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On the Conservation Laws of Modified KdV-KP Equation

Emrullah Yaşar[a], Sait San[b]

[a] *Uludag university, Faculty of Arts and Sciences, Department of Mathematics, Bursa, Turkey*
[b] *Eskişehir Osmangazi University, Faculty of Arts and Sciences, Department of Mathematics-Computer, Eskişehir, Turkey*

Abstract. In this paper we study the conservation laws of modified Korteweg-de Vries-Kadomtsev Petviashvili equation (mKdV-KP). As the considered equation is of evolution type, no recourse to a Lagrangian formulation is made. However, we show that using the partial Lagrangian approach and the multiplier method one can obtain a number of local and nonlocal conservation laws for underlying equation.

1. Introduction

As stated in [22] conservation laws are very important tools in the study of differential equations from mathematical as well as physical point of view. If the under study system has conservation laws then its integrability is quite possible [2, 6]. They are also used for existence, uniqueness and Lyapunov stability analysis and construction of numerical schemes. Moreover, conservation laws are used in obtaining the new nonlocal symmetries, nonlocal conservation laws and linearization [10].

The first study in the literature for obtaining the conservation laws is given by E. Noether [21]. In this study, Noether states that for Euler–Lagrange differential equations, to each Noether symmetry associated with the Lagrangian there corresponds a conservation law which can be determined explicitly by a formula. The application of Noether's theorem depends upon the knowledge of a suitable Lagrangian ([18, 21]).

There are some new approaches in the literature about construction of conservation laws such as direct method, partial Lagrangian method, the characteristic method, the variational approach (multiplier approach), nonlocal conservation theorem method ([5, 12, 14–16, 19, 22, 23]). Moreover there are some sofwares for constructing the conservation laws [9, 27].

The celebrated Korteweg-de Vries (KdV) equation [1, 17]

$$u_t + 6uu_x + u_{xxx} = 0$$

governs the dynamics of solitary waves. It was derived to describe shallow water waves of long wavelength and small amplitude. It is an important equation from the view point of integrable systems because it has infinite number of conservation laws, gives multiple soliton solutions, has bi-Hamiltonian structures, a Lax pair, and has many other physical properties [25].

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Email addresses: eyasar@uludag.edu.tr (Emrullah Yaşar), ssan@ogu.edu.tr (Sait San)

Kadomtsov and Petviashivilli obtained the Kadomtsov-Petviashivilli (KP) equation [4, 13] as an improvement of the KdV equation

$$(u_t + 6uu_x + u_{xxx})_x + u_{yy} = 0.$$

Using the idea of Kadomtsev and Petviashvili, who relaxed the restriction that the waves be strictly one-dimensional in the Korteweg-de Vries (KdV) equation, leads to the (2+1)-dimensional modified KdV–KP equation [8, 26]:

$$u_{tx} - \frac{3}{2}u_{xx} + 12u_x^2 u + 6u^2 u_{xx} + u_{xxxx} + u_{yy} = 0 \tag{1}$$

This equation was investigated in the literature because it is used to model a variety of nonlinear phenomena. In recent years, the exact travelling wave solutions of the Eq. (1) have been studied by many authors [3, 4, 8, 20, 24, 26].

In this paper, we aim to construct the conservation laws of Eq. (1) with two distinct methods. First, we use the partial Lagrangian approach. We resort to this method when the underlying equation does not has a Lagrangian or finding the Lagrangian is the difficult. Second, we use the multiplier method with the help of homotopy operator.

The organization of the paper is as follows: In Section 2, we give the necessary operators and definitions. In Section 3, we give the brief description of the partial Lagrangian and it's application to Eq. (1) . Section 4 is devoted to the multiplier method and finding the conservation laws of Eq.(1) . In Section 5, we give some concluding remarks.

2. Fundemental Operators

We first present notation to be used and recall basic definitions and theorems which can be found in the literature cited [9, 11, 18]. The summation convention is adopted in which there is summation over repeated upper and lower indices. Consider a s-th-order system of partial differential equations (PDEs) of n independent variables $x = (x^1, x^2, ..., x^n)$ and m dependent variables $u = (u^1, u^2, ..., u^m)$

$$E^\alpha(x, u, u_{(1)}, ..., u_{(s)}), \quad \alpha = 1, ..., m, \tag{2}$$

where $u_{(1)}, u_{(2)}, ..., u_{(s)}$ denote the collections of all first, second,...,s-th-order partial derivatives. The derivatives of u^α with respect to x^i are $u_i^\alpha = D_i(u^\alpha)$, $u_{ij} = D_j D_i(u^\alpha)$ where

$$D_i = \frac{\partial}{\partial x^i} + u_i^\alpha \frac{\partial}{\partial u^\alpha} + u_{ij}^\alpha \frac{\partial}{\partial u_j^\alpha} + ...$$

is the total derivative operator with respect to x^i. The collection of sth-order derivatives, $s \geq 1$, is denoted by $u_{(s)}$. As usual $\mathcal{A}$ is the vector space of differential functions of finite orders. The basic operators defined in $\mathcal{A}$ are stated below.

The Euler-Lagrange operator given by

$$\frac{\delta}{\delta u^\alpha} = \frac{\partial}{\partial u^\alpha} + \sum_{s\geq 1}^{\infty} (-1)^s D_{i_1} \dots D_{i_s} \frac{\partial}{\partial u^\alpha_{i_1 \dots i_s}}, \quad \alpha = 1, ..., m.$$

and *The Lie-Backlund operator* is

$$X = \xi^i \frac{\partial}{\partial x_i} + \eta^\alpha \frac{\partial}{\partial u^\alpha}, \quad \xi^i, \eta^\alpha \in \mathcal{A} \tag{3}$$

The operator (3) is an abbreviated form of the infinite formal sum

$$X = \xi^i \frac{\partial}{\partial x_i} + \eta^\alpha \frac{\partial}{\partial u^\alpha} + \sum_{s\geq 1} \zeta^\alpha_{i_1 i_2 \dots i_s} \frac{\partial}{\partial u^\alpha_{i_1 i_2 \dots i_s}},$$

where the additional coefficients are determined uniquely by the prolongation formulae

$$\begin{aligned}\zeta_i^\alpha &= D_i\left(W^\alpha\right)+\xi^j u_{ji}^\alpha \\ \zeta_{i_1\ldots i_s}^\alpha &= D_{i_1}\ldots D_{i_s}\left(W^\alpha\right)+\xi^j u_{ji_1\ldots i_s}^\alpha, \quad s>1,\end{aligned}$$

in which W^α is the *Lie characteristic function*

$$W^\alpha=\eta^\alpha-\xi^j u_j^\alpha .$$

The Noether operators associated with a Lie–Bäcklund operator X is given by

$$N^i=\xi^i+W^\alpha \frac{\delta}{\delta u_i^\alpha}+\sum_{s=1}^{\infty}(-1)^s D_{i_1}...D_{i_s}(W^\alpha)\frac{\delta}{\delta u_{ii_1\ldots i_s}^\alpha}.$$

The $n-$tuple vector $T=\left(T^1,T^2,...,T^n\right), T^j\epsilon A,\ j=1,...,n$ is a conserved vector of eq(2) if T^i satisfies

$$D_i T^i\big|_{(2)}=0. \tag{4}$$

Equation (4) defines a local conservation law of system (2).

3. Partial Noether Approach

If the standard Lagrangian does not exist or is difficult to find, then we write its partial Lagrangian and derive the conservation laws by the partial Noether approach introduced by Kara and Mahomed [15].

Suppose that the system of Eq.(2) are written as

$$E_\alpha=E_\alpha^0+E_\alpha^1=0. \tag{5}$$

If there exists a function $L=L(x,u,u_{(1)},u_{(2)},...,u_{(l)}),\ l\leq s$ and nonzero functions $f_\alpha^\beta\in\mathcal{A}$ such that (5) can be written as $\frac{\delta L}{\delta u^\alpha}=f_\alpha^\beta E_\beta^1$ provided $E_\beta^1\neq 0$ for some β. L is known as a partial Lagrangian of (5), otherwise it is the standard Lagrangian.

The differential equations of the form $\frac{\delta L}{\delta u^\alpha}=f_\alpha^\beta E_\beta^1$ are called a system of partial Euler–Lagrange equations. The partial Noether operator X corresponding to a partial Lagrangian $L(x,u_{(1)},u_{(2)},...,u_{(n-1)})$ of Eq.(2) is determined from

$$XL+L(D_i\xi^i)=W^\alpha\frac{\delta L}{\delta u^\alpha}+D_i(B^i),\quad i=1,...,N, \tag{6}$$

for a suitable gauge terms $B=(B^1,B^2,...,B^n),\ B^i\in\mathcal{A}$.

Theorem 3.1. *The conserved vector of the Eq. (2) associated with a partial Noether operator X corresponding to the partial Lagrangian L is determined from*

$$T^i=B^i-N^iL=B^i-\xi^iL-W^\alpha\frac{\delta L}{\delta u_i^\alpha}-\sum_{s=1}^{\infty}(-1)^s D_{i_1}...D_{i_s}(W^\alpha)\frac{\delta L}{\delta u_{ii_1\ldots i_s}^\alpha}$$

where W^α are the characteristics of the conservation law.

Eq. (1) has a partial Lagrangian

$$L=\frac{-u_tu_x}{2}+\frac{3}{4}u_x^2+\frac{1}{2}u_{xx}^2-\frac{1}{2}u_y^2-3u^2u_x^2$$

whose partial Noether operators $X = \xi\frac{\partial}{\partial x} + \phi\frac{\partial}{\partial y} + \tau\frac{\partial}{\partial t} + \eta\frac{\partial}{\partial u}$ satisfy (6) and the partial Euler–Lagrange-type equation is given by

$$\frac{\delta L}{\delta u} = u_{tx} - \frac{3}{2}u_{xx} + 6uu_x^2 + 6u^2u_{xx} + u_{xxxx} + u_{yy} \tag{7}$$

so that Eq.(7) can be written as

$$\frac{\delta L}{\delta u} = -6uu_x^2.$$

Determining equations for the partial Noether symmetry are given, by (6),

$$X^{[2]}L + (\xi_x + u_x\xi_u + \tau_t + u_t\tau_u + \phi_y + u_y\phi_u)L = (B_t^1 + B_u^1u_t + B_x^2 + B_u^2u_x + B_y^3 + B_u^3u_y) + \frac{\delta L}{\delta u}(\eta - \tau u_t - \xi u_x - \phi u_y). \tag{8}$$

Eq.(8) for $L = \frac{-u_tu_x}{2} + \frac{3}{4}u_x^2 + \frac{1}{2}u_{xx}^2 - \frac{1}{2}u_y^2 - 3u^2u_x^2$ give rise to

$$\begin{aligned}
&-\tfrac{u_t}{2}\eta^x - \tfrac{u_x}{2}\eta^t + \tfrac{3}{2}u_x\eta^x + u_{xx}\eta^{xx} - u_y\eta^y - 6uu_x^2\eta - 6u^2u_x\eta^x \\
&+(\xi_x + u_x\xi_u + \tau_t + u_t\tau_u + \phi_y + u_y\phi_u)\left(\tfrac{-u_tu_x}{2} + \tfrac{3}{4}u_x^2 + \tfrac{1}{2}u_{xx}^2 - \tfrac{1}{2}u_y^2 - 3u^2u_x^2\right) \\
&= (B_t^1 + B_u^1u_t + B_x^2 + B_u^2u_x + B_y^3 + B_u^3u_y) - 6uu_x^2(\eta - \tau u_t - \xi u_x - \phi u_y)
\end{aligned} \tag{9}$$

where $B^1 = B^1(x, y, t, u)$, $B^2 = B^2(x, y, t, u)$ and $B^3 = B^3(x, y, t, u)$ are the gauge terms. Separating Eq.(9) with respect to derivatives of u yield the following overdetermined linear system

$$\begin{array}{lcl}
u_x & : & -B_u^2 - \frac{1}{2}\eta_t + \frac{3}{2}\eta_x - 6u^2\eta_x \\
u_y & : & -B_u^3 - \eta_y \\
u_t & : & -B_u^1 - \frac{1}{2}\eta_x \\
u_xu_t & : & -\eta_u + 6u^2\tau_x - \frac{3}{2}\tau_x - \frac{1}{2}\phi_y \\
u_xu_yu_t & : & \frac{1}{2}\phi_u \\
u_x^2u_t & : & 3u^2\tau_u - 6u\tau + \frac{1}{2}\xi_u - \frac{3}{4}\tau_u \\
u_xu_y & : & 6u^2\phi_x + \frac{1}{2}\phi_t - \frac{3}{2}\phi_x + \xi_y \\
u_x^2u_y & : & 3u^2\phi_u - 6u\phi - \frac{3}{4}\phi_u \\
u_x^2 & : & \frac{1}{2}\xi_t + \frac{3}{2}\eta_u - \frac{3}{4}\xi_x + \frac{3}{4}\tau_t + \frac{3}{4}\phi_y - 6u^2\eta_u + 3u^2\xi_x - 3u^2\tau_t + 3u^2\phi_y \\
u_t^2 & : & \frac{1}{2}\tau_x \\
u_x^3 & : & -\frac{3}{4}\xi_u + 3u^2\xi_u - 6u\xi \\
u_y^2 & : & -\eta_u + \frac{1}{2}\phi_y - \frac{1}{2}\tau_t - \frac{1}{2}\xi_x \\
u_y^3 & : & \frac{1}{2}\phi_u \\
u_xu_t^2 & : & \frac{1}{2}\tau_u \\
u_tu_y & : & \frac{1}{2}\phi_x + \tau_y \\
u_y^2u_t & : & \frac{1}{2}\tau_u \\
u_xu_y^2 & : & \frac{1}{2}\xi_u \\
u_{xx}^2u_y & : & -\frac{1}{2}\phi_u \\
u_{xx}u_tu_x^2 & : & -\tau_{uu} \\
u_{xx}u_yu_x^2 & : & -\phi_{uu} \\
u_{xx}u_{xy}u_x & : & -2\phi_u
\end{array}$$

$$
\begin{array}{lll}
u_{xx}u_{tx}u_x & : & -2\tau_u \\
u_{xx}u_xu_y & : & -2\phi_{ux} \\
u_{xx}u_xu_t & : & -2\tau_{ux} \\
u_{xx}^2 & : & \eta_u - \frac{3}{2}\xi_x + \frac{1}{2}\tau_t + \frac{1}{2}\phi_y \\
u_{xx} & : & \eta_{xx} \\
u_{xx}u_{xy} & : & -2\phi_x \\
u_{xx}u_{tx} & : & -2\tau_x \\
u_{xx}u_x & : & 2\eta_{xu} - \xi_{xx} \\
u_{xx}u_x^2 & : & -2\xi_{ux} + \eta_{uu} \\
u_{xx}u_y & : & -\phi_{xx} \\
u_{xx}u_t & : & -\tau_{xx} \\
u_{xx}u_x^3 & : & -\xi_{uu} \\
u_{xx}^2u_x & : & -\frac{5}{2}\xi_u \\
u_{xx}^2u_t & : & -\frac{1}{2}\tau_u \\
1 & : & -B_t^1 - B_x^2 - B_y^3.
\end{array}
$$

The solution of this system yields the following partial Noether operator and gauge terms:

$$X = \eta(x,y,t)\frac{\partial}{\partial u} \text{ with } \xi(x,y,t,u) = \phi(x,y,t,u) = \tau(x,y,t,u) = 0, \eta = f(x,y,t)$$

where η satisfies the equation $\eta_{xx} = 0$, and

$$
\begin{aligned}
B^1(x,y,t,u) &= -\frac{1}{2}f_xu + \alpha(x,y,t),\ B^2(x,y,t,u) = -\frac{1}{2}f_tu + \frac{3}{2}f_xu - 2u^3f_x + \beta(x,y,t), \\
B^3(x,y,t,u) &= -f_yu + \gamma(x,y,t).
\end{aligned}
$$

We set $\alpha(x,y,t) = \beta(x,y,t) = \gamma(x,y,t) = 0$ as they contribute to the trivial part of the conserved vector. The formula for conserved vectors for the second order partial Lagrangian are

$$T^x = B^2 - \xi L - W\left[\frac{\partial L}{\partial u_x} - D_x\frac{\partial L}{\partial u_{xx}}\right] - D_x(W)\frac{\partial L}{\partial u_{xx}} \tag{10}$$

$$T^y = B^3 - \phi L - W\frac{\partial L}{\partial u_y} \tag{11}$$

$$T^t = B^1 - \tau L - W\frac{\partial L}{\partial u_t} \tag{12}$$

Using the Eqs. (10)-(12) yield the following independent conserved vectors for Eq.(1):

$$T^x = -\frac{1}{2}f_tu + \frac{3}{2}f_xu - 2u^3f_x - f\left[\frac{-u_t}{2} + \frac{3}{2}u_x - 6u^2u_x - u_{xxx}\right] - f_xu_{xx},$$

$$T^t = -\tfrac{1}{2}f_xu + \tfrac{u_x}{2}f, \qquad T^y = -f_yu + fu_y.$$

The corresponding conservation law is the following:

$$D_xT^x + D_yT^y + D_tT^t = f\left(u_{tx} - \frac{3}{2}u_{xx} + 12uu_x^2 + 6u^2u_{xx} + u_{xxxx} + u_{yy}\right)$$

where f is the characteristic of the partial Noether symmetry operator X. Thus, if we take for instance $f(x,y,t) = x$ then one can obtain the following conserved vector whose components are given by

$T^x = \frac{3}{2}u - 2u^3 + \frac{xu_t}{2} - \frac{3}{2}xu_x + 6xu^2u_x + xu_{xxx} - u_{xx}$,
$T^y = xu_y$,
$T^t = -\frac{u}{2} + \frac{xu_x}{2}$.

4. The Multiplier Method

Another approach to determining conserved flows involves the well known result that the Euler–Lagrange operator $\delta/\delta u^{\alpha}$ annihilates the total divergence [5, 9, 10, 22]. A multiplier $\Lambda_{\alpha}(x, u, u_t, ...)$ has the property that

$$\Lambda_{\alpha} E^{\alpha} = D_i T^i \tag{13}$$

holds identically. Here we get multipliers of fourth order, that is

$$\Lambda_{\alpha} = \Lambda_{\alpha}(x, y, t, u, u_x, u_{xt}, u_{yy}, u_{xx}, u_{xxxx}).$$

By calculating the variational derivative of (13) the determining equations for the multipliers

$$\frac{\delta(\Lambda_{\alpha} E^{\alpha})}{\delta u^{\alpha}} = 0$$

are obtained. Solving the above over-determined system, multipliers are found. Then using the multipliers conservation laws are obtained systematically.

Now, we will derive the conservation laws of the mKdV-KP equation by the multiplier method. The fourth order multiplier for (1) is,

$$\Lambda(x, y, t, u, u_x, u_{xt}, u_{yy}, u_{xx}, u_{xxx}, u_{xxxx})$$

and the corresponding determining equation is

$$\begin{aligned}
&\Lambda_u(u_{tx} - \frac{3}{2}u_{xx} + 12uu_x^2 + 6u^2u_{xx} + u_{xxxx} + u_{yy}) + \Lambda(12u_x^2 + 12uu_{xx}) \\
&-D_x(\Lambda_{u_x}(u_{tx} - \frac{3}{2}u_{xx} + 12uu_x^2 + 6u^2u_{xx} + u_{xxxx} + u_{yy}) + \Lambda(24uu_x)) \\
&+D_{xx}(\Lambda_{u_{xx}}(u_{tx} - \frac{3}{2}u_{xx} + 12uu_x^2 + 6u^2u_{xx} + u_{xxxx} + u_{yy}) + \Lambda(-\frac{3}{2} + 6u^2)) \\
&+D_{xt}(\Lambda_{u_{xt}}(u_{tx} - \frac{3}{2}u_{xx} + 12uu_x^2 + 6u^2u_{xx} + u_{xxxx} + u_{yy}) + \Lambda) \\
&+D_{yy}(\Lambda_{u_{yy}}(u_{tx} - \frac{3}{2}u_{xx} + 12uu_x^2 + 6u^2u_{xx} + u_{xxxx} + u_{yy}) + \Lambda) \\
&+D_{xxxx}(\Lambda_{u_{xxxx}}(u_{tx} - \frac{3}{2}u_{xx} + 12uu_x^2 + 6u^2u_{xx} + u_{xxxx} + u_{yy}) + \Lambda) = 0
\end{aligned} \tag{14}$$

After expansion and lengthy calculations of (14), with respect to different combinations of derivatives of u and solving the over determined system, we find with the aid of Maple package program [9] the following six multipliers:

$$\begin{aligned}
\Lambda_1 &= xy, \\
\Lambda_2 &= x, \\
\Lambda_3 &= -\frac{1}{6}y(6F(t)x - F_t y^2), \\
\Lambda_4 &= -F(t)x + \frac{1}{2}F_t y^2, \\
\Lambda_5 &= F(t)y, \\
\Lambda_6 &= F(t).
\end{aligned}$$

where $F(t)$ is arbitrary function of t. Corresponding to the above multipliers we obtain six local conserved vectors of (1).

For instance, we can find the conserved vectors corresponding to $\Lambda_1 = xy$. Separating to total derivatives, we get

$$\begin{aligned}(xy)(u_{tx} - \frac{3}{2}u_{xx} + 12uu_x^2 + 6u^2u_{xx} + u_{xxxx} + u_{yy}) &= D_x(6xyu_xu^2 - 2yu^3 - \frac{3}{2}xyu_x + \frac{3}{2}yu + xyu_{xxx} - yu_{xx}) \\ &\quad +D_y(-xu + xyu_y) + D_t(xyu_x).\end{aligned}$$

Therefore, we can easily write following conserved vector corresponding to the multiplier of Λ_1:

$$\begin{aligned}T_1^x &= 6xyu_xu^2 - 2yu^3 - \tfrac{3}{2}xyu_x + \tfrac{3}{2}yu + xyu_{xxx} - yu_{xx},\\ T_1^y &= -xu + xyu_y,\\ T_1^t &= xyu_x.\end{aligned}$$

Repeating the similar procedures in other multipliers respectively, five conserved vectors are found. We give the results in the following:

$$\begin{aligned}T_2^x &= 6xu_xu^2 - 2u^3 - \tfrac{3}{2}xu_x + \tfrac{3}{2}u + xu_{xxx} - u_{xx}\\ T_2^y &= xu_y\\ T_2^t &= xu_x\end{aligned}$$

$$\begin{aligned}T_3^x &= \tfrac{1}{12}y[-72u_xu^2F(t)x + 12u_xu^2F_ty^2 + 24F(t)u^3 + 18xF(t)u_x + 3u_xF_ty^2\\ &\quad +12xuF_t - 2uF_{tt}y^2 - 18uF(t) - 12u_{xxx}F(t)x + 2u_{xxx}F_ty^2 + 12u_{xx}F(t)]\\ T_3^y &= xuF(t) - \tfrac{1}{2}F_ty^2u - u_yF(t)xy + \tfrac{1}{6}u_yF(t)y^3\\ T_3^t &= -\tfrac{1}{6}yu_x(6F(t)x - F_ty^2)\\ T_4^x &= \text{-}6u_xu^2F(t)x + 3u_xu^2F_ty^2 + 2F(t)u^3 + \tfrac{3}{2}u_xF(t)x - \tfrac{3}{4}u_xF_ty^2\\ &\quad +uF_tx - \tfrac{1}{2}uy^2F_{tt} - \tfrac{3}{2}uF(t) - u_{xxx}F(t)x + \tfrac{1}{2}u_{xxx}F_ty^2 + u_{xx}F(t)\\ T_4^y &= -uF_ty - u_yF(t)x + \tfrac{1}{2}u_yF_ty^2\\ T_4^t &= -\tfrac{1}{2}u_x(2F(t)x - F_ty^2)\\ T_5^x &= -\tfrac{1}{2}y\left(-12u_xF(t)u^2 + 3u_xF(t)\right) + 2uF_t - 2u_{xxx}F(t)\\ T_5^y &= -uF(t) + u_yF(t)y\\ T_5^t &= u_xF(t)y\end{aligned}$$

$$\begin{aligned}T_6^x &= 6u_xF(t)u^2 - \tfrac{3}{2}u_xF(t) - uF_t + u_{xxx}F(t)\\ T_6^y &= u_yF(t)\\ T_6^t &= u_xF(t)\end{aligned}$$

where $F(t)$ is arbitrary function of t. Thus, we obtained nontrivial six conserved vectors for the mKdV-KP equation.

5. Concluding Remarks

In this study we have constructed conservation laws of the mKdV-KP equation which is not derivable from a variational principle. We obtained an infinitely nonlocal conservation laws using the the partial Lagrangian method. Also we applied the multiplier method on mKdV-KP equation. We yield six multipliers and thus six local conserved vectors were obtained.

The conserved vectors obtained here can be used in reductions and solutions of the underlying equation [7]. In future work, with the aid of conservation laws of the equation, nonlocal symmetries such as potential and nonclassical potential symmetries will be obtain. As these symmetries enable one to obtain new interesting solutions of the considered equation.

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