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Nonlinear Schrödinger equations with spatio-temporal dispersion in Kerr, parabolic, power and dual power law media: A novel extended Kudryashov's algorithm and soliton solutions

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ABSTRACT

In this study, we perform the extended Kudryashov method to nonlinear Schrödinger equation (NLSE) with spatio-temporal dispersion that arises in a propagation of light in nonlinear optical fibers, planar waveguides, Bose–Einstein condensate theory. Four types of nonlinearity – Kerr law, power law, parabolic law and dual-power law – are being considered for the model. By using this scheme, the topological, singular soliton and rational solutions are obtained. In addition, some graphical simulations of solutions are provided.

It is demonstrated that the proposed algorithm is effective and can be handled for many other nonlinear complex differential equations.

Introduction

It is well known that there exist various regimes that describe the dynamics of soliton propagation such as Korteweg-de Vries (KdV) equation and Kadomtsev-Petviashvili (KP) hierarchies. One of most visible is the nonlinear Schrödinger's equation (NLSE) with spatio-temporal dispersion

$$iq_t + aq_{tx} + bq_{xx} + cF\left(|q|^2\right)q = 0. \tag{1}$$

The nonlinear Schrödinger's equation (NLSE) is a very important equation that arises in physics, engineering and mathematical physics [1]. It typically shows up in the study of optical fibers, planar waveguides, Bose–Einstein condensate theory, fluid dynamics, plasma physics, mathematical biology and many more. There are various types of solution to Eq. (1). They are include the soliton solution, cnoidal wave solutions, periodic solutions, doubly periodic waves and many more [2–7].

Optical solitons form the basic molecules for signal propagation across long distances [13]. Also, optical solitons is one of the main topics of research in modern telecommunications industry. Several advances in this field of research, during the past few decades, led to modern day marvel in telecom systems. Internet activity, facebook, twitter and all such communication means are the outcomes of research in solitons and other related topics in quantum and nonlinear optics [12].

In this paper, our aim is to reveal soliton and other counterpart solutions for NLSE with spatio-temporal dispersion in the four types of nonlinear forms which are Kerr law, power law, parabolic law and dual-power law.

The paper is structured thusly. In Section "The extended Kudryashov's method", we present extended Kudryashov method's for complex nonlinear evolution equations (NLEEs). Section "Exact solutions of model equation" is devoted to application of the method to Eq. (1) for four distinct cases including Kerr law, power law, parabolic law and dual-power law.

In final section, we give some concluding remarks.

The extended Kudryashov's method

We now briefly present the main steps of method. For the details and applications, we refer to [9].

We consider the following nonlinear evolution equation (NLEE)

$$F(u, u_t, u_x, u_{xx}, u_{xt}, \ldots) = 0, \tag{2}$$

where $u = u(x,t)$ is an unknown function, F is a polynomial in u and its various partial derivatives u_t, u_x with respect to t, x respectively, in which the highest order derivatives and nonlinear terms are involved.

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Step 1 Inserting the traveling wave transformation which corresponds the linear combination of time and space translations of Lie symmetry generators

$$u(x,t) = U(\xi), \quad \xi = k(x - ct), \tag{3}$$

into Eq. (2) then it transforms to a nonlinear ordinary differential equation (NLODE) of the form

$$P\left(U, -kcU', kU', k^2U'', \dots\right) = 0 \tag{4}$$

where k, c are constants to be determined later.

Step 2 We assume that the solution of Eq. (4) can be expressed in the form

$$U(\xi) = A_0 + \sum_{k=1}^{N}\sum_{i+j=k} A_{ij}\phi^i(\xi)\psi^j(\xi) + \sum_{k=1}^{N}\sum_{i+j=k} B_{ij}\phi^{-i}(\xi)\psi^{-j}(\xi) \tag{5}$$

where A_0, B_{ij}, $A_{ij}(i,j = 0,1,2,\dots,N)$ are constants to be determined, and the functions $\phi(\xi)$ and $\psi(\xi)$ holds the Bernoulli and Riccati equations, respectively

$$\phi'(\xi) = R_2\phi^2(\xi) - R_1\phi(\xi), \quad R_2 \neq 0, \tag{6}$$

$$\psi'(\xi) = S_2\psi^2(\xi) + S_1\psi(\xi) + S_0, \quad S_2 \neq 0 \tag{7}$$

where R_2, R_1, S_2, S_1 and S_0 are constants.

It is well known that the elementary solutions of the Bernoulli and Riccati equations with respect to arbitrary coefficients of R_2, R_1, S_2, S_1, S_0 are given respectively

$$\phi(\xi) = \begin{cases} \frac{R_1}{R_2 + R_1\exp(R_1\xi + \xi_0)}, & R_1 \neq 0 \\ \frac{-1}{R_2\xi + \xi_0}, & R_1 = 0 \end{cases} \tag{8}$$

and

$$\psi(\xi) = \begin{cases} \frac{-S_1}{2S_2} - \frac{\sqrt{\mu}}{2S_2}\tanh\left(\frac{\sqrt{\mu}}{2}\xi + \xi_0\right), & \mu > 0 \\ \frac{-S_1}{2S_2} - \frac{\sqrt{\mu}}{2S_2}\coth\left(\frac{\sqrt{\mu}}{2}\xi + \xi_0\right), & \mu > 0 \\ \frac{-S_1}{2S_2} + \frac{\sqrt{-\mu}}{2S_2}\tan\left(\frac{\sqrt{-\mu}}{2}\xi + \xi_0\right), & \mu < 0 \\ \frac{-S_1}{2S_2} - \frac{\sqrt{-\mu}}{2S_2}\cot\left(\frac{\sqrt{-\mu}}{2}\xi + \xi_0\right), & \mu < 0 \\ \frac{-S_1}{2S_2} - \frac{1}{S_2\xi + \xi_0}, & \mu = 0 \end{cases} \tag{9}$$

where $\mu = S_1^2 - 4S_0S_2$ and ξ_0 is an arbitrary real constant.

Step 3 The main point of the application is based on the constant N. This positive integer number N in Eq. (5) is determined by balancing the highest order derivatives and the nonlinear terms in Eq. (4).

Step 4 We obtain polynomial forms of $\phi(\xi)\psi(\xi)$ after substituting the series of (5) into (4) along with (6) and (7). In this polynomial form, we collect all terms of same powers and equating them to be zero, we get an over-determined system of algebraic equations which can be assisted by the Maple to reveal the unknown parameters k, c, R_1, R_2, S_0, S_1, S_2, A_0, A_{ij} and B_{ij}, $(i,j = 0,1,\dots)$. Consequently, we construct the exact solutions of original Eq. (2).

Exact solutions of model equation

The dimensionless form of NLSE with spatio-temporal dispersion is given by [7]

$$iq_t + aq_{tx} + bq_{xx} + cF\left(|q|^2\right)q = 0 \tag{10}$$

where x represents the non-dimensional distance along the fiber, t represents temporal variable in dimensionless form and a, b and c are real valued constants. The dependent variable $q(x,t)$ is a complex valued wave profile.

The coefficient of the constant a is spatio-temporal dispersion and the coefficient of constant b is group-velocity dispersion. In addition, c is the coefficient of the nonlinear term where the functional F represents the non-Kerr law nonlinearity in general. The nonlinear function $F\left(|q|^2\right)q$ is k times continuously differentiable, so that [10]

$$F\left(|q|^2\right)q \in \bigcup_{m,n=1}^{\infty} C^k((-n,n)\times(-m,m);\ R^2).$$

Eq. (10) reduces to the standart NLSE when $c = 0$, and it arises as a model for nonlinear pulse propagation in monomode optical fibers [8].

In order to solve Eq. (10), we use the following wave transformation

$$q(x,t) = U(\xi)e^{i\Phi(x,t)} \tag{11}$$

where $U(\xi)$ represents the shape of the pulse and

$$\xi = k(x - vt), \tag{12}$$

$$\Phi(x,t) = -\kappa x + \omega t + \theta. \tag{13}$$

The function $\Phi(x,t)$ is the phase component of the soliton, κ is the soliton frequency, ω is the wave number of the soliton, θ is the phase constant and v is the velocity of the soliton.

Plugging Eq. (11) into Eq. (10) and then decomposing into imaginary and real parts, the following equations are deduced

$$v = \frac{a\omega - 2b\kappa}{1 - a\kappa}, \tag{14}$$

$$k^2(b - av)U'' - (\omega - a\omega\kappa + b\kappa^2)U + cF\left(U^2\right)U = 0, \tag{15}$$

respectively.

Kerr-law nonlinearity

The Kerr-law of nonlinearity occurs in the case of if a light wave in an optical fiber faces nonlinear responses from nonharmonic motion of electrons bound in molecules, caused by an external electric field [11].

The Kerr law nonlinearity is the case when $F(s) = s$. For Kerr law nonlinearity, (10) reads

$$iq_t + aq_{tx} + bq_{xx} + c|q|^2q = 0 \tag{16}$$

and Eq. (15) converts to

$$k^2(b - av)U'' - (\omega - a\omega\kappa + b\kappa^2)U + cU^3 = 0. \tag{17}$$

Balancing U'' with U^3 in Eq. (17) yields $N = 1$. Therefore the solution form of Eq. (17) has the following expression

$$U(\xi) = A_0 + A_{1,0}\phi(\xi) + A_{0,1}\psi(\xi) + B_{1,0}\phi^{-1}(\xi) + B_{0,1}\psi^{-1}(\xi) \tag{18}$$

Substituting $U(\xi)$ and its necessary derivatives into (17) and equating all the coefficients of $\phi(\xi)\psi(\xi)$ to zero, thus we get highly complicated a system of algebraic equations. Handling this system with the help of Maple or Mathematica, we reach to the following results:

Result 1.

$$A_0 = \pm\frac{kS_1(av - b)}{c\sqrt{\frac{2av-2b}{c}}}, \quad A_{0,1} = 0, \quad A_{1,0} = 0,$$

$$B_{0,1} = \pm\sqrt{\frac{2av - 2b}{c}}S_0k, \quad B_{1,0} = 0,$$

$$S_2 = -\frac{1}{4}\frac{-k^2S_1^2av + k^2S_1^2b + 2b\kappa^2 - 2a\omega\kappa + 2\omega}{S_0k^2(av-b)}. \tag{19}$$

Then, the solutions of (16) corresponding to (19) are

$$q(x,t) = \left(\pm\frac{kS_1(av-b)}{c\sqrt{\frac{2av-2b}{c}}} \pm \sqrt{\frac{2av-2b}{c}}S_0k\psi^{-1}(\xi)\right)e^{i(-\kappa x+\omega t+\theta)} \tag{20}$$

where

$$\psi(\xi) = \frac{-S_1}{2S_2} - \frac{\sqrt{\mu}}{2S_2}\tanh\left(\frac{\sqrt{\mu}}{2}k(x-vt)+\xi_0\right), \quad \mu>0, \tag{21}$$

$$\psi(\xi) = \frac{-S_1}{2S_2} - \frac{\sqrt{\mu}}{2S_2}\coth\left(\frac{\sqrt{\mu}}{2}k(x-vt)+\xi_0\right), \quad \mu>0, \tag{22}$$

$$\psi(\xi) = \frac{-S_1}{2S_2} + \frac{\sqrt{-\mu}}{2S_2}\tan\left(\frac{\sqrt{-\mu}}{2}k(x-vt)+\xi_0\right), \quad \mu<0, \tag{23}$$

$$\psi(\xi) = \frac{-S_1}{2S_2} - \frac{\sqrt{-\mu}}{2S_2}\cot\left(\frac{\sqrt{-\mu}}{2}k(x-vt)+\xi_0\right), \quad \mu<0, \tag{24}$$

$$\psi(\xi) = \frac{-S_1}{2S_2} - \frac{1}{S_2k(x-vt)+\xi_0}, \quad \mu=0, \tag{25}$$

and

$$\mu = S_1^2 - 4S_0S_2 = \frac{2(\kappa^2b - \kappa a\omega + \omega)}{k^2(av-b)}.$$

One can see that the solutions of (20) include the solutions of Riccati equation. They physically represent dark soliton, singular soliton, singular periodic solutions and a family of rational soliton solutions, respectively.

Result 2.

$$A_0 = \pm\frac{\sqrt{\frac{2\kappa^2b-2\kappa a\omega+2\omega}{av-b}}(av-b)}{c\sqrt{\frac{2av-2b}{c}}}, \quad A_{0,1} = 0,$$

$$A_{1,0} = \pm\sqrt{\frac{2av-2b}{c}}R_2k,$$

$$B_{0,1} = 0,$$

$$B_{1,0} = 0, \quad R_1 = \pm\frac{1}{k}\sqrt{\frac{2\kappa^2b - 2\kappa a\omega + 2\omega}{av-b}}. \tag{26}$$

Then, the solutions of (16) corresponding to (26) are

$$q(x,t) = \left(\pm\frac{\sqrt{\frac{2\kappa^2b-2\kappa a\omega+2\omega}{av-b}}(av-b)}{c\sqrt{\frac{2av-2b}{c}}} \pm \sqrt{\frac{2av-2b}{c}}R_2k\phi(\xi)\right)e^{i(-\kappa x+\omega t+\theta)} \tag{27}$$

where

$$\phi(\xi) = \frac{R_1}{R_2 + R_1\exp(R_1k(x-vt)+\xi_0)}, \quad R_1 \neq 0 \tag{28}$$

$$\phi(\xi) = \frac{-1}{R_2k(x-vt)+\xi_0}, \quad R_1 = 0. \tag{29}$$

One can see that the solutions of (27) include the solutions of Bernoulli equation. They physically represent a new family of soliton solutions and a family of rational soliton solutions, respectively.

Result 3.

$$A_0 = 0, \quad A_{0,1} = \pm\sqrt{\frac{2av-2b}{c}}S_2k, \quad A_{1,0} = 0,$$

$$B_{0,1} = \pm\frac{1}{2}\frac{b\kappa^2 - a\omega\kappa + \omega}{c\sqrt{\frac{2av-2b}{c}}S_2k}, \quad B_{1,0} = 0,$$

$$S_0 = \frac{1}{4}\frac{b\kappa^2 - a\omega\kappa + \omega}{k^2S_2(av-b)}, \quad S_1 = 0. \tag{30}$$

Then, the solutions of (16) corresponding to (30) are

$$q(x,t) = \left(\pm\sqrt{\frac{2av-2b}{c}}S_2k\psi(\xi) \pm \frac{1}{2}\frac{(b\kappa^2 - a\omega\kappa + \omega)}{c\sqrt{\frac{2av-2b}{c}}S_2k}\psi^{-1}(\xi)\right)e^{i(-\kappa x+\omega t+\theta)} \tag{31}$$

where $\psi(\xi)$ holds Eq. (9) and

$$\mu = S_1^2 - 4S_0S_2 = -\frac{\kappa^2b - \kappa a\omega + \omega}{k^2(av-b)}.$$

One can see that the solutions of (31) include the solutions of Riccati equation. They physically represent dark soliton, singular soliton, singular periodic solutions and a family of rational soliton solutions, respectively.

Result 4.

$$A_0 = 0, \quad A_{0,1} = \pm\sqrt{\frac{2av-2b}{c}}S_2k, \quad A_{1,0} = 0,$$

$$B_{0,1} = \pm\frac{1}{4}\frac{b\kappa^2 - a\omega\kappa + \omega}{c\sqrt{\frac{2av-2b}{c}}S_2k}, \quad B_{1,0} = 0,$$

$$S_0 = -\frac{1}{8}\frac{b\kappa^2 - a\omega\kappa + \omega}{k^2S_2(av-b)}, \quad S_1 = 0. \tag{32}$$

Then, the solutions of (16) corresponding to (32) are

$$q(x,t) = \left(\pm\sqrt{\frac{2av-2b}{c}}S_2k\psi(\xi) \pm \frac{1}{4}\frac{(b\kappa^2 - a\omega\kappa + \omega)}{c\sqrt{\frac{2av-2b}{c}}S_2k}\psi^{-1}(\xi)\right)e^{i(-\kappa x+\omega t+\theta)} \tag{33}$$

where $\psi(\xi)$ holds Eq. (9) and

$$\mu = S_1^2 - 4S_0S_2 = \frac{\kappa^2b - \kappa a\omega + \omega}{2k^2(av-b)}.$$

One can see that the solutions of (33) include the solutions of Riccati equation. They physically represent dark soliton, singular soliton, singular periodic solutions and a family of rational soliton solutions, respectively.

Result 5.

$$A_0 = \pm\frac{kS_1(av-b)}{c\sqrt{\frac{2av-2b}{c}}}, \quad A_{0,1} = \pm\sqrt{\frac{2av-2b}{c}}S_2k, \quad A_{1,0} = 0, \quad B_{0,1} = 0,$$

$$B_{1,0} = 0, \quad S_0 = -\frac{1}{4}\frac{-k^2S_1^2av + k^2S_1^2b + 2b\kappa^2 - 2a\omega\kappa + 2\omega}{k^2S_2(av-b)}. \tag{34}$$

Then, the solutions of (16) corresponding to (34) are

$$q(x,t) = \left(\pm\frac{kS_1(av-b)}{c\sqrt{\frac{2av-2b}{c}}} \pm \sqrt{\frac{2av-2b}{c}}S_2k\psi(\xi)\right)e^{i(-\kappa x+\omega t+\theta)} \tag{35}$$

where $\psi(\xi)$ holds Eq. (9) and

$$\mu = S_1^2 - 4S_0S_2 = \frac{2(\kappa^2 b - \kappa a\omega + \omega)}{k^2(av-b)}.$$

One can see that the solutions of (35) include the solutions of Riccati equation. They physically represent dark soliton, singular soliton, singular periodic solutions and a family of rational soliton solutions, respectively.

In this case, to the extent of our knowledge the solutions of (20), (27), (31) and (33) are new exact solutions. Additionally, we obtain a family of rational soliton solutions that is given by Eq. (35).

Power-law nonlinearity

The power-law nonlinearity is exhibited in various process such as semiconductors, higher-order photons and in nonlinear plasmas. It is readily seen that this law is a generalization to the Kerr-law nonlinearity [11].

In this case,

$$F(s) = s^n$$

so that Eq. (10) modifies to

$$iq_t + aq_{tx} + bq_{xx} + c|q|^{2n}q = 0 \tag{36}$$

where the parameter n is referred to as the nonlinearity parameter which is in the range $0 < n < 2$, and in particular $n \neq 2$ since this case leads to a self-focusing singularity. Then, Eq. (15) simplifies to

$$k^2(b-av)U'' - (\omega - a\omega\kappa + b\kappa^2)U + cU^{2n+1} = 0. \tag{37}$$

In order to balancing, we make the following ansatz

$$U = V^{\frac{1}{n}} \tag{38}$$

so that (37) transforms to

$$k^2(b-av)\left(nVV'' + (1-n)(V')^2\right) - n^2V^2(-a\omega\kappa + b\kappa^2 + \omega) + cn^2V^4 = 0. \tag{39}$$

Balancing VV'' with V^4 in Eq. (39) gives $N = 1$. Thus, we have the following series expansion

$$V(\xi) = A_0 + A_{1,0}\phi(\xi) + A_{0,1}\psi(\xi) + B_{1,0}\phi^{-1}(\xi) + B_{0,1}\psi^{-1}(\xi). \tag{40}$$

Plugging $V(\xi)$ and its necessary derivatives into (39) and equating all the coefficients of $\phi(\xi)\psi(\xi)$ to zero, we obtain a highly complicated system of algebraic equations. Solving this system with the help of Maple or Mathematica, we yield the following result:

$$A_0 = 0, \quad A_{0,1} = \pm\frac{1}{4}\frac{n(\omega - a\omega\kappa + b\kappa^2)\sqrt{\frac{anv+av-bn-b}{c}}}{S_0k(av-b)}, \quad A_{1,0} = 0,$$

$$B_{1,0} = 0,$$

$$B_{0,1} = \pm\frac{\sqrt{\frac{anv+av-bn-b}{c}}}{n}S_0k, \quad S_1 = 0, \quad S_2 = \frac{n^2(\omega - a\omega\kappa + b\kappa^2)}{4(av-b)S_0k^2}. \tag{41}$$

Then, the solutions of (36) corresponding to (42) are

$$q(x,t) = \left(\pm\frac{1}{4}\frac{n(\omega - a\omega\kappa + b\kappa^2)\sqrt{\frac{anv+av-bn-b}{c}}}{S_0k(av-b)}\psi(\xi) \pm\frac{\sqrt{\frac{anv+av-bn-b}{c}}}{n}S_0k\psi^{-1}(\xi)\right)^{\frac{1}{n}} e^{i(-\kappa x+\omega t+\theta)} \tag{42}$$

where $\psi(\xi)$ holds Eq. (9) and

$$\mu = S_1^2 - 4S_0S_2 = -\frac{n^2(\kappa^2 b - \kappa a\omega + \omega)}{k^2(av-b)}.$$

One can see that the solutions of (42) include the solutions of Riccati equations. They physically represent dark soliton, singular soliton, singular periodic solutions and a family of rational soliton solutions, respectively.

In this case, to the extent of our knowledge the solutions of (42) are new exact solutions. It is interesting to note that any solution could not be detected in [3] while we could obtain a plenty of new exact physically important solutions.

Parabolic-law nonlinearity

This law arises in the nonlinear interaction between Langmuir waves and electrons. It describes the nonlinear interaction between the high frequency Langmuir waves and the ion-acoustic waves by ponder-motive forces [11].

For parabolic-law nonlinearity,

$$F(s) = \alpha s + \beta s^2$$

where α and β are constants. The form of the Eq. (10) in this case is

$$iq_t + aq_{tx} + bq_{xx} + c\left(\alpha|q|^2 + \beta|q|^4\right)q = 0, \tag{43}$$

and Eq. (15) converts to

$$k^2(b-av)U'' - (\omega - a\omega\kappa + b\kappa^2)U + \alpha U^3 + \beta U^5 = 0. \tag{44}$$

In order to balancing, we make the following ansatz

$$U = V^{\frac{1}{2}} \tag{45}$$

so that (44) transforms to

$$k^2(b-av)\left(2VV'' - (V')^2\right) - 4V^2(-a\omega\kappa + b\kappa^2 + \omega) + 4\alpha V^3 + 4\beta V^4 = 0. \tag{46}$$

Balancing VV'' with V^4 in Eq. (46) yields $N = 1$. Thus, we have the following series expansion

$$V(\xi) = A_0 + A_{1,0}\phi(\xi) + A_{0,1}\psi(\xi) + B_{1,0}\phi^{-1}(\xi) + B_{0,1}\psi^{-1}(\xi). \tag{47}$$

Inserting $V(\xi)$ and its necessary derivatives into (46) and equating all the coefficients of $\phi(\xi)\psi(\xi)$ to zero, we obtain a highly complicated system of algebraic equations. Solving this system with the help of Maple or Mathematica, we reach to the following results:

Result 1.

$$A_0 = -\frac{3\alpha}{4\beta}, \quad A_{0,1} = \pm\frac{3}{64}\frac{16\kappa^2 b\beta - 16\kappa a\beta\omega + 3\alpha^2 + 16\beta\omega}{\sqrt{\frac{3av-3b}{4\beta}}S_0k\beta^2}, \quad A_{1,0} = 0,$$

$$B_{0,1} = \pm\sqrt{\frac{3av-3b}{4\beta}}S_0k, \quad B_{1,0} = 0, \quad S_1 = \pm\frac{3\alpha}{4\beta k\sqrt{\frac{3av-3b}{4\beta}}},$$

$$S_2 = \frac{1}{16}\frac{16\kappa^2 b\beta - 16\kappa a\beta\omega + 3\alpha^2 + 16\beta\omega}{(av-b)\beta S_0k^2}. \tag{48}$$

Then, the solutions of (43) corresponding to (48) are

$$q(x,t)=\left(-\frac{3\alpha}{4\beta}\pm\frac{3}{64}\frac{16\kappa^2b\beta-16\kappa a\beta\omega+3\alpha^2+16\beta\omega}{\sqrt{\frac{3av-3b}{4\beta}}S_0k\beta^2}\psi(\xi)\pm\sqrt{\frac{3av-3b}{4\beta}}S_0k\psi^{-1}(\xi)\right)^{\frac{1}{2}}e^{i(-\kappa x+\omega t+\theta)} \quad (49)$$

where $\psi(\xi)$ holds Eq. (9) and

$$\mu=S_1^2-4S_0S_2=-\frac{4(\kappa^2b-\kappa a\omega+\omega)}{k^2(av-b)}.$$

One can see that the solutions of (49) include the solutions of Riccati equation. They physically represent dark soliton, singular soliton, singular periodic solutions and a family of rational soliton solutions, respectively.

Result 2.

$$A_0=\frac{-\alpha\pm2\sqrt{4\kappa^2b\beta-4\kappa a\beta\omega+\alpha^2+4\beta\omega}}{4\beta},$$

$$A_{1,0}=0,\quad B_{0,1}=\pm\sqrt{\frac{3av-3b}{4\beta}}S_0k,\quad B_{1,0}=0,$$

$$S_1=\pm\frac{\alpha\pm4\sqrt{4\kappa^2b\beta-4\kappa a\beta\omega+\alpha^2+4\beta\omega}}{4\beta\sqrt{\frac{3av-3b}{4\beta}}k},$$

$$A_{0,1}=\pm\frac{\lambda_1}{64\lambda_2},$$

$$\begin{aligned}\lambda_1=&32\kappa^2b\beta\left(-\alpha\pm2\sqrt{4\kappa^2b\beta-4\kappa a\beta\omega+\alpha^2+4\beta\omega}\right)\\&-32\kappa a\beta\omega\left(-\alpha\pm2\sqrt{4\kappa^2b\beta-4\kappa a\beta\omega+\alpha^2+4\beta\omega}\right)\\&-16\kappa^2\alpha b\beta+16\kappa a\alpha\beta\omega+8\alpha^2\left(-\alpha\pm2\sqrt{4\kappa^2b\beta-4\kappa a\beta\omega+\alpha^2+4\beta\omega}\right)\\&+32\beta\omega\left(-\alpha\pm2\sqrt{4\kappa^2b\beta-4\kappa a\beta\omega+\alpha^2+4\beta\omega}\right)-3\alpha^3-16\omega\beta\alpha,\end{aligned}$$

$$\lambda_2=\sqrt{\frac{3av-3b}{4\beta}}S_0k\beta^2\left(\alpha\pm4\sqrt{4\kappa^2b\beta-4\kappa a\beta\omega+\alpha^2+4\beta\omega}\right),$$

$$S_2=\frac{\lambda_3}{48\lambda_4},$$

$$\begin{aligned}\lambda_3=&32\kappa^2b\beta\left(-\alpha\pm2\sqrt{4\kappa^2b\beta-4\kappa a\beta\omega+\alpha^2+4\beta\omega}\right)\\&-32\kappa a\beta\omega\left(-\alpha\pm2\sqrt{4\kappa^2b\beta-4\kappa a\beta\omega+\alpha^2+4\beta\omega}\right)\\&-16\kappa^2\alpha b\beta+16\kappa a\alpha\beta\omega+8\alpha^2\left(-\alpha\pm2\sqrt{4\kappa^2b\beta-4\kappa a\beta\omega+\alpha^2+4\beta\omega}\right)\\&+32\beta\omega\left(-\alpha\pm2\sqrt{4\kappa^2b\beta-4\kappa a\beta\omega+\alpha^2+4\beta\omega}\right)-3\alpha^3-16\omega\beta\alpha,\end{aligned}$$

$$\lambda_4=(av-b)\beta S_0k^2\left(\alpha\pm4\sqrt{4\kappa^2b\beta-4\kappa a\beta\omega+\alpha^2+4\beta\omega}\right). \quad (50)$$

Then, the solutions of (43) corresponding to (50) are

$$q(x,t)=\left(\frac{-\alpha\pm2\sqrt{4\kappa^2b\beta-4\kappa a\beta\omega+\alpha^2+4\beta\omega}}{4\beta}\pm\frac{\lambda_1}{64\lambda_2}\psi(\xi)\pm\sqrt{\frac{3av-3b}{4\beta}}S_0k\psi^{-1}(\xi)\right)^{\frac{1}{2}}e^{i(-\kappa x+\omega t+\theta)} \quad (51)$$

where $\psi(\xi)$ holds Eq. (9) and

$$\mu=S_1^2-4S_0S_2=\frac{\lambda_5}{\lambda_6},$$

$$\begin{aligned}\lambda_5=&56\kappa^2\alpha b\beta+16\kappa^2b\beta\left(-\alpha\pm2\sqrt{4\kappa^2b\beta-4\kappa a\beta\omega+\alpha^2+4\beta\omega}\right)\\&-56\kappa a\alpha\beta\omega-16\kappa a\beta\omega\left(-\alpha\pm2\sqrt{4\kappa^2b\beta-4\kappa a\beta\omega+\alpha^2+4\beta\omega}\right)\\&+15\alpha^3+5\alpha^2\left(-\alpha\pm2\sqrt{4\kappa^2b\beta-4\kappa a\beta\omega+\alpha^2+4\beta\omega}\right)\\&+56\omega\beta\alpha+16\beta\omega\left(-\alpha\pm2\sqrt{4\kappa^2b\beta-4\kappa a\beta\omega+\alpha^2+4\beta\omega}\right),\end{aligned}$$

$$\lambda_6=2k^2\beta(av-b)\left(\alpha\pm4\sqrt{4\kappa^2b\beta-4\kappa a\beta\omega+\alpha^2+4\beta\omega}\right).$$

One can see that the solutions of (51) include the solutions of Riccati equation. They physically represent dark soliton, singular soliton, singular periodic solutions and a family of rational soliton solutions, respectively.

In this case, to the extent of our knowledge the solutions of (49) and (51) are new exact solutions (Figs. 1–7).

Dual-power law nonlinearity

Dual-power law nonlinearity is used to explain the saturation of the nonlinear refractive index. Moreover, this law serves as a basic model to describe the solitons in photovoltaic-photorefractive materials such as LiNbO3 [11].

In this case,

$$F(s)=\alpha s^n+\beta s^{2n}$$

where α and β are constants. As can be seen that this law is a generalization of the parabolic law nonlinearity. Therefore, for this constraint of $F(s)$, Eq. (10) reduces to

$$iq_t+aq_{tx}+bq_{xx}+c\left(\alpha|q|^{2n}+\beta|q|^{4n}\right)q=0, \quad (52)$$

and Eq. (15) converts to

$$k^2(b-av)U''-(\omega-a\omega\kappa+b\kappa^2)U+\alpha U^{2n+1}+\beta U^{4n+1}=0. \quad (53)$$

In order to balancing, we make the following ansatz

$$U=V^{\frac{1}{2n}} \quad (54)$$

so that (53) transforms to

$$\begin{aligned}&k^2(b-av)\left(2nVV''+(1-2n)(V')^2\right)\\&-4n^2V^2(-a\omega\kappa+b\kappa^2+\omega)+4\alpha n^2V^3+4\beta n^2V^4=0. \quad (55)\end{aligned}$$

Balancing VV'' with V^4 in Eq. (55) yields $N=1$. Thus, we have the following series expansion

$$V(\xi)=A_0+A_{1,0}\phi(\xi)+A_{0,1}\psi(\xi)+B_{1,0}\phi^{-1}(\xi)+B_{0,1}\psi^{-1}(\xi). \quad (56)$$

Inserting $V(\xi)$ and its necessary derivatives into (55) and equating all the coefficients of $\phi(\xi)\psi(\xi)$ to zero, we obtain a highly complicated system of algebraic equations. Solving this system with the help of Maple or Mathematica, we get the following results:

Result 1.

$$A_0=0,\quad A_{0,1}=0,$$

$$A_{1,0}=\frac{2R_2(\kappa^2bn-\kappa an\omega+\kappa^2b-\kappa a\omega+n\omega+\omega)}{\alpha R_1},$$

$$B_{0,1}=0,$$

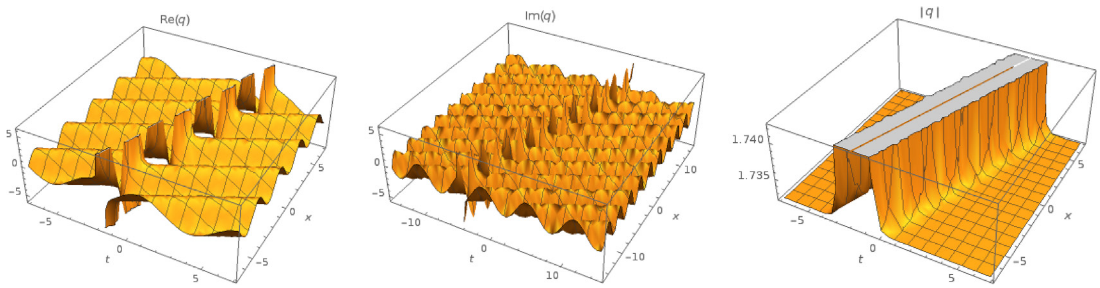

Fig. 1. The plot of (20) with (21) when $K = 2, b = a = \omega = k = c = S_1 = S_0 = \theta = \xi_0 = 1$.

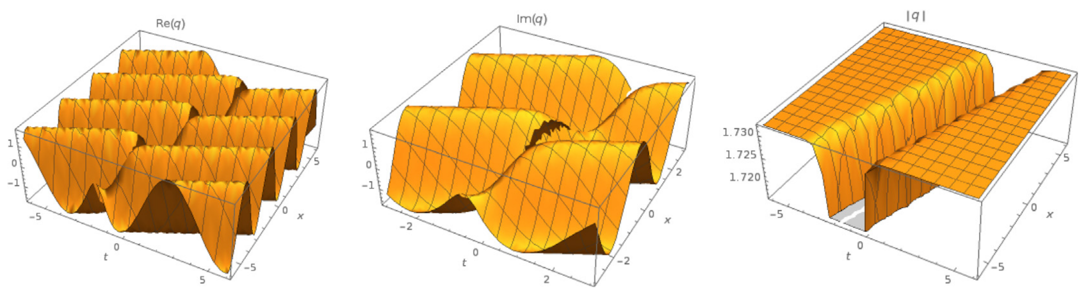

Fig. 2. The plot of (20) with (22) when $K = 2, b = a = \omega = k = c = S_1 = S_0 = \theta = \xi_0 = 1$.

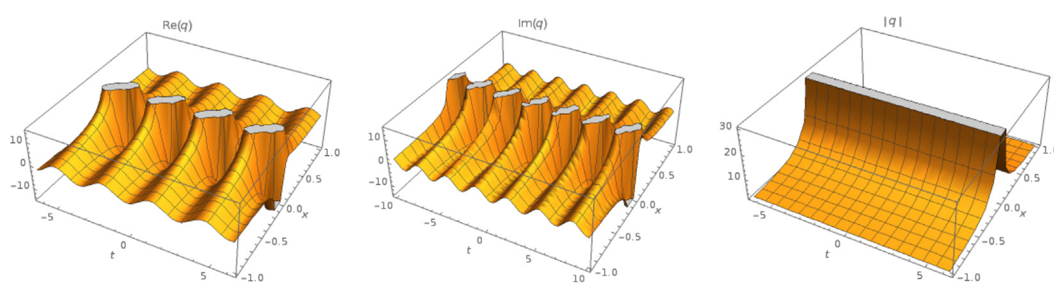

Fig. 3. The plot of (20) with (23) when $K = -\frac{1}{2}, b = 2, a = -1, \omega = 2, k = -2, c = S_1 = S_0 = \theta = \xi_0 = 1$.

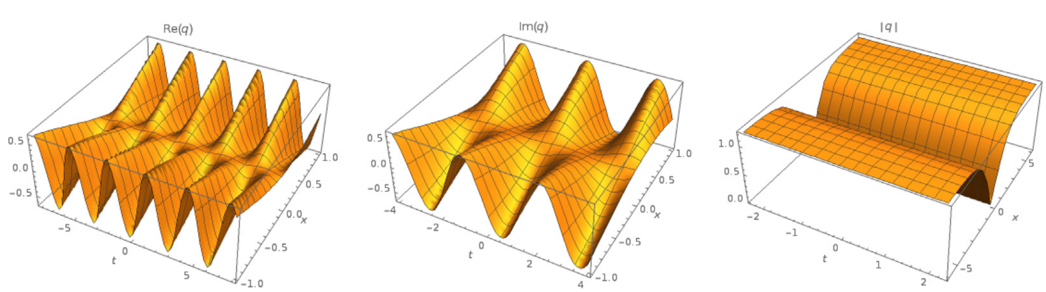

Fig. 4. The plot of (20) with (24) when $K = -\frac{1}{2}, b = 2, a = -1, \omega = 2, k = -2, c = S_1 = S_0 = \theta = \xi_0 = 1$.

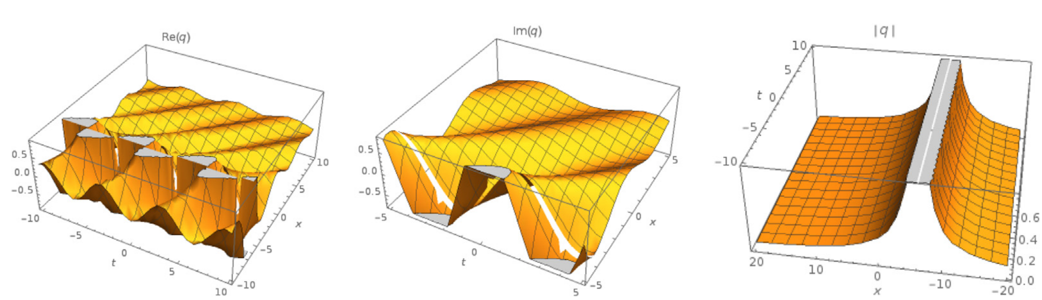

Fig. 5. The plot of (20) with (25) when $a = 2, K = b = \omega = k = c = S_1 = S_0 = \theta = \xi_0 = 1$.

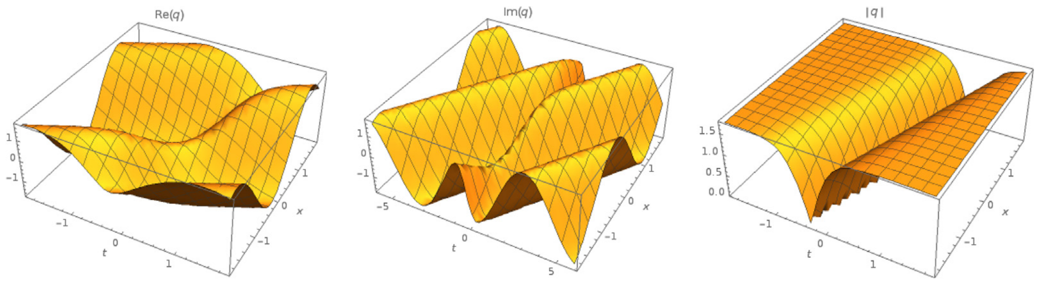

Fig. 6. The plot of (27) with (28) when $K = 2, b = a = \omega = k = c = R_2 = \theta = \xi_0 = 1$.

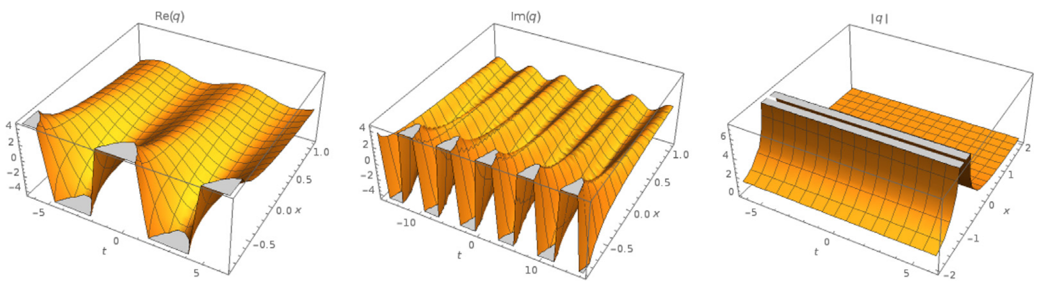

Fig. 7. The plot of (27) with (29) when $a = 2, K = b = \omega = k = c = R_2 = \theta = \xi_0 = 1$.

$$B_{1,0} = 0, \quad R_1 = \pm\frac{2n\sqrt{-(av-b)(\kappa^2 b - \kappa a\omega + \omega)}}{k(av-b)},$$

$$\beta = -\frac{(2\kappa^2 bn - 2\kappa an\omega + \kappa^2 b - \kappa a\omega + 2n\omega + \omega)\alpha^2}{4(\kappa^2 bn - \kappa an\omega + \kappa^2 b - \kappa a\omega + n\omega + \omega)^2}. \tag{57}$$

Then, the solutions of (52) corresponding to (57) are

$$q(x,t) = \left(\frac{2R_2(\kappa^2 bn - \kappa an\omega + \kappa^2 b - \kappa a\omega + n\omega + \omega)}{\alpha R_1}\phi(\xi)\right)^{\frac{1}{2n}} e^{i(-\kappa x + \omega t + \theta)} \tag{58}$$

where $\psi(\xi)$ holds Eq. (8).

One can see that the solutions (58) include the solutions of Bernoulli equation. They physically represent a new family of soliton solutions and a family of rational soliton solutions, respectively.

Result 2.

$$A_0 = \frac{2(\kappa^2 bn - \kappa an\omega + \kappa^2 b - \kappa a\omega + n\omega + \omega)}{\alpha}, \quad A_{0,1} = 0,$$

$$A_{1,0} = -\frac{2R_2(\kappa^2 bn - \kappa an\omega + \kappa^2 b - \kappa a\omega + n\omega + \omega)}{\alpha R_1},$$

$$B_{0,1} = 0,$$

$$B_{1,0} = 0, \quad R_1 = \pm\frac{2n\sqrt{-(av-b)(\kappa^2 b - \kappa a\omega + \omega)}}{k(av-b)},$$

$$\beta = -\frac{(2\kappa^2 bn - 2\kappa an\omega + \kappa^2 b - \kappa a\omega + 2n\omega + \omega)\alpha^2}{4(\kappa^2 bn - \kappa an\omega + \kappa^2 b - \kappa a\omega + n\omega + \omega)^2}. \tag{59}$$

Then, the solutions of (52) corresponding to (59) are

$$q(x,t) = \left((\kappa^2 bn - \kappa an\omega + \kappa^2 b - \kappa a\omega + n\omega + \omega)\left(\frac{2}{\alpha} - \frac{2R_2}{\alpha R_1}\phi(\xi)\right)\right)^{\frac{1}{2n}} \times e^{i(-\kappa x + \omega t + \theta)} \tag{60}$$

where $\psi(\xi)$ holds Eq. (8).

One can see that the solutions of (60) include the solutions of Bernoulli equation. They physically represent a new family of soliton solutions and a family of rational soliton solutions, respectively.

In this case, to the extent of our knowledge, the solutions of (58) and (60) are new exact solutions.

Concluding remarks

In this paper, we have applied the extended Kudryashov method that has more advantageous than classical simplest equation method to NLSE equation with spatio-temporal dispersion which include the cases of Kerr, parabolic, power and dual power law nonlinearity effects. We successfully obtained the exact solutions which include topological, singular soliton, rational solutions and singular periodic solutions. Comparing our results with El-Borai et al.'s results [3], we conclude two important aspects. First, our solutions contain solutions of the Bernoulli equation. Second, classical solutions, inverse solutions and linear combination solutions of the Riccati equations which are $\psi^{-1}, a\psi + b\psi^{-1}, \psi$ is presented. However, the obtained solutions in [3] contain only solutions of the Riccati equation which is ψ.

We also emphasize that in Kerr-law nonlinearity case the solutions of [3] are exactly the coincide with our solutions (Results 1 and 5) when $S_1 = 0$.

The proposed method is direct, concise, effective and can be extended to other types of complex NLEEs. Yet the applicability of the method to NLSE depends on the balancing of reduced ODE. If one can not reveal the balance number then the aforementioned method does not work.

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