Effects of rotational restraints on the thermal buckling of carbon nanotube

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The thermal buckling response of nanotubes with rotational restraints is investigated using a Timoshenko beam model with non-local elasticity theory. Two trigonometric (Fourier) series are selected to analyse the thermal buckling of the non-local Timoshenko nanotube with rotational restraints. Explicit equations are obtained for the boundary values with a coefficient matrix. In particular, the new method can be degenerated to the non-local Euler beam model by assigning proper value to the shear correction factor. The main advantage of the present technique is its capability of dealing with deformable or rigid supporting conditions. Several numerical examples are solved to asses proposed approach reliability. The results show that thermal buckling including the thermal effects are lower than those without the thermal effects when the temperature rises. The rotational restraint springs have significant effects on the buckling response of nanotubes.

1. Introduction: Nanotechnology and nanoscience have opened a new area in engineering, materials science, medicine, chemical, energy production, biomaterials and electronics leading to innovation and change. Graphene sheets and single-walled carbon nanotubes are two of the nanosized materials that have great potential in delineating of composite materials, gas detection, and new sensors and arouse attention among scientific communities. Single-walled carbon nanotubes are nanomaterials which have tremendous potential in designs of new structures, machines, composite materials, sensors, and gas detection. In recent years, several researchers have investigated carbon nanotubes and their correlation, using different mathematical theories and definitions [1, 2]. To design carbon nanotubes, nanomachines, and nanostructures, size-dependent different elasticity theories have been utilised such as strain, stress-based elastic models, peridynamics, and modified couple stress theory.

Small size effects are related to molecules, particles, and atoms that constitute the nanomaterials. Classical elasticity theories are not considered the size effects and these theories lack the accountability of the size effects arising from the atoms and molecules. One of the promising higher-order elasticity theories is Eringen's non-local elasticity theory [3], which considers the size effects and underlying physics within the integral formulation of this small size effect. Recently size-dependent continuum theories have been used to execute the strategies about the effect of the small size [4–12]. Due to the fact that classical elasticity theory cannot predict the mechanical behaviours of small-sized structures and machines, several researchers have proposed managing to predict the mechanical properties of this type of structure in recent years [13–21].

Literature review reveals that the conducted theoretical and experimental studies on thermal buckling of nanotubes are based on the assumptions that the supporting conditions are classical (simply supported, clamped, and free). A very limited literature is available for nanotubes with deformable boundary conditions. The attempt of this study is to present a semi-analytical method for investigating the thermal buckling of rotationally restrained nanotubes with non-local elasticity theory. For this purpose, the theoretical formulation of the non-local Timoshenko beam model with rotational restraints is presented at first. Two Fourier infinite series with Stokes' transformation are being utilised and analytical solutions for the thermal buckling load are obtained. The influence of rotational restraints and non-local effects of the nanotubes on the thermal buckling temperature are discussed and investigated in detail. The main subject of this study is that the possibility of enhancing the buckling temperature of carbon nanotubes by using rotational restraints.

2. Non-local Timoshenko beam theory: Fig. 1 shows a carbon nanotube with rotational restraints of length L. For the non-local Timoshenko beam theory. The following partial differential equation is used [3]:

$$\sigma_{ij}^{\mathrm{nl}} - (e_0 a)^2 \nabla^2 \sigma_{ij}^{\mathrm{nl}} = C{:}\epsilon, \tag{1}$$

where ϵ expresses the fourth-order strain tensor, C represents the elasticity tensor, and e_0 and a represent the material constant and internal characteristic length, respectively. Equation (1) may be approximated to the following compact form:

$$\sigma_{xx}^{\mathrm{nl}} - (e_0 a)^2 \frac{\partial^2 \sigma_{xx}^{\mathrm{nl}}}{\partial x^2} = E\epsilon_{xx}, \tag{2}$$

$$\tau_{xz}^{\mathrm{nl}} - (e_0 a)^2 \frac{\partial^2 \tau_{xz}^{\mathrm{nl}}}{\partial x^2} = G\gamma_{xz}, \tag{3}$$

where E represents the elasticity modulus and G denotes the shear modulus. σ_{xx} is the axial stress, ϵ_{xx} is the axial strain, τ_{xz} is the shear stress, and γ_{xz} is the shear strain. The following relations can be written:

$$\epsilon_{xx} = z\frac{\partial \phi}{\partial x}, \tag{4}$$

$$\gamma_{xz} = \frac{\partial w}{\partial x} - \phi, \tag{5}$$

where ϕ is the rotation and w is the transverse displacement caused by bending. The following differentiation relations are often used in Timoshenko beam theory:

$$\frac{\partial V}{\partial x} = -\psi_{\mathrm{T}} \frac{\partial^2 w}{\partial x^2}, \tag{6}$$

$$\frac{\partial M}{\partial x} + V = 0, \tag{7}$$

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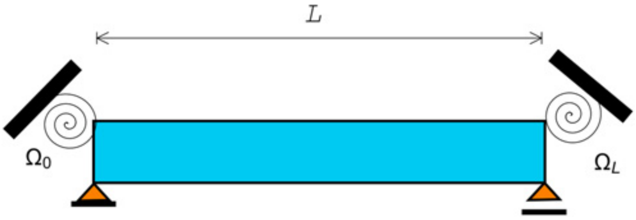

Fig. 1 *Carbon nanotube with rotational springs at both ends*

where V is the shear force, M is the bending moment and ψ_T is the thermal force, which may be expressed as following form [22]:

$$\psi_T = -\frac{E\alpha TA}{1-2\nu}, \tag{8}$$

where α denotes the thermal expansion coefficient, A is the cross-sectional area, T is the temperature change and ν is the Poisson's ratio. The shear force and bending moment can be shown as follows:

$$V = \int_A z\sigma_{xx}\,\mathrm{d}A, \tag{9}$$

$$M = \int_A \tau_{xy}\,\mathrm{d}A. \tag{10}$$

The following relations are obtained by using the above equations [22]:

$$M - (e_0 a)^2 \frac{\partial^2 M}{\partial x^2} = EI\frac{\partial \phi}{\partial x}, \tag{11}$$

$$V - (e_0 a)^2 \frac{\partial^2 V}{\partial x^2} = \kappa AG\left(\frac{\partial \phi}{\partial x} - \phi\right), \tag{12}$$

where κ denotes the shear correction factor. The moment of inertia can be written the following relation:

$$I = \int_A z^2\,\mathrm{d}A. \tag{13}$$

The following relation can be obtained from (7) and (11):

$$M = EI\frac{\partial \phi}{\partial x} + (e_0 a)^2\left(-\frac{\partial V}{\partial x}\right). \tag{14}$$

Using (6) and (12), the following equation can be found in terms of thermal normal force:

$$M = EI\frac{\partial \phi}{\partial x} + (e_0 a)^2\left(\psi_T\frac{\partial^2 w}{\partial x^2}\right). \tag{15}$$

Based on (6) and (12), it can be derived that

$$V = \kappa AG\left(\frac{\partial \phi}{\partial x} - \phi\right) - (e_0 a)^2\left(-\psi_T\frac{\partial^3 w}{\partial x^3}\right). \tag{16}$$

Inserting (16) into (6), we can obtain

$$\kappa AG\left(\frac{\partial^2 w}{\partial x^2} - \frac{\partial \phi}{\partial x}\right) + (e_0 a)^2\left(\psi_T\frac{\partial^4 w}{\partial x^4}\right) = -\psi_T\frac{\partial^2 w}{\partial x^2}. \tag{17}$$

3. Transverse displacement and rotation: The following solution function is selected for the lateral displacement:

$$w_0 \quad x = 0, \tag{18}$$

$$w_L \quad x = L, \tag{19}$$

$$w(x) = \sum_{k=1}^{\infty} A_k \sin\left(\frac{k\pi x}{L}\right) \quad 0 < x < L. \tag{20}$$

Similarly, the rotation function is taken as

$$\phi_0 \quad x = 0, \tag{21}$$

$$\phi_L \quad x = L, \tag{22}$$

$$\phi(x) = \sum_{k=1}^{\infty} B_k \cos\left(\frac{k\pi x}{L}\right) \quad 0 < x < L. \tag{23}$$

The Fourier coefficient in (20) can be written as follows:

$$A_k = \frac{2}{L}\int_0^L \phi(x)\sin\left(\frac{n\pi x}{L}\right)\mathrm{d}x, \tag{24}$$

Taking the first derivative of (20) yields

$$w'(x) = \sum_{k=1}^{\infty} \frac{k\pi x}{L} C_k \cos\left(\frac{k\pi x}{L}\right), \tag{25}$$

The function $w'(x)$ is represented by a Fourier cosine series

$$w'(x) = \frac{b_0}{L} + \sum_{k=1}^{\infty} b_k \cos\left(\frac{k\pi x}{L}\right). \tag{26}$$

The Fourier coefficients in (26) are given by

$$b_0 = \frac{2}{L}\int_0^L w'(x)\,\mathrm{d}x = \frac{2}{L}[w(L) - w(0)], \tag{27}$$

$$b_k = \frac{2}{L}\int_0^L w'(x)\cos\left(\frac{k\pi x}{L}\right)\mathrm{d}x \quad k = 1, 2, \ldots \tag{28}$$

We will need to use integration by parts in the following form:

$$b_k = \frac{2}{L}\left[w(x)\cos\left(\frac{k\pi x}{L}\right)\right]_0^L + \frac{2}{L}\left[\frac{k\pi}{L}\int_0^L w(x)\sin\left(\frac{k\pi x}{L}\right)\mathrm{d}x\right]. \tag{29}$$

The above equation can be shown as a following compact form:

$$b_k = \frac{2}{L}[(-1)^k w(L) - w(0)] + \frac{k\pi}{L}C_k. \tag{30}$$

The second-, third- and higher-order derivatives may be calculated with the use of a similar procedure. The higher-order derivatives of

$w(x)$ can be written as follows:

$$\frac{dw(x)}{dx} = \frac{w_L - w_0}{L} + \sum_{k=1}^{\infty} \cos(\beta_k x)\left(\frac{2((-1)^k w_L - w_0)}{L} + \beta_k A_k\right), \tag{31}$$

$$\frac{d^2w(x)}{dx^2} = -\sum_{k=1}^{\infty} \beta_k \sin(\beta_k x)\left(\frac{2((-1)^k w_L - w_0)}{L} + \beta_k A_k\right), \tag{32}$$

$$\frac{d^3w(x)}{dx^3} = \frac{w''_L - w''_0}{L} + \sum_{k=1}^{\infty} \cos(\beta_k x)\left(\frac{2((-1)^k w''_L - w''_0)}{L} - \beta_k^2\left(\frac{2((-1)^k w_L - w_0)}{L} + \beta_k A_k\right)\right), \tag{33}$$

$$\frac{d^4w(x)}{dx^4} = -\sum_{k=1}^{\infty} \alpha_k \sin(\beta_k x)\left(\frac{2((-1)^k w_{L''} - w_{0''})}{L} - \alpha_k^2\left(\frac{2((-1)^n w_L - w_0)}{L} + \beta_k A_k\right)\right), \tag{34}$$

where

$$\beta_k = \frac{k\pi}{L}. \tag{35}$$

Similar applications can be made to rotation function. Taking the first derivative of rotation function with respect to x

$$\phi'(x) = -\sum_{k=1}^{\infty} \beta_k B_k \sin\left(\frac{k\pi x}{L}\right). \tag{36}$$

To write the Fourier sine series as a Fourier sine series, one more derivative should be computed

$$\phi''(x) = -\sum_{k=1}^{\infty} \beta_k^2 B_k \cos\left(\frac{k\pi x}{L}\right). \tag{37}$$

If the similar steps are repeated in [23–27], the following relations are derived:

$$\frac{d\phi(x)}{dx} = -\sum_{k=1}^{\infty} \beta_k B_k \sin(\beta_k x), \tag{38}$$

$$\frac{d^2\phi(x)}{dx^2} = (\phi'_L - \phi'_0) + \sum_{k=1}^{\infty} \cos(\beta_k x)\left(\frac{2((-1)^k \phi'_L - \phi'_0)}{L} - \beta_k^2 B_k\right), \tag{39}$$

$$\frac{d^3\phi(x)}{dx^3} = \sum_{k=1}^{\infty} \beta_k \sin(\beta_k x)\left(\frac{2((-1)^k \phi'_L - \phi'_0)}{L} - \beta_k^2 B_k\right). \tag{40}$$

The following Fourier coefficients are obtained from Fourier sine, cosine series, and Stokes' transformation:

$$A_k = -\frac{2AG\kappa(M_0 + (-1)^{k+1}M_L)}{EIL\psi_T\beta_k^3 + AGL\kappa(\psi_T + (-EI + \psi_T(e_0a)^2)\beta_k^2)}, \tag{41}$$

$$B_k = -\frac{2(\psi_T - AG\kappa)(M_0 + (-1)^{k+1}M_L)}{EIL\psi_T\beta_k^3 + AGL\kappa(\psi_T + (-EI + \psi_T(e_0a)^2)\beta_k^2)}, \tag{42}$$

where

$$M_0 = EI\phi'_0 + \psi_T(e_0a)^2 w''_0, \tag{43}$$

$$M_L = EI\phi'_L + \psi_T(e_0a)^2 w''_L. \tag{44}$$

The non-local boundary conditions can be written as follows by using the effect of rotational restraints:

$$EI\frac{d\phi(0)}{dx} + \psi_T(e_0a)^2\frac{d^2w(0)}{dx^2} = \Omega_0\phi_0, \tag{45}$$

$$EI\frac{d\phi(L)}{dx} + \psi_T(e_0a)^2\frac{d^2w(L)}{dx^2} = \Omega_L\phi_L. \tag{46}$$

Fourier coefficients in (41) and (42) are substituted into (32) and (38) to obtain

$$\left(-1 - \sum_{k=1}^{\infty}\frac{2L\Omega_0\Lambda_1}{EI\psi_T k^2\pi^2 + AG\Lambda_2\kappa}\right)M_0 + \left(-1 - \sum_{k=1}^{\infty}\frac{2L(-1)^k\Omega_0\Lambda_1}{EI\psi_T k^2\pi^2 + AG\Lambda_2\kappa}\right)M_L = 0, \tag{47}$$

$$\left(-1 - \sum_{k=1}^{\infty}\frac{2L(-1)^k\Omega_L\Lambda_1}{EI\psi_T k^2\pi^2 + AG\Lambda_2\kappa}\right)M_0 + \left(-1 - \sum_{k=1}^{\infty}\frac{2L\Omega_L\Lambda_1}{EI\psi_T k^2\pi^2 + AG\Lambda_2\kappa}\right)M_L = 0, \tag{48}$$

where

$$\Lambda_1 = \psi_T - AG\kappa, \tag{49}$$

$$\Lambda_2 = \psi_T L^2 + k^2\pi^2(-EI + \psi_T(e_0a)^2). \tag{50}$$

The following eigenvalue problem is obtained by the means of end moments

$$\begin{bmatrix} \delta_{11} & \delta_{12} \\ \delta_{21} & \delta_{22} \end{bmatrix}\begin{bmatrix} M_0 \\ M_L \end{bmatrix} = 0, \tag{51}$$

where

$$\delta_{11} = -1 - \sum_{k=1}^{\infty}\frac{2L\Omega_0\Lambda_1}{EI\psi_T k^2\pi^2 + AG\Lambda_2\kappa}, \tag{52}$$

$$\delta_{12} = -1 - \sum_{k=1}^{\infty}\frac{2L(-1)^k\Omega_0\Lambda_1}{EI\psi_T k^2\pi^2 + AG\Lambda_2\kappa}, \tag{53}$$

$$\delta_{21} = -1 - \sum_{k=1}^{\infty}\frac{2L(-1)^k\Omega_L\Lambda_1}{EI\psi_T k^2\pi^2 + AG\Lambda_2\kappa}, \tag{54}$$

$$\delta_{22} = -1 - \sum_{k=1}^{\infty}\frac{2L\Omega_L\Lambda_1}{EI\psi_T k^2\pi^2 + AG\Lambda_2\kappa}. \tag{55}$$

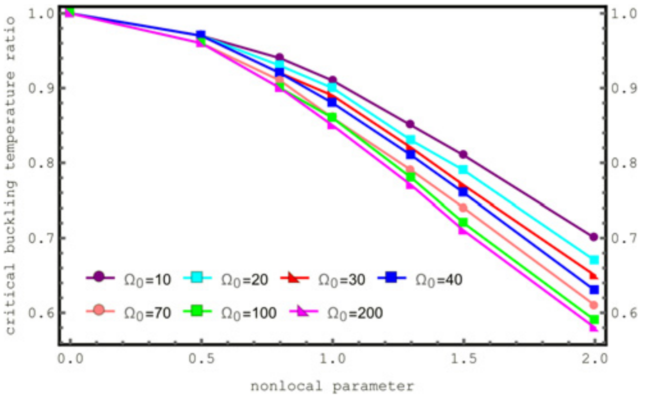

Fig. 2 *Effects of the non-local parameter on the critical non-dimensional thermal buckling temperature for various rotational restraint parameters*

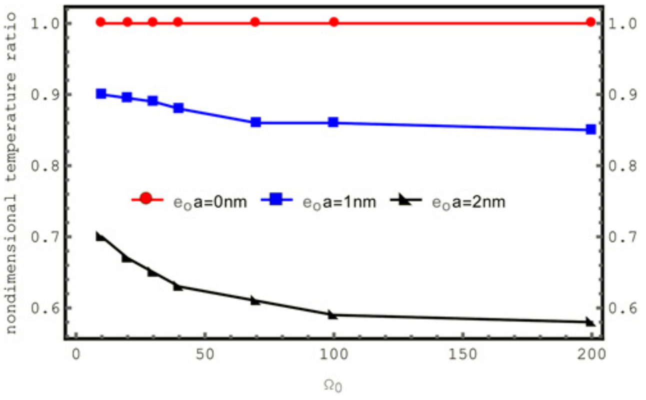

Fig. 4 *Effect of asymmetrical spring parameters on the critical non-dimensional thermal buckling temperature*

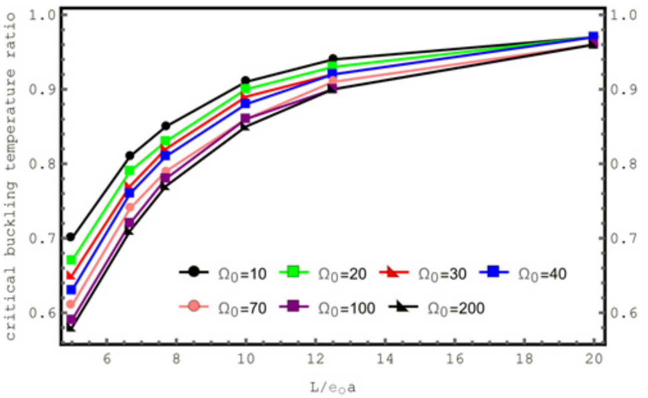

Fig. 3 *Effects both small-scale parameter and length on the critical non-dimensional thermal buckling temperature for various rotational restraint parameters*

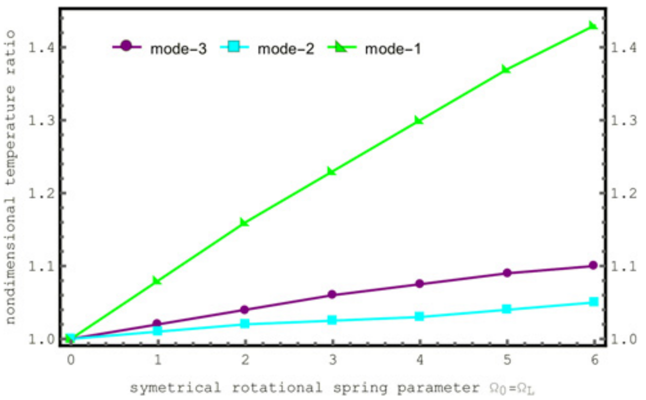

Fig. 5 *Effect of symmetrical spring parameters on the first three non-dimensional thermal buckling temperature*

The eigenvalues of the coefficient matrix can be derived by setting the following determinant in (51) to zero:

$$\left|\psi_{ij}\right| = 0 \quad (i, j = 1, 2). \tag{56}$$

4. Results and discussions: In this section, thermal buckling analysis of a carbon nanotube with rotational restraints under thermal axial loading is carried out. The non-local elasticity theory with Timoshenko beam theory is applied in order to capture size effects. The material parameters utilised in the computation are Poisson's ratio $\nu = 0.3$, the mass density $\rho = 2.3\,\text{g/cm}^3$, Young's modulus $E = 1\,\text{TPa}$, the shear coefficient $\kappa = 8/10$, the shear modulus $G = 0.4\,\text{TPa}$, and the temperature expansion coefficient $\alpha = 1.1 \times 10^{-6}\,\text{K}^{-1}$. It is pointed out that non-local parameter [28, 29] e_0a must be <2 nm for single-walled carbon nanotubes [30].

The variation of the critical non-dimensional thermal buckling temperature for various rotational restraint parameters with respect to different non-local parameters is illustrated in Fig. 2 at $\Omega_L = 0.0$. It is seen that by increasing the value of the non-local parameter, the magnitude of the critical non-dimensional thermal buckling temperature increases. It means that the non-local elasticity theory introduces a stiffness–softening effect, which is consistent with the literature. This decreasing trend is more obvious in higher values of rotational spring parameters.

It can be seen from Fig. 3 that when the ratio of the length to the non-local parameter is small, non-local effects are more prominent. However, the non-local effects on the critical non-dimensional thermal buckling temperature can be controlled by rotational restraints.

Fig. 4 presents the non-dimensional critical buckling temperature for a nanotube with asymmetrical spring parameters. This parametric example points out to the possibility of enhancing the buckling temperature of nanotubes for the different non-local parameter. It also shows the effectiveness of the present method to capture the significance of the rotational restraints at the ends, non-local parameters, and asymmetrical boundary conditions on the thermal buckling response of carbon nanotubes.

The dependence of the critical thermal buckling temperature on the symmetrical spring parameters is shown in Fig. 5. The non-local parameters $e_0a = 0$ and $e_0a = 0.5$ and the mode numbers 1, 2, and 3 are considered. It can be seen from Fig. 5 that the ranges of the buckling temperature for the first three mode number are quite different. It should be noted that the non-dimensionless buckling temperature here represents the ratio of with and without rotational restraint parameter. The present solution method can be extended to calculate the dynamical buckling load.

5. Conclusion: Thermal buckling analysis of a carbon nanotube subjected to axial thermal loading is explored by employing Timoshenko beam theory. The size effects are taken into account by using Eringen's non-local elasticity theory which contains one non-local parameter. Two Fourier series are used to represent the deflection and rotation. By implementing Stokes' transformation, a coefficient matrix is obtained including rotational restraint, non-local, and thermal parameters. Finally, the effect of different parameters such as non-locality parameters, length and rotational restrained parameters on the critical non-dimensional thermal buckling load of the carbon nanotube is investigated. It can be seen that the critical non-dimensional thermal buckling load decreases with the increases of the non-local parameter. Also, the small-scale parameter has a stiffness–softening effect and thermal buckling temperature. Also, rotational restraints at the ends possess a hardening effect and increase the critical buckling load and temperature.

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