Hindawi
Complexity
Volume 2017, Article ID 7101927, 11 pages
https://doi.org/10.1155/2017/7101927

Research Article

Synchronization and Electronic Circuit Application of Hidden Hyperchaos in a Four-Dimensional Self-Exciting Homopolar Disc Dynamo without Equilibria

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Received 3 February 2017; Accepted 19 March 2017; Published 31 May 2017

Academic Editor: Karthikeyan Rajagopal

We introduce and investigate a four-dimensional hidden hyperchaotic system without equilibria, which is obtained by augmenting the three-dimensional self-exciting homopolar disc dynamo due to Moffatt with an additional control variable. Synchronization of two such coupled disc dynamo models is investigated by active control and sliding mode control methods. Numerical integrations show that sliding mode control provides a better synchronization in time but causes chattering. The solution is obtained by switching to active control when the synchronization errors become very small. In addition, the electronic circuit of the four-dimensional hyperchaotic system has been realized in ORCAD-PSpice and on the oscilloscope by amplitude values, verifying the results from the numerical experiments.

1. Introduction

Hyperchaos is a feature of a chaotic system having more than one positive Lyapunov exponent [1]. Because of potential theoretical and practical applications in technology, such as secure communications, lasers, nonlinear circuits, neural networks, generation, control, and synchronization, hyperchaos has featured as an important research area in nonlinear science [2–5]. The theory about hidden hyperchaos with either only stable or no equilibrium states is still in its infancy and has only recently been understood by mathematicians [6–9].

In 1979, Moffatt identified inconsistencies in the modeling of a simple self-exciting homopolar disc dynamo because of the neglect of induced azimuthal eddy currents, which can be resolved by introducing a segmented disc dynamo [10]. Here we investigate hidden hyperchaos, synchronization, and electronic circuit realization for a higher-dimensional version of the self-exciting homopolar disc dynamo, which was not yet completely well understood.

Since Pecora and Carroll [11] investigated synchronization in chaotic systems in 1990, such behavior has become an important research area in nonlinear science, not only for understanding the complicated phenomena in various fields but also for its potential applications especially in secure communication and image encryption. Two indistinguishable chaotic systems, starting from nonidentical initial values, would evolve in time to completely different trajectories because of the sensitive dependence of chaotic systems to their initial values. The aim of synchronizing chaos is to ensure that the states track the desired trajectory. Many effective methods exist to deal with synchronization of chaotic and hyperchaotic systems. These include active control [12–20], passive control [21], sliding mode control [22–31], adaptive control [32], and backstepping design [33]. Of these, active control is an important simple method used in the synchronization of nonlinear systems. It maintains asymptotic stability at zero error by eliminating the nonlinear terms and making all the eigenvalues have negative real parts. The other commonly preferred method, sliding mode control, maintains the synchronization by enforcing the error system to stay on a constructed sliding surface.

The first model with a simple electronic application was realized by Chua et al. [34]. In the following years, many electronic circuit applications such as simple RLC, RC circuits [35–37], oscillators [38, 39], power circuits [40, 41], and capacitor circuits which show chaotic features were improved upon. On the one hand, numerous electronic circuit realizations with interesting features, which mimic novel chaotic and hyperchaotic systems, have been proposed in recent years [3, 42–45].

Current interest in hidden hyperchaotic attractors motivates us to study an extension about the self-exciting homopolar disc dynamo [10] to 4D homopolar dynamo without equilibria. The existence of hidden hyperchaotic attractors in this new disc dynamo is confirmed. Synchronization of two such coupled 4D self-exciting homopolar disc dynamo systems is analyzed with active and sliding mode control methods. Moreover, we have designed an electronic circuit and have used an oscilloscope to view the hyperchaotic rescaled dynamo without equilibria, implemented in real time.

2. Model and Hidden Hyperchaos of 4D Self-Exciting Homopolar Disc Dynamo System without Equilibria

Dynamo models have been the object of much interest in order to understand both the generation of magnetic fields and their reversals in astrophysics. Moffatt [10] extended the simplest self-exciting Bullard dynamo to include radial magnetic diffusion, to produce the disc dynamo model, written nondimensionally as

$$\begin{aligned} \dot{x} &= r(y-x),\\ \dot{y} &= mx-(1+m)y+xz,\\ \dot{z} &= g\left[1+mx^2-(1+m)xy\right], \end{aligned} \tag{1}$$

where x, y, and z are state variables and r, m, and g are positive parameters. x and y represent the magnetic fluxes due to radial and azimuthal current distributions, respectively. z denotes the angular velocity of the disc. g is the applied torque; r and m are the constants which depend on the inductances and the electrical resistance of the dynamo.

By modifying the characteristics of the segmented disc dynamo (1), hidden chaotic or hyperchaotic spiral attractors have been observed numerically under special initial conditions with two symmetric stable node-foci. This leads to the interesting and striking observation of multiple attractors for a broad range of parameters. As in many nonlinear dynamical systems, the occurrence of multiple attractors implies the existence of multistability in the self-exciting dynamo, with the long-term behavior being fundamentally different depending on which basin of attraction the initial conditions belong. Now, we introduce a dislocated feedback controller to system (1) as a new state w and translate z to $z-m$ to result in following 4D system:

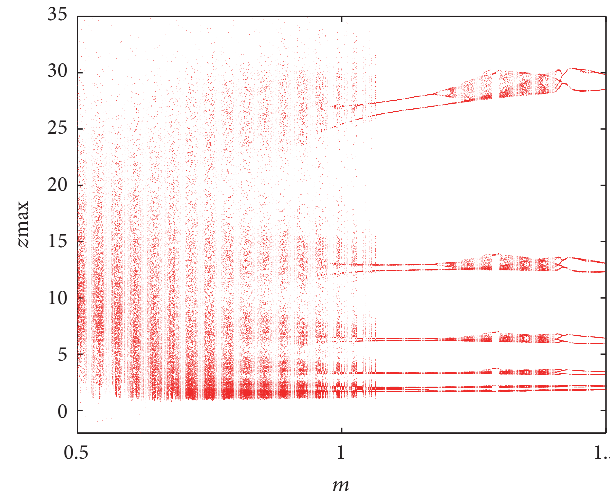


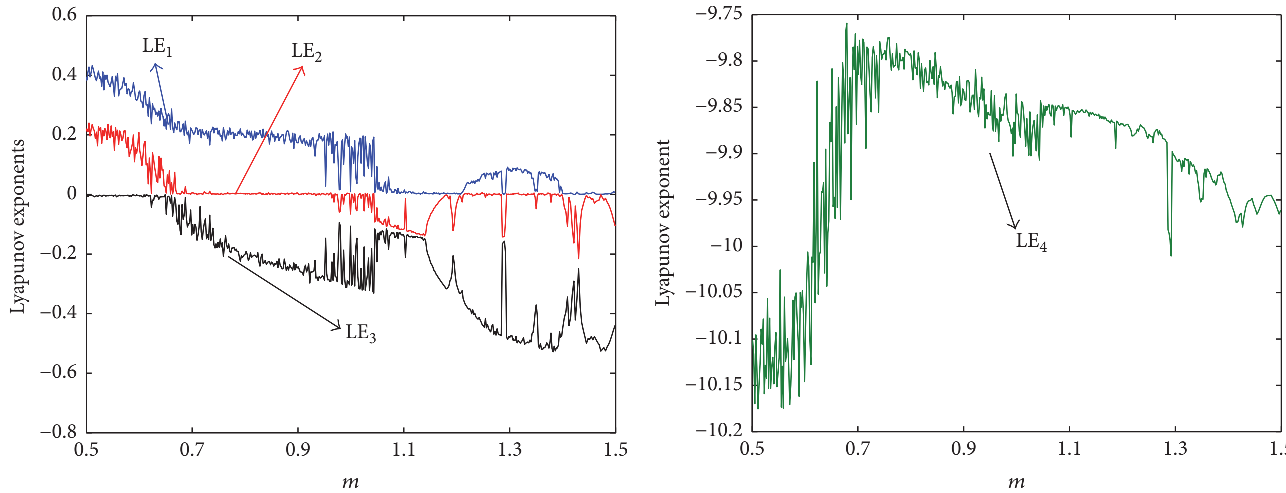
Figure 1: Bifurcation diagram of system (2) without equilibria versus parameter m in $[0.5, 1.5]$.

$$\begin{aligned} \dot{x} &= r(y-x)+w,\\ \dot{y} &= -(1+m)y+xz,\\ \dot{z} &= g\left[1+mx^2-(1+m)xy\right],\\ \dot{w} &= -ky, \end{aligned} \tag{2}$$

where k is a positive parameter. Although system (2) is similar to the algebraic forms of hyperchaotic Lorenz, Chen, Lü, and unified systems, they are not topologically equivalent [46–52]. The proposed system (2) will be a way of understanding the generation of magnetic fields and their reversals in the Earth, the Sun, and other astrophysical bodies. Figure 1 shows a bifurcation diagram exhibiting a period-doubling route to chaos of the peak of z (zmax) of system (2) versus the parameters $m \in [0.5, 1]$, $r = 8$, $g = 35$, and $k = 3$. There are some periodic windows in the chaotic region. Plots of the Lyapunov exponents about $m \in [0.5, 1.5]$ are shown in Figure 2. Figure 3 indicates that system (2) is indeed hidden hyperchaotic for initial states (1.13, 0.5, 0.8, and 1.5) and parameter $m = 0.5$. Its Lyapunov exponents are 0.4113, 0.2233, 0.0000, and −10.1345) and Kaplan-Yorke dimension is $D_{KY} = 3.0626$.

To find the equilibrium states of system (2), we set $\dot{x} = \dot{y} = \dot{z} = \dot{w} = 0$ and solve

$$\begin{aligned} r(y-x)+w &= 0,\\ -(1+m)y+xz &= 0,\\ g\left[1+mx^2-(1+m)xy\right] &= 0,\\ -ky &= 0. \end{aligned} \tag{3}$$



(a)

(b)

FIGURE 2: Lyapunov exponents LE_1, LE_2, LE_3, and LE_4 of system (2) without equilibria versus parameter m in [0.5, 1.5].

There seems to be two equilibria: $E_1(1/\sqrt{-m}, 0, 0, r/\sqrt{-m})$ and $E_2(-1/\sqrt{-m}, 0, 0, -r/\sqrt{-m})$; however, because m is positive, hyperchaotic self-exciting homopolar disc dynamo system (2) has no real equilibria.

3. Synchronization of the 4D Self-Exciting Homopolar Disc Dynamo System

For synchronization, two self-exciting homopolar disc dynamo hyperchaotic systems are coupled together with different initial values. The driver system, x, controls the response system, y. They are given, respectively, by

$$\begin{aligned}
\dot{x}_1 &= r(x_2 - x_1) + x_4, \\
\dot{x}_2 &= -(1+m)x_2 + x_1x_3, \\
\dot{x}_3 &= g\left[1 + mx_1^2 - (1+m)x_1x_2\right], \\
\dot{x}_4 &= -kx_2,
\end{aligned} \tag{4}$$

$$\begin{aligned}
\dot{y}_1 &= r(y_2 - y_1) + y_4, \\
\dot{y}_2 &= -(1+m)y_2 + y_1y_3 + u_1, \\
\dot{y}_3 &= g\left[1 + my_1^2 - (1+m)y_1y_2\right] + u_2, \\
\dot{y}_4 &= -ky_2,
\end{aligned} \tag{5}$$

where u_1 and u_2 in system (5) are the control functions to be determined. The synchronization errors are obtained by subtracting the driver and response systems. Thus, they are defined as $e_i = y_i - x_i$ (for $i = 1, 2, 3, 4$) and the error dynamics become as follows:

$$\begin{aligned}
\dot{e}_1 &= r(e_2 - e_1) + e_4, \\
\dot{e}_2 &= -(1+m)e_2 + y_1y_3 - x_1x_3 + u_1, \\
\dot{e}_3 &= g\left[m\left(y_1^2 - x_1^2\right) - (1+m)(y_1y_2 - x_1x_2)\right] + u_2, \\
\dot{e}_4 &= -ke_2.
\end{aligned} \tag{6}$$

Our objective is to make system (6) asymptotically stable at the zero error state.

3.1. Active Control. The nonlinear terms in system (6) can be eliminated by defining the control functions u_1 and u_2 as in the following:

$$\begin{aligned}
u_1 &= -y_1y_3 + x_1x_3 + v_1, \\
u_2 &= -g\left[m\left(y_1^2 - x_1^2\right) - (1+m)(y_1y_2 - x_1x_2)\right] + v_2.
\end{aligned} \tag{7}$$

Then, substituting (7) into system (6) gives

$$\begin{aligned}
\dot{e}_1 &= r(e_2 - e_1) + e_4, \\
\dot{e}_2 &= -(1+m)e_2 + v_1, \\
\dot{e}_3 &= v_2, \\
\dot{e}_4 &= -ke_2.
\end{aligned} \tag{8}$$

This implies that the equations in system (8) are linear. Provided that the proper choices of control inputs v_1 and v_2 stabilize the error system (8), then e_1, e_2, e_3, and e_4 will converge to zero as $t \to +\infty$. Then, the synchronization of two identical self-exciting homopolar disc dynamo hyperchaotic

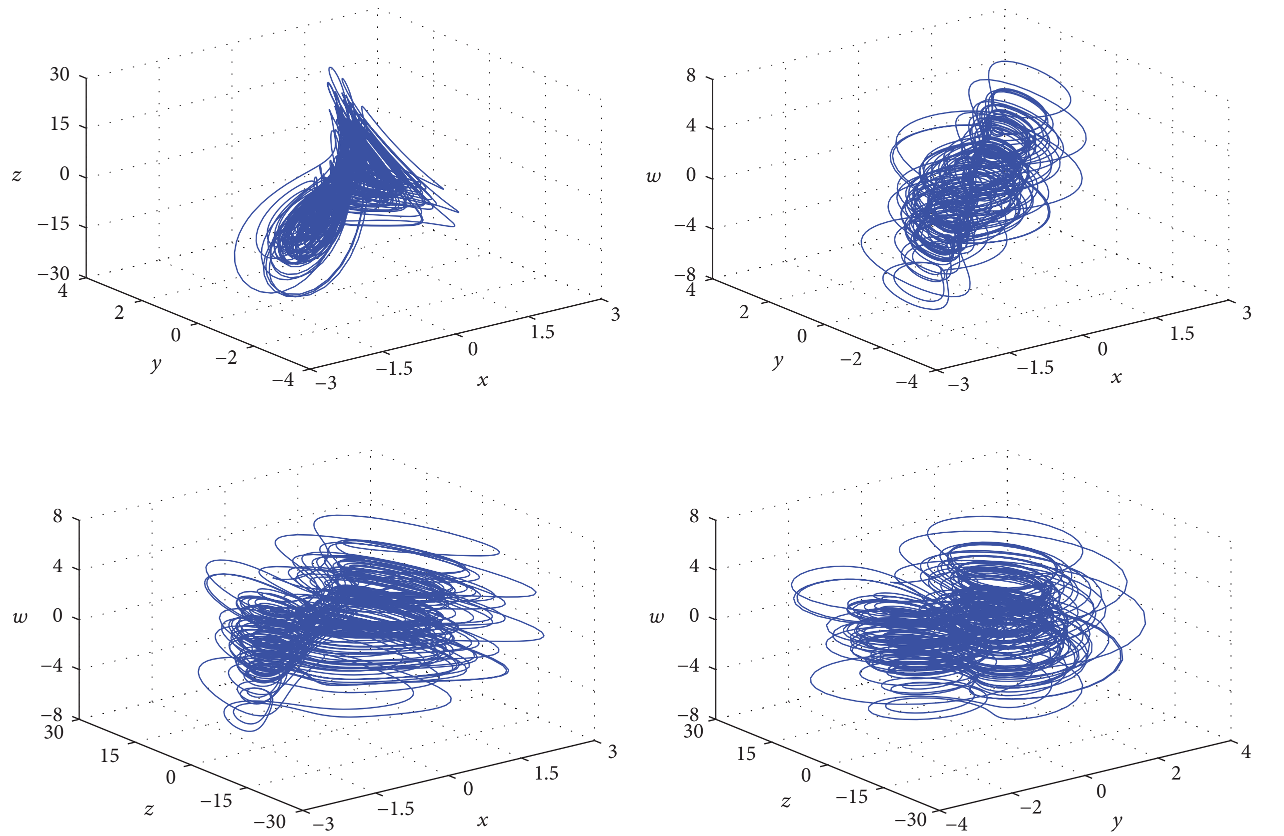


Figure 3: Hyperchaotic attractor of four-dimensional self-exciting homopolar disc dynamo system (2) without equilibria for $r = 8$, $g = 35$, $k = 3$, and $m = 0.5$.

systems will be achieved. A number of choices are possible for control functions v_1 and v_2. They are simply taken as

$$\begin{aligned} v_1 &= (1 + m - k_1) e_2 - re_1 + ke_4, \\ v_2 &= -k_2 e_3, \end{aligned} \tag{9}$$

where k_1 and k_2 are positive control gains. Substituting (9) to system (8) gives the following synchronization error dynamics:

$$\begin{aligned} \dot{e}_1 &= r(e_2 - e_1) + e_4, \\ \dot{e}_2 &= -re_1 - k_1 e_2 + ke_4, \\ \dot{e}_3 &= -k_2 e_3, \\ \dot{e}_4 &= -ke_2. \end{aligned} \tag{10}$$

The associated characteristic matrix of system (10) is

$$A = \begin{pmatrix} -r & -r & 0 & 1 \\ -r & -k_1 & 0 & k \\ 0 & 0 & -k_2 & 0 \\ 0 & -k & 0 & 0 \end{pmatrix}. \tag{11}$$

For the particular choice of control functions in (9), the closed loop system (10) has all of its eigenspectrum in the negative half plane since all of the parameters r, k, k_1, and k_2 are positive. So, this choice leads to a stable system where the error states e_1, e_2, e_3, and e_4 tend to zero as time t tends to infinity. Synchronization of two identical hyperchaotic self-exciting homopolar disc dynamos is therefore completed with the active control method.

3.2. Sliding Mode Control. It can be seen from system (6) that when e_2 and e_3 become zero, $\dot{e}_4$ will be zero and then $\dot{e}_1 = -re_1$. Therefore, when time goes to infinite, e_1 will converge to zero, too. Appropriate sliding surfaces for e_2 and e_3 can be, respectively, designed as

$$\begin{aligned} s_1 &= e_2 - k_3 e_4, \\ s_2 &= e_3 - k_4 e_4, \end{aligned} \tag{12}$$

where k_3 and k_4 are positive control gains.

The attainability condition for sliding mode is $s\dot{s} < 0$. Provided that this condition is satisfied, the sliding mode control functions are

$$\begin{aligned} u_1 &= (1 + m) e_2 - y_1 y_3 + x_1 x_3 - k_3 k e_2 - k_5 s_1 \\ &\quad - q_1 \operatorname{sign}(s_1), \\ u_2 &= -g \left[m \left(y_1^2 - x_1^2 \right) - (1 + m) \left(y_1 y_2 - x_1 x_2 \right) \right] \\ &\quad - k_4 k e_2 - k_6 s_2 - q_2 \operatorname{sign}(s_2), \end{aligned} \tag{13}$$

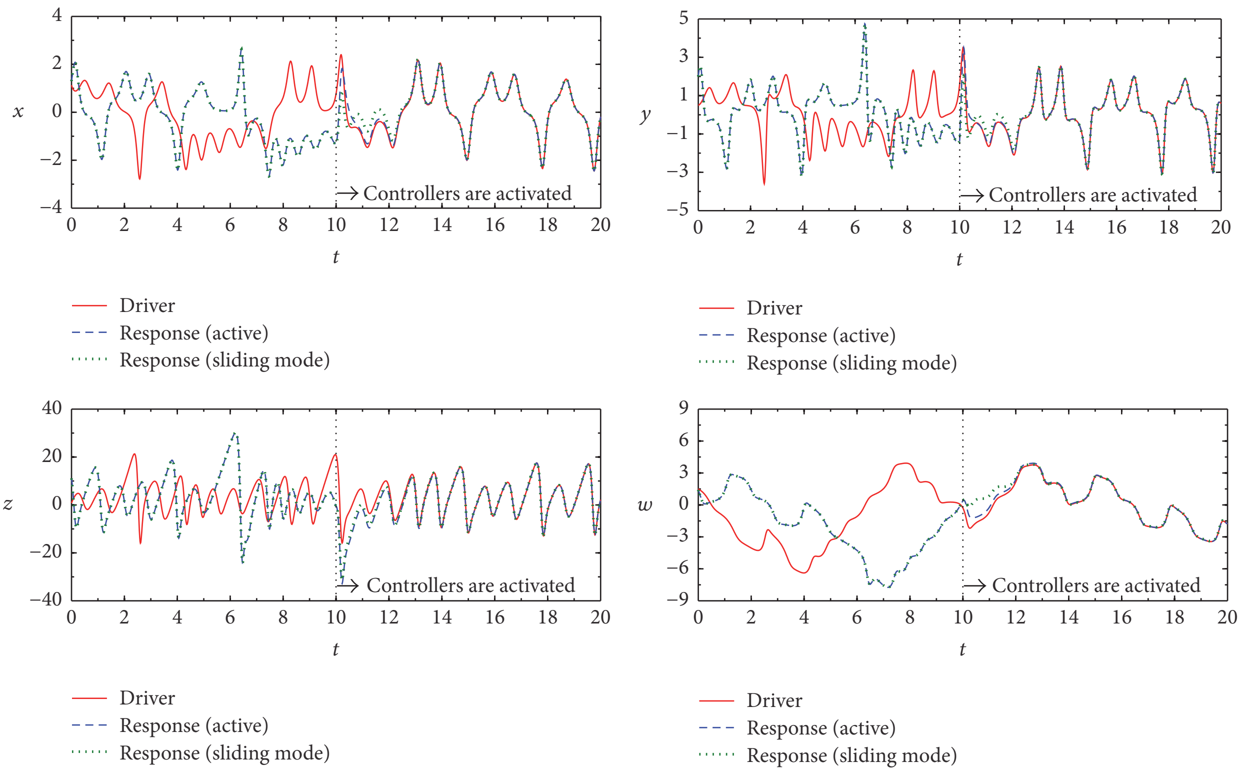


FIGURE 4: Time series of driver and response four-dimensional self-exciting homopolar disc dynamo hyperchaotic systems with the controllers are activated at $t = 10$.

where k_5 and k_6 are positive control gains. Large values of k_5 and k_6 decrease the time t taken to reach the sliding surface but lead to chattering; small values of q_1 and q_2 reduce chattering but increase the time to reach the sliding surface. Here "sign" means the signum function.

The designed control functions (13) guarantee that system (6) is held on the sliding surface $s = 0$. The time derivations of sliding surfaces are

$$\dot{s_1} = -k_5 s_1 - q_1 \operatorname{sign}(s_1),$$
$$\dot{s_2} = -k_6 s_2 - q_2 \operatorname{sign}(s_2). \tag{14}$$

For the Lyapunov function

$$V = \frac{1}{2}\left(s_1^2 + s_2^2\right), \tag{15}$$

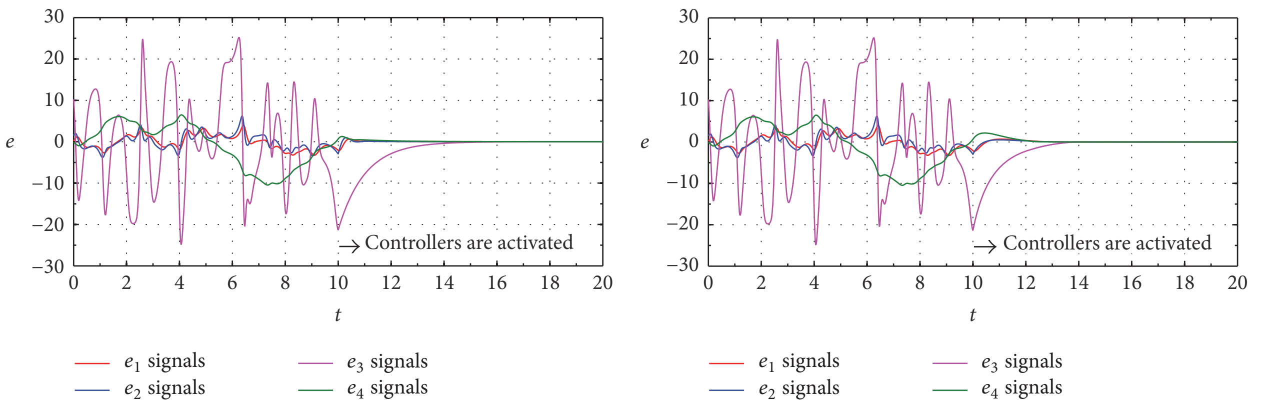
the time derivative of V becomes

$$\dot{V} = -k_5 s_1^2 - q_1 s_1 \operatorname{sign}(s_1) - k_6 s_2^2 - q_2 s_2 \operatorname{sign}(s_2) \le 0, \tag{16}$$

where $k_{5,6} \ge 0$. These conditions guarantee that the constructed sliding surfaces s_1 and s_2 would asymptotically stabilize to the zero synchronization error state, and we obtain synchronization between the two identical hyperchaotic self-exciting homopolar disc dynamos via the sliding mode control method.

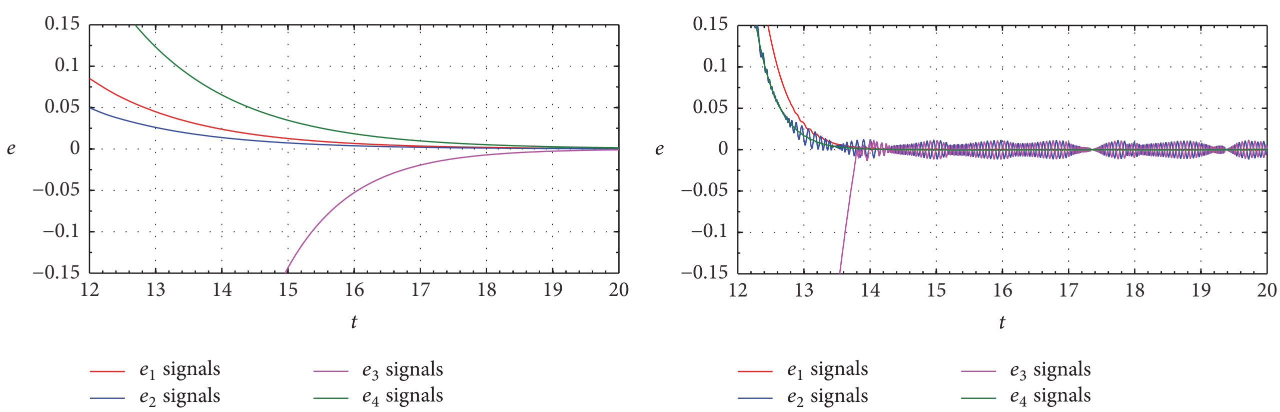
3.3. Numerical Simulations. We now perform some numerical experiments to show that synchronization occurs. We use an ode45 integration solver function with a variable step size. We take $r = 8$, $m = 0.5$, $g = 35$, and $k = 3$ to ensure that hyperchaotic behavior occurs. The gains of the active controllers are taken to be $k_1 = 1$ and $k_2 = 1$. The gains of the sliding mode controllers are taken to be $k_3 = 1$, $k_4 = 1$, $k_5 = 1$, $k_6 = 1$, $q_1 = 0.5$, and $q_2 = 0.5$. We choose the initial conditions for the driver and response systems to be $(1.13, 0.5, 0.8, 1.5)$ and $(1.3, 2, 11.1, 1.4)$, respectively. The controllers are activated when $t = 10$, and we plot the synchronization simulations in Figure 4, while the synchronization errors are plotted in Figure 5.

Figure 4 shows that both active and sliding mode controllers achieve the synchronization of the four-dimensional self-exciting homopolar disc dynamo hyperchaotic system. Figure 5 also shows that, after the activation of controllers at $t = 10$, the synchronization errors approach zero asymptotically, thereby validating the theoretical analyses. Synchronization is complete for $t \ge 16$ with active control and for $t \ge 14$ with sliding mode control. Activation of control at different times gives similar synchronization performances. The comparisons point out that the sliding mode control scheme has the advantage of a faster synchronization time. However, when the results are viewed more closely, as in Figure 6, the chattering problem of sliding mode controllers is evident. A solution is to switch to active control when the mean squared error is less than 0.00001. The new results from switching controllers are presented in Figure 7. It has no chattering and the synchronization performance is similar to that with the sliding mode control scheme.



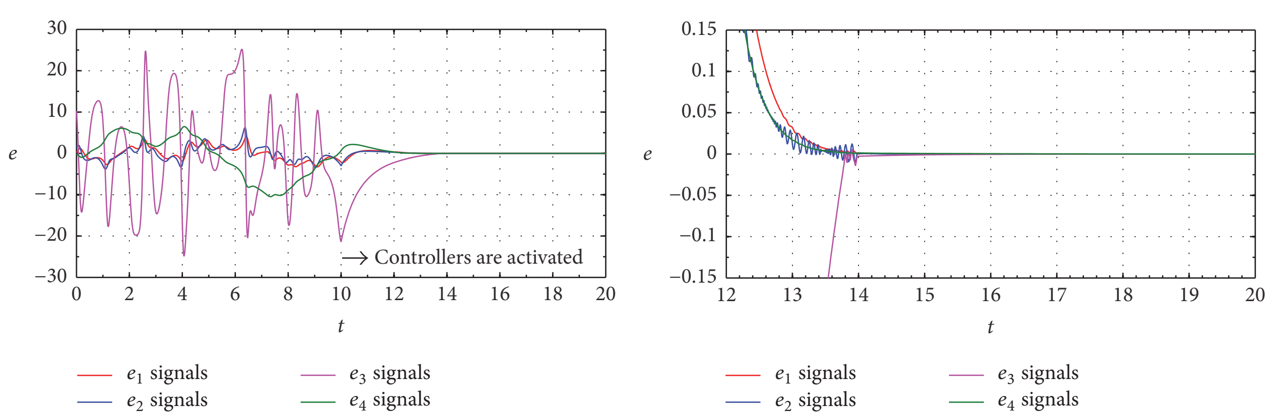
(a) (b)

Figure 5: Synchronization errors between driver and response four-dimensional self-exciting homopolar disc dynamo hyperchaotic systems with the controllers are activated at $t = 10$: (a) active controllers and (b) sliding mode controllers.



(a) (b)

Figure 6: Synchronization errors between $t \geq 12$ and $t \leq 20$: (a) active controllers and (b) sliding mode controllers.



(a) (b)

Figure 7: Synchronization errors with the proposed switching controllers are activated: (a) $t \leq 20$; (b) $t \geq 12$, $t \leq 20$.

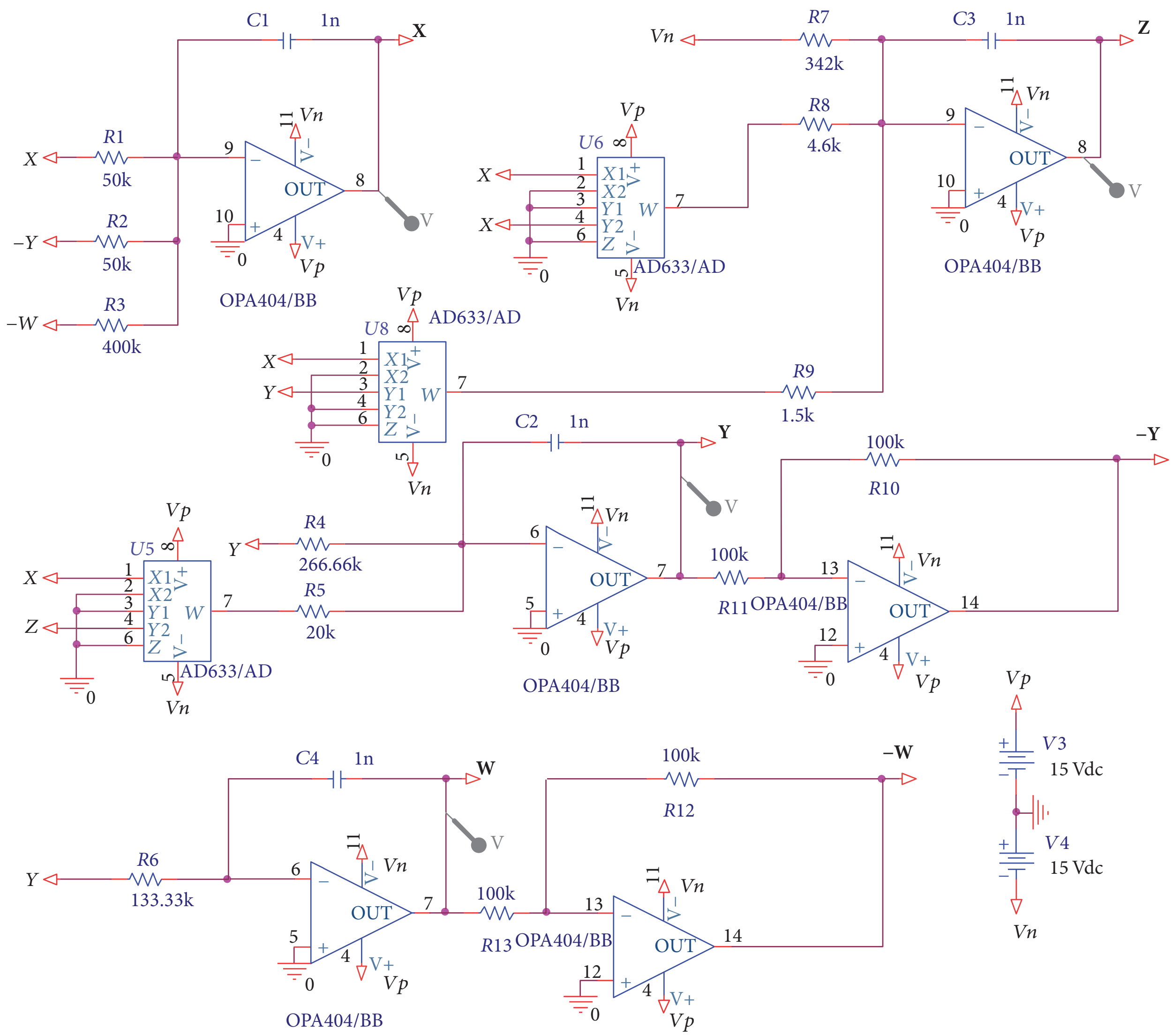


Figure 8: The electronic circuit schematic of the scaled hyperchaotic system (17).

4. Electronic Circuit Implementation of the 4D Self-Exciting Homopolar Disc Dynamo System

Because the values for x, y, and w fall within the interval of $(-15, 15)$, while z falls within the interval of $(-20, 25)$, z must be rescaled for observations using the oscilloscope. We let $X = x$, $Y = y$, $Z = z/2$, and $W = w$ and consider the system:

$$\begin{aligned} \dot{X} &= r(Y - X) + W, \\ \dot{Y} &= -(1 + m)Y + 2XZ, \\ \dot{Z} &= \frac{1}{2}g\left[1 + mX^2 - (1 + m)XY\right], \\ \dot{W} &= -kY. \end{aligned} \tag{17}$$

We can now design an electronic circuit for the scaled hyperchaotic model (17) with parameters $r = 8$, $m = 0.5$, $g = 35$, and $k = 3$ and initial conditions $(1.13, 0.5, 0.4, 1.5)$ using ORCAD-PSpice. The schematic of the electronic circuit is shown in Figure 8. We used resistors, capacitors, TL081 opamps, and AD633 multipliers with the values $R1 = R2 = 50$ Kohm, $R3 = 400$ Kohm, $R4 = 266.66$ Kohm, $R5 = 20$ Kohm, $R6 = 133.33$ Kohm, $R7 = 342$ Kohm, $R8 = 4.6$ Kohm, $R9 = 1.5$ Kohm, $R10 = R11 = R12 = R13 = 100$ Kohm, $C1 = C2 = C3 = C4 = 1$ nF, $Vn = -15$ V, and $Vp = 15$ V. Since the AD633 multiplier IC is limited to between −10 V and +10 V, the output voltage must be divided by 10 V. Real-time application of system (17) was realized with electronic components and shown on the electronic card in Figure 9.

Outputs from the ORCAD-PSpice simulation and oscilloscope phase portraits for the scaled hyperchaotic system (17) with parameters $r = 8$, $m = 0.5$, $g = 35$, and $k = 3$ are given in Figures 10 and 11, respectively. The outputs verify those of the hyperchaotic system, which was modeled using MATLAB.

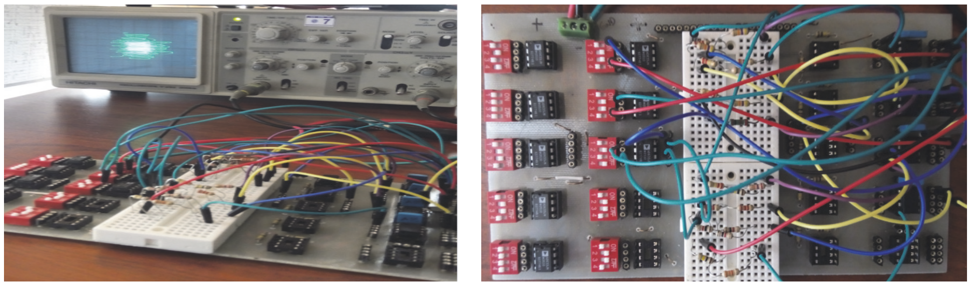


Figure 9: The experimental circuit of the hyperchaotic circuit.

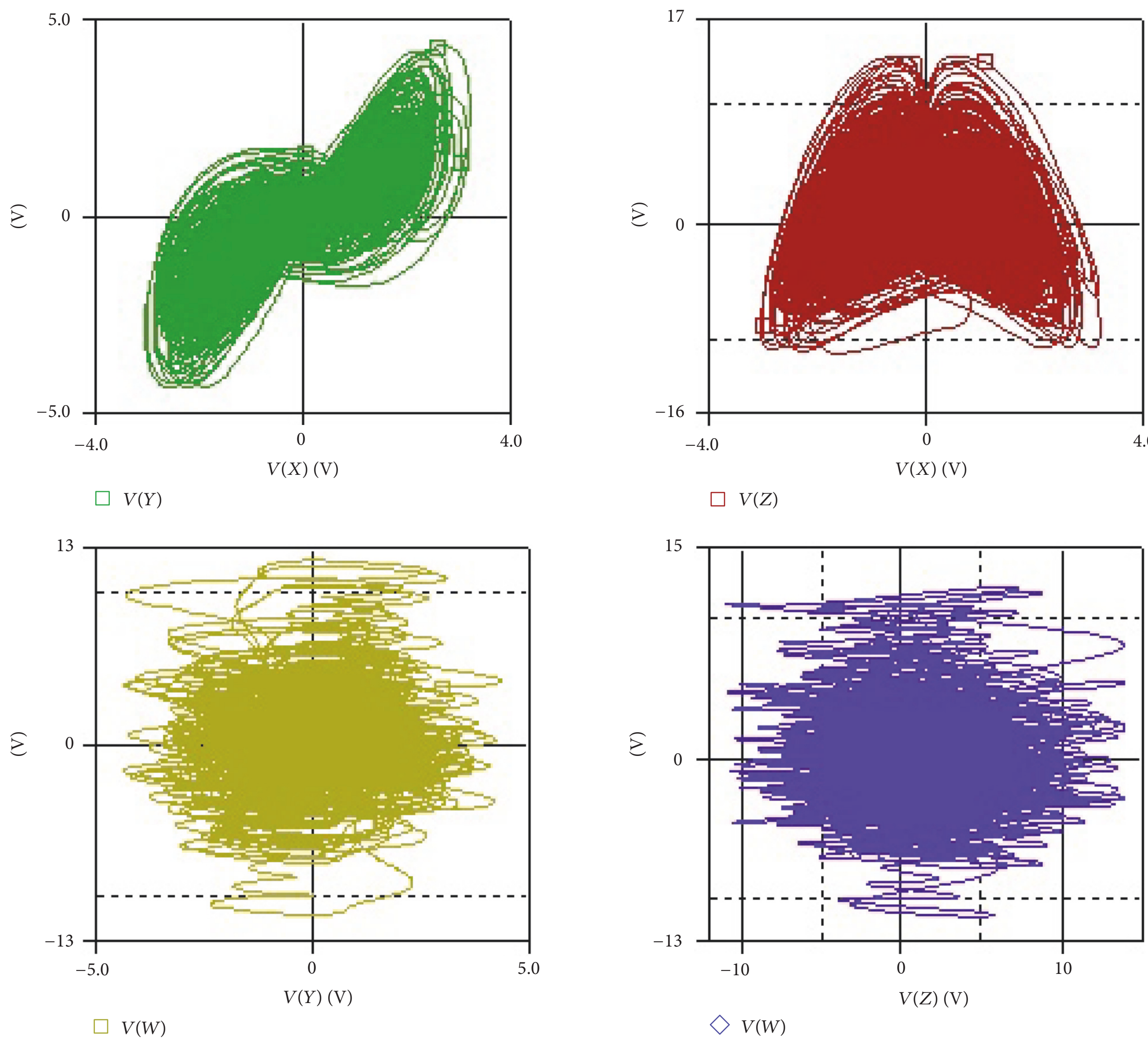


Figure 10: The phase portraits of scaled hyperchaotic system (17) in ORCAD-PSpice.

5. Conclusion

In this paper, we propose a novel four-dimensional self-exciting homopolar disc dynamo without equilibria, but exhibiting hidden hyperchaos. Furthermore, active control and sliding mode control methods are applied to synchronize two coupled four-dimensional dynamo systems. The feasibility of active controllers is ensured by requiring that the spectrum of eigenvalues of the synchronized error system falls in the left half plane. A Lyapunov function is proposed to

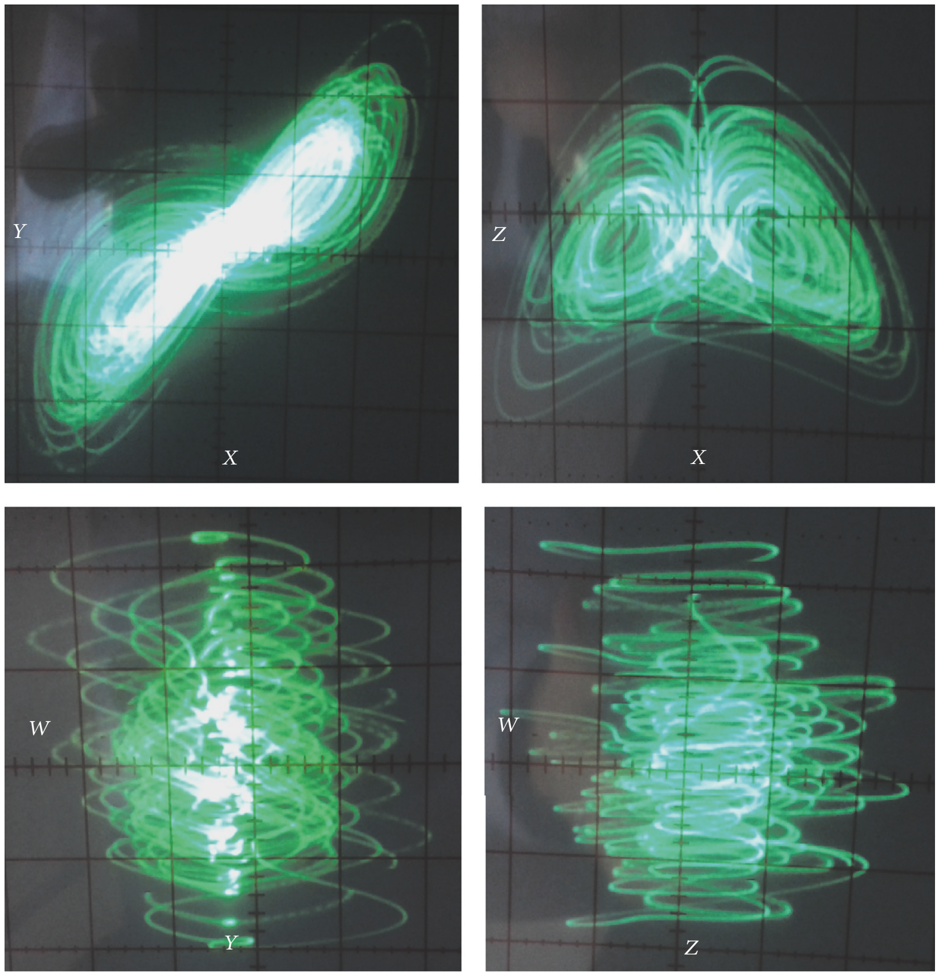


Figure 11: The phase portraits of scaled hyperchaotic system (17) with $r = 8$, $m = 0.5$, $g = 35$, and $k = 3$ on the oscilloscope.

guarantee the asymptotic stability of sliding surfaces and convergence to zero synchronization error. Numerical integrations are presented to compare the performances of the two controllers. Sliding mode control scheme gives better results but has chattering. Solution is provided by a switch to active controllers when chattering starts. An electronic circuit for the rescaled system is implemented via the ORCAD-PSpice program. Numerical simulations validated the theoretical analyses. This physical example will form the basis of more systematic studies of hyperchaos without equilibria in a future study.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

Acknowledgments

This work was supported by the National Natural Science Foundation of China (Grant no. 11561069), the Guangxi Natural Science Foundation of China (Grant no. 2016GXNSFBA380170), the Scientific Research Foundation of the Higher Education Institutions of Guangxi Province of China (Grant no. KY2016YB364), the Open Foundation for Guangxi Colleges and Universities Key Lab of Complex System Optimization and Big Data Processing (nos. 2016CSOBDP0003 and 2016CSOBDP0202), and Sakarya University Scientific Research Projects Unit (no. 201609-00-008).

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