Volume 3, No. 1, 1989

Journal of the Faculties of
Engineering of Uludağ University

THE INVERSION METHOD IN QUASI-OPTICAL SCATTERING OF ELECTROMAGNETIC WAVES FROM PERFECTLY CONDUCTING CURVED SURFACES*

H. Ergun BAYRAKÇI**

ABSTRACT

In this study ($\vec{E}$, $\vec{H}$) fields have been calculated by applying the inversion method on the ray paths in the quasi-optical scattering phenomena of infinite and finite sourced electromagnetic waves from perfectly conducting curved surfaces.

ÖZET

Elektromagnetik Dalgaların Düzgün Eğrisel Mükemmel İletken Yüzeylerden Optik Benzeri Saçılmasında Enversiyon Yöntemi

Bu çalışmada, sonsuz ve sonlu kaynaklı elektromagnetik dalgaların düzgün eğrisel mükemmel iletken yüzeylerden optik benzeri saçılmasında ($\vec{E}$, $\vec{H}$) alanlarına ait büyüklükler, optik ışın yolları üzerine enversiyon yöntemi uygulanarak bulunmuştur.

INTRODUCTION

A quasi-optical scattering problem, by using of the inversion method can be transformed in to a problem which can be easily solved or a know solution in

---

* *The thesis of Ass. Prof., 1981.*

** *Uludağ University, Engineering Faculty, Department of Electronic Engineering BURSA.*

illuminated region. After having solved the simple problem the solutions of the original problem can be found by an inverse transformation.

On the other hand, a quasi-optical scattering problem, a geometrical inversion and a physical inversion are applicable in order to find the field expressions of reflected waves from a perfectly conducting scattering surfaces. This method is applied for the first time in this study for quasi-optical scattering prob lems and physical inversion is based upon the equiphase curves on specific plane of equiphase surfaces. The physical inversion, in the quasi-optical solution is based upon the property of the reflected waves from a perfectly conducting curved surface. The curved surface does not change their direction and magnitude although the coordinates of the coordinate of the source is transformed or changed.

The coordinate transformation which is mentioned in this study is being realized according to the integral transformation in the application of Poisson-Summation formula for the exact solution. The physical inversion can be considered boundary conditions in the quasi-optical solution. Briefly, in the physical inversion, it has been shown that linear and circular equiphase curves occur in the direction of reflected wave. As a result of the field companent pertaining to waves reflected from the perfectly conducting surface has been obtained.

1. TWO DIMENSIONAL GEOMETRIC INVERSION

In this study, two dimentional inversion is considered. On the other hand this may also be called "Inversion on the plane". For example, the two dimensional geometrical inversion is obtained by the making use of a circle at he (oxy) plane of circular cylindirical coordinates system and at the plane Φ = constant is spherical coordinates system. As shown in Figure 1 a is defined as the distance between the points. A and 0 (the center of the circle) and then relationship R^2 = ab holds, b is the distance between B and 0, then the points A and B are the inverses of each other respect to the inversion center 0. The point 0 is called as the radius R is defined as the inversion radius[1].

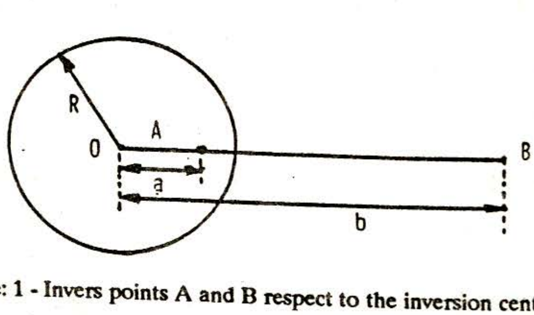

Figure: 1 - Invers points A and B respect to the inversion center 0

The inversion of a circle with respect to an inner tangent circle:

The inversion of a circle with respect to an inner tangent circle is a circle which is also tangent from inside.

In Figure 2 three circles with given geometry and inner tangent are considered. By the using the geometry in Figure 2.

$$\frac{2}{R_2} = \frac{1}{R_1} + \frac{1}{R_3} \tag{1.1}$$

is obtained. Consequently, the inversion of the circle A_n with respect to the circle D_n with respect to the center O as R taken as the inversion radius is the circle B_n. Therefore, the inversion of a circle with respect to another circle tangent from inside is a circle which is tangent at the tangent point.

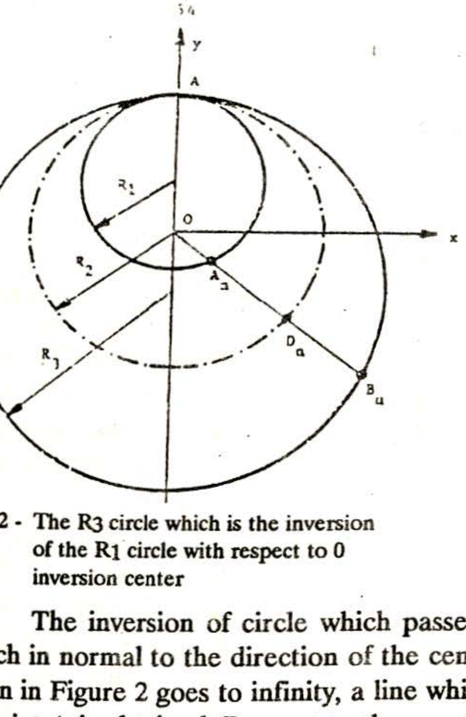

Fig. 2 - The R3 circle which is the inversion of the R1 circle with respect to 0 inversion center

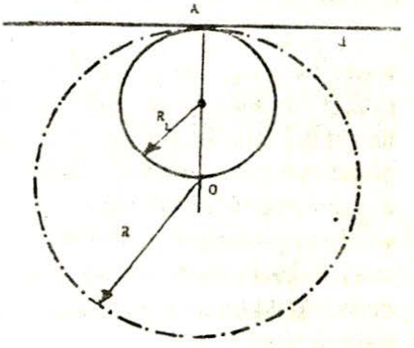

Fig. 3 - The lined being the inversion of the circle R1 which is passing through the 0 inversion center

The inversion of circle which passes from the inversion center is a line which in normal to the direction of the center. When the radius of the circle R_3 given in Figure 2 goes to infinity, a line which is perpendicular to the center line at point A is obtained. Respect to the geometry $R_3 \rightarrow \infty$ is given in Figure 3. At this case, as R_3 goes to infinity

$$\frac{2}{R_2} = \frac{1}{R_1} \tag{1.2}$$

is obtained. Therefore, it has been shown that the inverse of a circle which passes from the inversion center is a line which is tangent to that circle.

Briefly, the inversion of a circle passing from the inversion center is a line and reversely the inversion of a line is a curve passing from the inversion center.

In fact in two dimensional inversion infinite cylinder and infinite plane are considered. But, in this study, by defining the equiphase curves, the circle and the cylindre axis have been considered on the two dimentional inversion.

2. THE PHYSICAL INVERSION OF WAVES IN QUASI-OPTICAL SCATTERING

Physical inversion with geometric inversion must be included in order to solve a problem by inversion.

Physical inversion is based upon the property reflected waves from a perfectly counducting surface. The coordinates transformation mentioned in this study have been realized in the integral transformation which belongs to the Poisson-Summation formula in the exact-solution. By this coordinates transformation circular equiphase curves occur in the direction of reflected wave and pass through reflection point and which have the center at a specific point and this reflected ray.

In quasi-optical scattering problems, TEM, TE and TM modes occur localized plane waves for reflection from perfectly conducting scattering surfaces.

It can be shown by the examples in the exact solution that localized plane waves have equiphase curve lines. For the ray propagate in only the one direction, there must be an equiphase line normal to this ray or normal to the center line at the intersection point of the ray and the circle. Therefore, a circular equiphase curve can be transformed in to an equiphase curve only at this case waves can propagate in the desired direction of reflection. For the localized plane waves to propagate in the direction of reflection, the circle which is the inversion of this circle with respect to the tangent must be consired as the equiphase curve. In fact, this is the inverse transform when goes back to the original coordinates system.

On the other hand, the theorem of reflection states that: The incident ray, the reflected ray and the normal of the surface all lie on the plane. The angle between the incident ray and the normal of the surface equals the angle between the reflected ray and the normal of the surface. Also the inversion at the TE and TM modes is taken in to consideration at the two dimentional plane. In quasi-optical scattering problem which mean an application of Poisson-Summation formula in the exact solution, the reflection from the scattering surface is based upon the existance of equiphase curves produced on a plane.

3. THE PHYSICAL INVERSION OF THE REFLECTED WAVES IN QUASI-OPTICAL SCATTERING PROBLEMS WITH INFINITE SOURCE

By using the geometry in Figure 4 and defining a as the radius of curvature of the surface; F^i as the tangential component of the magnetic or electric field of the incident plane waves to the reflection point A of the curved surface. The equation relating there is as follows:

$$F^{i} = e^{-ikacos\alpha} \qquad 3.1$$

Here, the magnitude of the wave has been taken as a unit.

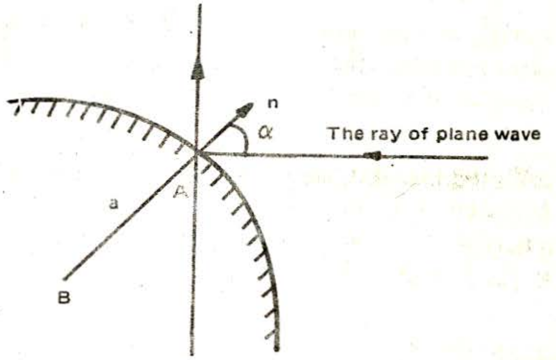

Figure: 4 - The reflection of the plane wave ray from the regular curved surface.

By considering the reflection boundary condition for the rays reflected from the perfectly conducting surface the value of the tangential component of the reflected field over the surface can be expressed as follows:

$$F_1 = e^{ikacos\alpha} \qquad 3.2$$

Since the expression denotas for the propagation of a plane wave, it indicates a line normal to the direction of the ray reflected as an equiphase curve from the reflection point.

On the other hand, with respect the inversion of this equiphase line, having its center on the reflected ray passing from the reflection point and with a radios of kacosα, is a circle of radios ka/z cosα as been given in the first part of this study. It is supposed that the new equiphase line is excited by the I_2 current. These equiphase liner are shown in Figure 5. For the tangential companent of $(\vec{E}, \vec{H})$ fields proportional to the quantity can be written as follows:

$$F_2 = I_2 \frac{e^{i\frac{ka}{2}\cos\alpha}}{\sqrt{\frac{ka}{2}\cos\alpha}} \qquad 3.3$$

This equivalent equiphase surface is supposed to produce the same field at the reflection point, it can be expressed as follows:

$$e^{-ika\cos\alpha} = I_2 \frac{e^{i\frac{ka}{2}\cos\alpha}}{\sqrt{\frac{ka}{2}\cos\alpha}} \tag{3.4}$$

This for I_2 current it is found that

$$I_2 = \frac{ka}{2}\cos\alpha \frac{e^{-i\frac{3ka}{2}\cos\alpha}}{\sqrt{\frac{ka}{2}\cos\alpha}}$$

For the reflected far field, with $f = a/2 \cos\alpha$ it holds true that

$$F^S = I_2 \frac{e^{ik(\ell + f)}}{\sqrt{k\ell}} \tag{3.5}$$

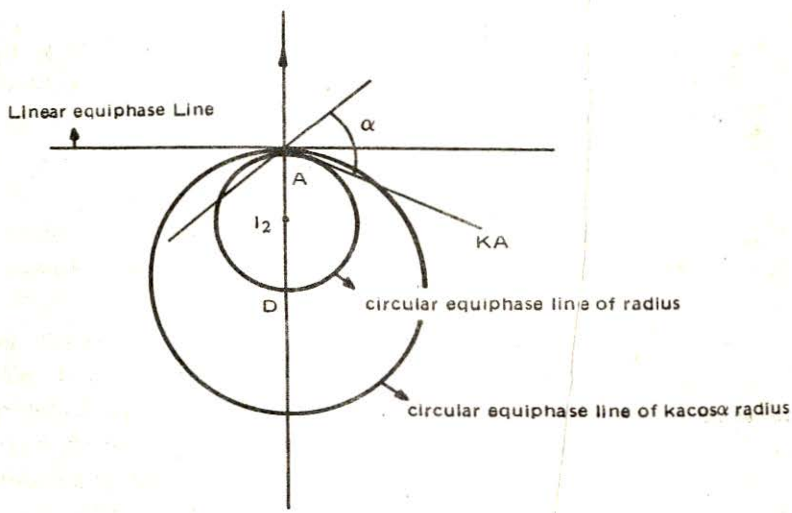

Figure: 5 - The inversion of linear equiphase line with respect to the circular equiphase line of kacosα radius.

Finally, the reflected far field in two dimensional problems is obtained as

$$F^S \sim \sqrt{\frac{ka\cos\alpha}{2}} \frac{e^{ik(\ell - ka\cos\alpha)}}{\sqrt{k\ell}} \tag{3.6}$$

Note that, in the asimptotic computation of inverse integral the current I_2 is taken as a constant.

4. THE PHYSICAL INVERSION OF THE REFLECTED WAVES IN QUASI-OPTICAL SCATTERING PROBLEMS WITH FINITE SOURCES

In quasi-optical problems with finite sources, the physical inversion used for obtaining the field equations concerning the reflected waves is based upon the interaction between the equiphase curve lines of these two simple problems which will be mentioned now:

The side of the reflection point of the scattering surface is filled with a perfect conductor, and direction of the ray which is reflected from the plane within the field of the finite source, passing from the reflection point A, and circle with a radius of $k\ell_0$ with the center on the reflected ray is composing the equiphase line.

In this case, the quantity which is proportional with the tangential compa-nent of the $(\vec{E}, \vec{H})$ fields is given as the localized plane waves as follows:

$$F^{i} = I_0 \sqrt{\frac{2}{\pi}} \frac{e^{ik\ell_0 - i\pi/4}}{\sqrt{k\ell_0}} \qquad 4.1$$

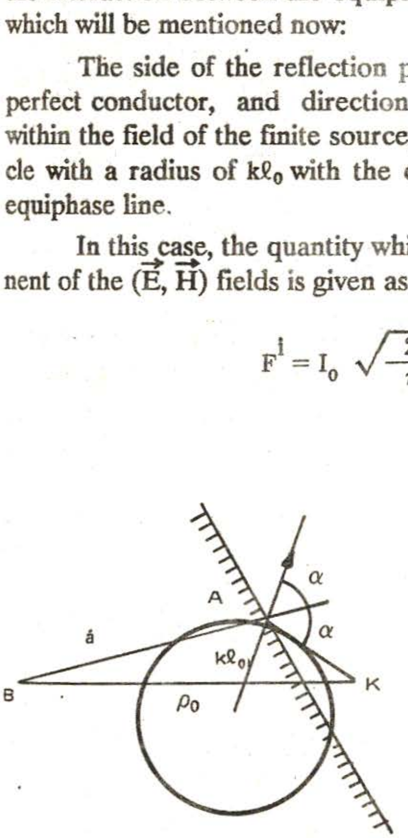

Figure: 6 - The circular equiphase line on the reflected ray in reflection of the perfectly conducting plane

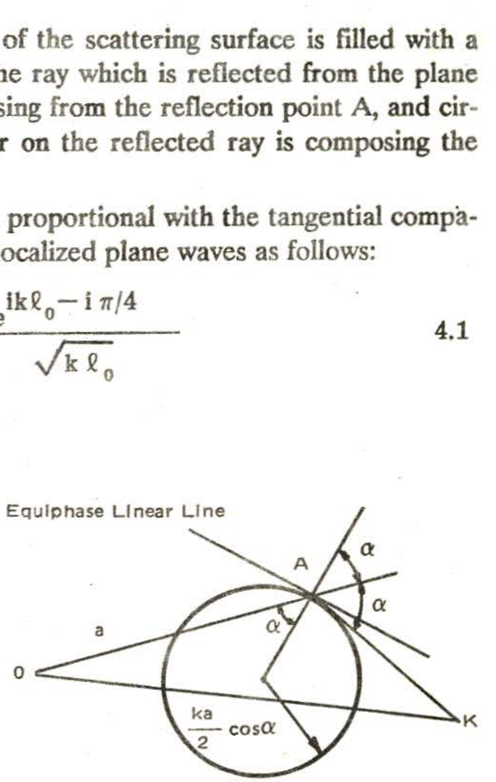

Figure: 7 - Equiphase curves at the reflection point which are linear and circular with radius ha/2 cosα

The incident ray from infinity to the reflection point A of the originally regular and curved perfectly conducting surface produces a circular equi-phase curve which passes from the point A and which has a center located on the reflected ray and having a radius ka/2 cosα (This case is valid for free-source)

On the other hand, when the field produced at the reflection point is considered by taking the magnitude of the incident ray as unit, the equiphase liner produced at the reflection point A tangential to the original surface with respect to the perfectly conducting infinite plane, is normal to the reflected ray. The field of this equiphase line can be expressed in the form:

$$F_1 = e^{ikacos\alpha} \qquad 4.2$$

The inversion of this equiphase line with respect to the circle having the radius kacosα and which is toward the inside of the curved surface beginning from the reflection point on the reflected ray, is again a circle which has radius ka/2 cosα and passes through the reflection of the reflected ray. This circle can be defined as circular equiphase curve in the direction of the reflected ray in the case of the regular curved surface with free source.

This circular equiphase curve has got the same meaning as the circular equiphase curve having the radius ka/2 cosα.

5. INTERACTION BETWEEN CIRCULAR EQUIPHASE LINES

Interaction lines with radius $k\ell_0$ and ka/2 cosα. The ray direction, thus the center is at the ray direction and passes through the reflection point obtained circular equiphase line is

$$kf_1 = \frac{2k\ell_0 . kacos\,\alpha}{2k\ell_0 + kacos\,\alpha} \qquad 5.1$$

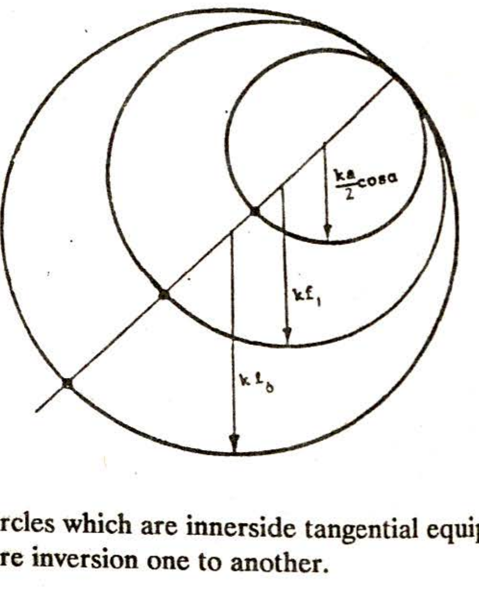

Figure: 8 - Interaction between equiphase lines

As shown in Figure 8 two circles which are innerside tangential equiphase lines appears with the radius kf_1, are inversion one to another.

Therefore the center of the new equiphase line is at the reflection ray point and passes through the reflection point A, which has a radius

$$kf = \frac{kf_1}{2} = \frac{k\ell_0 \cdot ka\cos\alpha}{2k\ell_0 + ka\cos\alpha} \qquad 5.2$$

it is a circular equiphase line.

On the other hand with the tangential field of the source at the reflection point, and the tangential field of the image source have been considered as equal F_1 and F_2 indicates the proportional quantity of the tangential fields of source and image respectively written as:

$$F_1 = I_0 \sqrt{\frac{2}{\pi}} \frac{e^{ik\ell_0 - i\pi/4}}{\sqrt{k\ell_0}}, \quad F_2 = I_2 \sqrt{\frac{2}{\pi}} \frac{e^{ikf - i\pi/4}}{\sqrt{kf}} \qquad 5.3$$

Where I_2 is a new value of the interaction between two equiphase circular lines.

At the reflection point $F_1 = F_2$, I_2 must be as follows:

$$I_2 = I_0 \sqrt{\frac{f}{\ell_0}} e^{ik(\ell_0 - f)} \qquad 5.4$$

As given in 5.4 image current of the obtained field of the reflection ray direction has the following form

$$F_2 = I_2 \sqrt{\frac{2}{\pi}} \frac{e^{ik(\ell + f) - i\pi/4}}{\sqrt{k(\ell + f)}} \qquad 5.5$$

In the above equation, inversion taken in the plane is invers transformation for the obtained the current.

Finally, the field at the reflection point can be found using the expression of f given in equation 5.2

$$F_2 = I_0 \sqrt{\frac{(\ell + \ell_0)\, a\cos\alpha}{2\ell\ell_0 + (\ell + \ell_0)\, a\cos\alpha}} \sqrt{\frac{2}{\pi}} \frac{e^{ik(\ell + \ell_0) - i\pi/4}}{\sqrt{k(\ell + \ell_0)}} \qquad 5.6$$

In case of far field $\ell \to \infty$ is expressed as

$$F_2 = I_0 D(\alpha) \sqrt{\frac{2}{\pi}} \frac{e^{ik(\ell + \ell_0) - i\pi/4}}{\sqrt{k\ell}} \qquad 5.7$$

Where $D(\alpha)$ divergence coefficient is

$$D(\alpha) = \sqrt{\frac{a\cos\alpha}{2\ell_0 + a\cos\alpha}} \qquad 5.8$$

CONCLUSION

In this paper, the description of the circular equiphase lines of the plane waves are defined. And also, in the condition given above, depending on the equiphase line, inversion method is applied on the ray paths which is the coordinate transformation. First surface of the perfect conducting curve surface at the field of infinite source is considered.

Taken the radius of the reflection ray, passes through the reflection point, circular equiphase lines are occured. Inverse transformation is realized when taking the equiphase circular lines are obtained with reflection far field from the curve surface.

In this case, reflection of the perfect conducting curve surface of the field of finite source is taken. It is proved that the transformation of the solution of the original problem is reduced into the two simple problems. One of the simpler problems at the reflection point and at the source, tangential to the original surface is reflection from the infinite perfectly conductive plane. The other simpler problem, the incident ray from the infinity, passes through the source point reflecting from the original curve surface as can be considered.

The integration of the equiphase circular lines are obtained. When taking the inverse transformation, respect to the normal to the reflection ray and the linear equiphase lines, inverse transformation has been realized.

LITERATURE

1. AKHUNLAR, A.: "Elektromagnetik Alanlar-Statik Elektrik Alanları I", İ.T.Ü. Yayınları, s. 297-310, 1965.
2. BAYRAKÇI, H.E.: "Dipolden Işıyan Elektromagnetik Dalgaların Silindirik Bir Yönlü Empedans Yüzeyinden Yüksek Frekansta Saçılması", TÜBİTAK MAG-394, 1976, 1977.
3. COLLIN, R.E., ZUCKER, F.J.: "Antenna Theory Part 2", Mc Graw Hill Book Co. 1969.
4. FELSEN, I.B., MARCUVITZ, N.: "Radiation and Scattering of Waves", Prentice-Hall, 1973.
5. ISHIHARA, T. and FELSEN, I.B.: "High Frequency Fields Excited by a line Source Located on a perfectly conducting Concave cylindrical Surface", IEEE Trans. on Antennas and Propagation, Nov. 1978.
6. PATBAK, P.H., KOUYOMAJIAN, R.G.: "The Radiation From Apertures in Curved Surfaces", NASA, July, 1973.
7. WAIT, R.J.: "Electromagnetic Radiation from Cylindrical Structures", Pergamon Press, 1959.