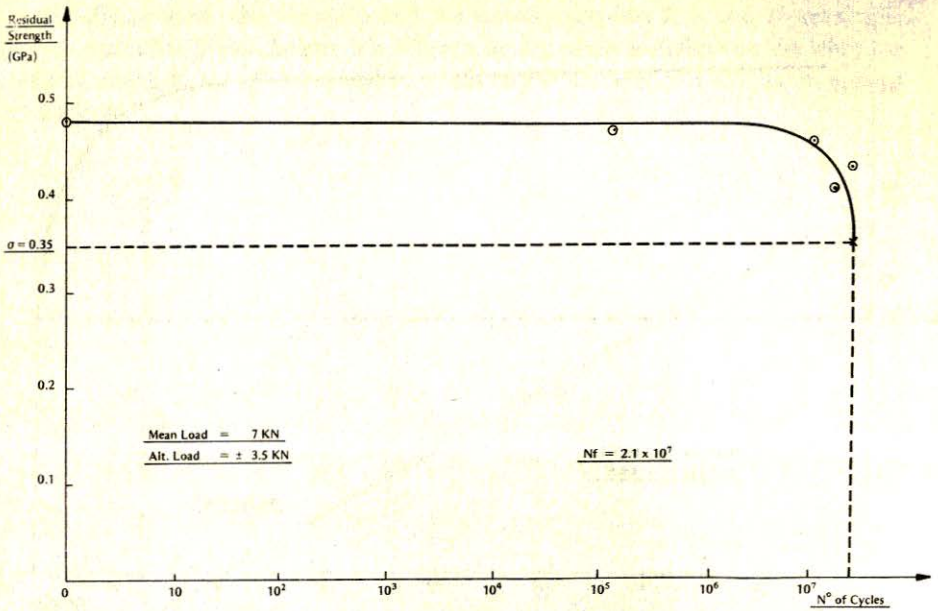


Figure: 9 – Graph of residual strength v's N^0 of cycles for FT 4

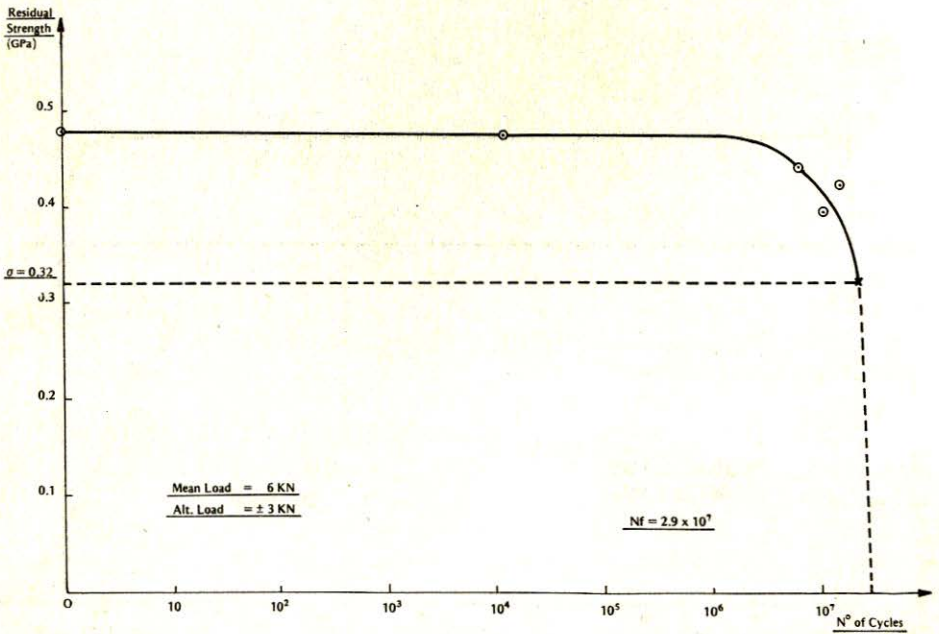


Figure: 10 – Graph of residual strength v's N^0 of cycles for FT 5

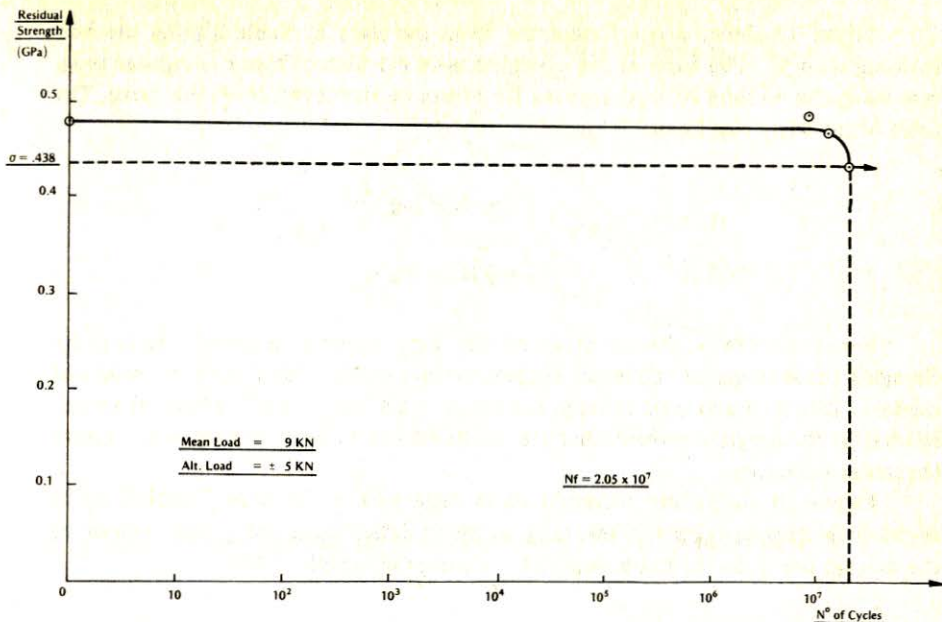


Figure: 11 – Graph of residual strength v's N^0 of cycles for FT 6

Table: 3

Test	σ_r GPa	σ GPa	σ_f GPa	r	N $\times 10^6$	Log N	Nf $\times 10^6$	Log Nf	t
FT3	0.47	0.37	0.48	0.909	0.105	5.02	20.6	7.32	0.698
	0.505			1.227	10.56	7.02			0.96
	0.472			0.927	13.0	7.11			0.982
	0.459			0.809	14.32	7.16			0.980
	0.37			0	20.63	7.31			1
FT4	0.468	0.345	0.48	0.911	0.12	5.079	21.0	7.32	0.706
	0.458			0.84	8.13	6.91			0.946
	0.409			0.474	13.23	7.12			0.974
	0.437			0.68	19.24	7.28			0.995
FT5	0.48	0.32	0.48	1	0.011	4.04	29.0	7.46	0.559
	0.44			0.69	8.22	6.91			0.929
	0.396			0.359	13.14	7.12			0.956
	0.424			0.573	19.62	7.28			0.978
FT6	0.478	0.438	0.48	0.952	0.04	4.6	20.5	7.31	0.644
	0.493			1.309	7.973	6.90			0.946
	0.471			0.786	13.01	7.11			0.974
	0.438			0	20.5	7.31			1

Figure 13 shows a curve obtained from the data in Table 3 using the normalising theory⁶. The form of the curve has been calculated from a computer program using the method of least squares for errors in the points from the curve. The form of the curve was found to be:

$$\left(\frac{\sigma_r - \sigma}{\sigma_f - \sigma}\right)^{12.4} + \left(\frac{\log N - \log \frac{1}{2}}{\log N_f - \log \frac{1}{2}}\right)^{1.9} = 1$$

Figure 14 shows the variation of the sum of errors squared obtained by changing those values of the initial approximations of N_f , found using the wear-out model [FIGS. 7, 8 and 10], through a range of -20% to $+60\%$. These errors calculated by the program showed that the minimum error occurs at a value 5% above the initial estimate.

Figure 15 shows the variation of damage with cycle ratio [TABLE 4]. It shows little damage occurs in the early stages of fatiguing and that the majority of the damage occurs in the latter stages of the composites fatigue life.

Table: 4

Stress Ratio r	Damage $d = (1 - r)$	Cycle Ratio t
0.909	0.091	0.698
1.227	- 0.227	0.960
0.927	0.073	0.972
0.809	0.191	0.980
0	1.000	1.000
0.911	0.089	0.706
0.840	0.160	0.946
0.474	0.526	0.974
0.680	0.320	0.995
1.000	0.0	0.559
0.690	0.310	0.929
0.359	0.641	0.956
0.573	0.427	0.978
0.952	0.048	0.644
1.309	- 0.309	0.946
0.786	0.214	0.974
0.0	1.00	1.000

It can be seen that up to 90% of its fatigue life, only 7% damage has occurred and after 98% of its life there is 60% damage.

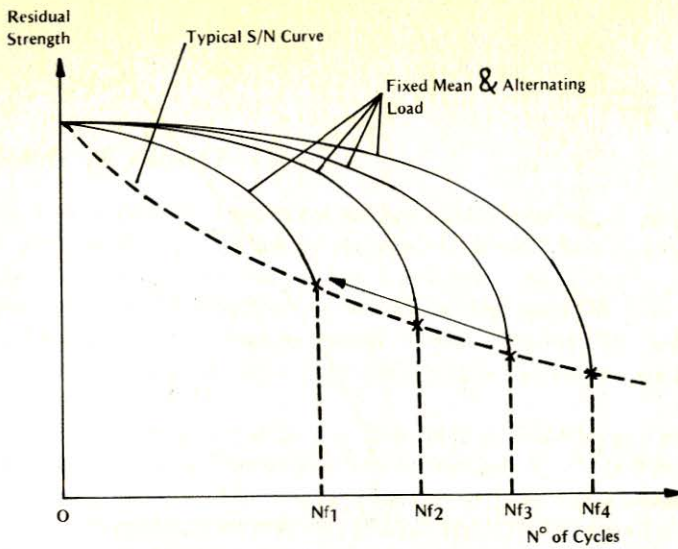
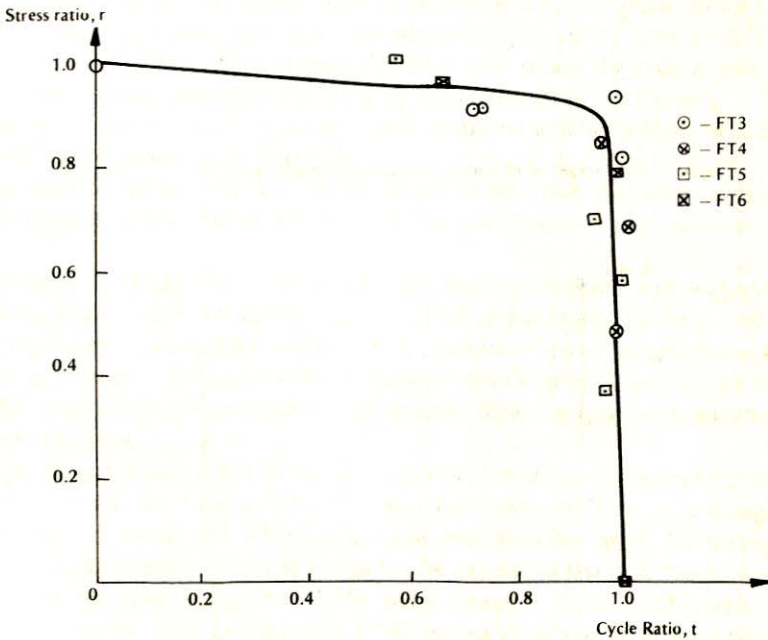


Figure: 12 – Typical graph of residual strength vs N^0 of cycles for a range of stress levels used to obtain the S/N curve



General form of interaction curve is,

$$\left(\frac{\sigma_r - \sigma}{\sigma_f - \sigma} \right)^{12.4} + \left(\frac{\log N - \log 1/2}{\log N_E - \log 1/2} \right)^{1.9} = 1$$

Figure: 13 – Normalised stress ratio vs cycle ratio for CFRP ($0^0/90^0$) fatigued at several stress levels

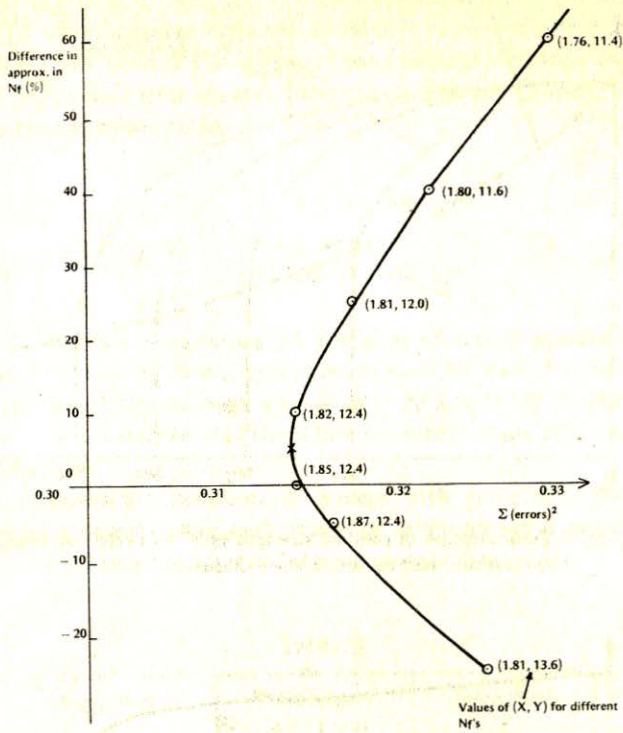


Figure: 14 – Graph of difference in approximations of N_f v's $\Sigma (\text{errors})^2$

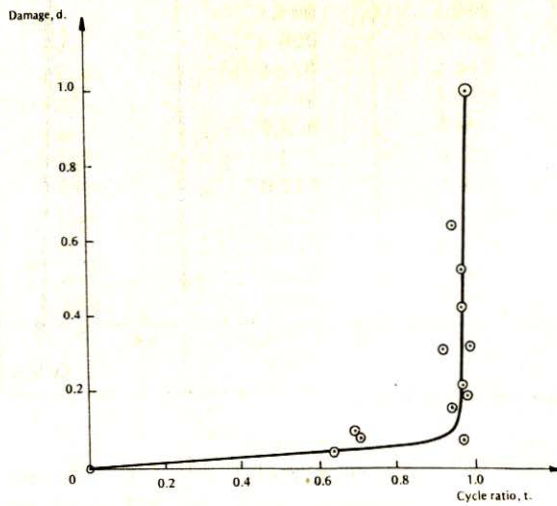


Figure: 15 – Graph of damage v 's cycle ratio

6. DISCUSSION OF RESULTS

Figure 3 (a) shows the theoretical loading cycle; however, in practice, the mean load is increased gradually up to the required level. There is a time lapse before applying the alternating load which is also increased gradually up to the required level [FIG. 3(b)]. Clearly, the loading on the specimen over the first few cycles, the run-up, is different in practice to that in theory. This would only have a significant effect on high stress, low cycle fatigue and so does not influence the results presented in this work.

From figures 5 and 6 it can be seen that there is no apparent yielding and this leads to a brittle failure. The values of tensile strength, $\sigma_F = 0.65$ GPa and average tensile stiffness, $E = 63.2$ GPa can be compared with the values obtained for a similar specimen of unidirectional lay-up [TABLE 1(a)], where $\sigma_F = 1.4$ GPa and $E = 130$ GPa. The reason for the difference is that the $0^\circ/90^\circ$ material has half the number of fibres in the 0° direction. Thus the tensile strength is halved although the strain, ϵ , is equal. Therefore, the tensile stiffness is also halved.

From the specification of the material in section 2.1, the tensile stiffness of the fibre is 140 GPa. From Table 1(a), the stiffness of the epoxy resin is 3.38 GPa and the overall stiffness of the composite is 62.9 GPa. Hence the resin lowers the overall stiffness of the composite which can be seen more clearly in Figure 1.

The wear out model gives values which compare well with those obtained from the fatigue failures [FIGS. 8 and 11].

The ratio of the applied stresses, σ , are not in the same proportion as the ratio of the applied loads. This is due to a variation in cross-sectional area over the sheet.

Figures 7 to 11 show that the carbon fibre laminate retains a high proportion of its strength until close to failure. Also, at high stress levels the fatigue life is reduced. In Figure 7, since there are no stress concentrations in the specimens, the applied stress was low compared with its ultimate tensile strength and so the estimate of N_f is significantly higher than those obtained from the specimens containing stress concentrations.

From figures 8 to 11, the stress concentration lowers the ultimate strength of the laminate. From the theory, however, the stress intensity factor of a homogeneous material (i.e. steel) is 3, whereas the value calculated for the $0^\circ/90^\circ$ laminate is 1.3. This is significantly lower. It is due to the individual fibres in the laminate resisting crack propagation and hence the stress intensity factor is effectively reduced. Thus, a cyclic stress applied to a CFRP laminate with stress concentrations can be a higher proportion of its ultimate tensile strength than for a homogeneous material with a similar stress concentration.

A stress concentration lowers the strength but should not affect the value of N_f^8 . However, in Figure 7, the value of N_f is twice that of N_f in Figure 8. This could just be due to the stress level in Figure 7 being a smaller proportion of its ultimate strength compared with the stress in Figure 8.

The effect of stress concentrations in unidirectional laminates is much more evident where the stress intensity at the edge of the hole caused shearing of the laminae and hence failure by 'shear-out'. This occurs because there are no cross-fibres and hence no resistance to crack propagation in the 0° directions. This is not the case in the $0^\circ/90^\circ$ laminate used during this research.

From the values of N_f in Figures 8 to 11, it should be possible to derive a relationship between the fatigue, N_f , and the maximum cyclic stress, σ [FIG. 12]. However, in Figures 8 to 11 all values of N_f occur in a small range between 2.05×10^7 and 2.9×10^7 cycles. Thus, to obtain a realistic relationship, more results are required for values of N_f between 10^5 cycles and 10^7 cycles.

Since there is a similarity between the curves in Figures 8 to 11, a normalising factor can be used which enables all points from the curves to be plotted on a single curve [SECTION 4.3]. The form of the curve fits all the residual strength data closely. The form of the curve is as expected from previous research work⁶.

Figure 14 is obtained from the theory of sum of least squares applied to data points in Figure 13. The estimates of N_f are shown to be in the right range. In fact, the optimum values of N_f are only 5 % greater than the initial estimates. From the design point of view, the estimates are more than satisfactory. The values of fatigue life, N_f , have been underestimated and, therefore, there is an inherent safety factor.

Figure 13, the damage curve, is possibly an alternative way of representing the results of a normalised curve. In this figure it is obvious that the majority of the damage is done in the latter stages of its life, which supports general theory that the CFRP retains its strength until close to the end of its life.

7. CONCLUSIONS

The reduction of residual static strength of carbon fibre reinforced plastic laminates due to fatiguing at different stress levels has been investigated. The residual strength results at various stress levels and different proportions of their fatigue lives have been normalised to give a relationship between the residual strength and the fatigue life of the composite. Using this relationship, for a given stress level, only one test is required of its residual strength at a certain number of cycles, to estimate the fatigue life at this particular stress level. However, with further research and more data, a relationship could also be derived between the maximum applied cyclic stress and the fatigue life.

For design applications of the material this would be very useful, especially as the collection of fatigue data is time-consuming and expensive.

From the research, it has also been shown that stress concentrations affect $0^\circ/90^\circ$ laminates to a lesser extent than homogeneous materials and unidirectional laminates, in both static and fatigue tests.

The static tensile strength is an average value obtained from several static tests which give a statistical distribution with a standard deviation of 0.03 GPa (± 6.5 %).

The errors from the normalised curve fit support the validity of the 'wear-out' model used for estimating the fatigue life of the composite.

8. CALCULATIONS

Calculation of tensile stiffness, E

(a) From Figure 5,

$$\begin{aligned}\text{Tensile stiffness, } E &= \frac{\text{Stress}}{\text{Strain}} \\ &= \frac{F/A}{e/l} \quad \begin{array}{l} A = 0.379 \times 10^{-4} \text{ m}^2 \\ l = \text{gauge length} \\ = 50 \times 10^{-3} \text{ m.} \end{array} \\ &= \frac{9.53 \times 10^3 / 0.379 \times 10^{-4}}{0.2 \times 10^{-3} / 50 \times 10^{-3}} \\ E &= 62.9 \times 10^9 \text{ N/m}^2\end{aligned}$$

(b) From Figure 6,

$$\begin{aligned}E &= \frac{17.5 \times 10^3 / 0.336 \times 10^{-4}}{0.41 \times 10^{-3} / 50 \times 10^{-3}} \\ E &= 63.5 \times 10^9 \text{ N/m}^2\end{aligned}$$

Average value of

$$E = 63.2 \times 10^9 \text{ N/m}^2$$

Calculation of Specific strength and stiffness

$$\begin{aligned}\text{Density of material, } \rho &= 1480 \text{ kg/m}^3 \\ &= 14.8 \text{ kN/m}^3\end{aligned}$$

$$\begin{aligned}\text{Specific strength, } \frac{\sigma}{\rho} &= \frac{0.65 \times 10^9}{14.8 \times 10^3} \\ &= 43.9 \times 10^3 \text{ m}\end{aligned}$$

$$\begin{aligned}\text{Specific stiffness, } \frac{E}{\rho} &= \frac{63.2 \times 10^9}{14.8 \times 10^3} \\ &= 4.25 \times 10^6 \text{ m}\end{aligned}$$

Calculation of average tensile strength

From Table 1(b)

$$\begin{aligned}\text{Average tensile strength, } \sigma_F &= 0.5 + 0.43 + 0.51 + 0.52 + 0.48 + 0.4 \\ \sigma_F &= 0.48 \times 10^9 \text{ N/m}^2\end{aligned}$$

Calculation of stress intensity factor

$$\text{For laminate without hole, } \sigma_F = 0.63 \times 10^9 \text{ N/m}^2$$

and with hole, $\sigma_F = 0.48 \times 10^9 \text{ N/m}^2$

$$\text{Stress intensity factor of hole, } k = \frac{0.63}{0.48}$$

$$k = 1.3$$

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