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Muon $g - 2$, 125 GeV Higgs and Neutralino Dark Matter in sMSSM

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Abstract

We discuss the sparticle (and Higgs) spectrum in a class of flavor symmetry–based minimal supersymmetric standard models, referred to here as sMSSM. In this framework the SUSY breaking Lagrangian takes the most general form consistent with a grand unified symmetry such as $SO(10)$ and a non-Abelian flavor symmetry acting on the three families with either a **2+1** or a **3** family assignment. Models based on gauged $SU(2)$ and $SO(3)$ flavor symmetry, as well as non-Abelian discrete symmetries such as S_3 and A_4, have been suggested which fall into this category. These models describe supersymmetry breaking in terms of seven phenomenological parameters. The soft supersymmetry breaking masses at $M_{\rm GUT}$ of all sfermions of the first two families are equal in sMSSM, which differ in general from the corresponding third family mass. In such a framework we show that the muon $g-2$ anomaly, the observed Higgs boson mass of ~ 125 GeV, and the observed relic neutralino dark matter abundance can be simultaneously accommodated. The resolution of the muon $g-2$ anomaly in particular yields the result that the first two generation squark masses, as well the gluino mass, should be $\lesssim 2$ TeV, which will be tested at LHC14.

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1 Introduction

The ATLAS and CMS experiments at the Large Hadron Collider (LHC) have independently reported the discovery [1, 2] of a Standard Model (SM)–like Higgs boson resonance of mass $m_h \simeq 125 - 126$ GeV using the combined 7 TeV and 8 TeV data. This discovery is compatible with low scale supersymmetry, since the Minimal Supersymmetric Standard Model (MSSM) predicts an upper bound of $m_h \lesssim 135$ GeV for the lightest CP-even Higgs boson, if the superparticle masses are assumed to not exceed several TeV [3]. On the other hand, no signals have shown up for supersymmetric particles at the LHC first run, and the current lower bounds on the colored sparticle masses

$$m_{\tilde{g}} \gtrsim 1.4 \text{ TeV (for } m_{\tilde{g}} \sim m_{\tilde{q}}) \quad \text{and} \quad m_{\tilde{g}} \gtrsim 0.9 \text{ TeV (for } m_{\tilde{g}} \ll m_{\tilde{q}}) \text{ [4, 5]} \tag{1}$$

have created some skepticism about naturalness arguments for the Higgs mass based on low scale supersymmetry.

Although the sparticle mass bounds in Eq. (1) are mostly derived for R-parity conserving constrained MSSM (cMSSM), they are applicable to a wider class of low scale supersymmetric models. There exist regions in the MSSM parameter space where the bounds in Eq. (1) can be relaxed by introducing R-parity violating couplings that break baryon number [6], but if the mass of the top quark superpartner, the stop, is below a TeV, the Higgs mass would be unacceptably small. Furthermore, neutralino dark matter will be lost in this case, owing to the violation of R-parity. Low scale supersymmetry can indeed accommodate a Higgs boson with mass $m_h \simeq 125$ GeV in the MSSM while preserving neutralino dark matter, but it requires either a large, $\mathcal{O}(\text{few} - 10)$ TeV, stop mass, or a relatively large soft supersymmetry breaking (SSB) trilinear A_t-term, along with a stop mass of around a TeV [7].

One of the most popular assumptions in low scale supersymmetric theories is that of universal soft supersymmetry breaking mass terms for the three generations of sfermions. This assumption is mainly motivated by the constraints obtained from flavor-changing neutral currents (FCNC) processes [8], with inspiration from minimal supergravity Lagrangian [9]. A practical outcome of three family universality of soft masses is that it would lead to heavy sleptons in the spectrum, since the stop should be heavy to fit the Higgs boson mass. Note, however, that the universality assumption does not follow from any symmetry principle, and as we elaborate here, may be relaxed in a controlled fashion based on underlying symmetries. Such a framework is referred to here as sMSSM, for flavor symmetry-based minimal supersymmetric standard model.

The Standard Model prediction for the anomalous magnetic moment of the muon, $a_\mu = (g-2)_\mu/2$ (muon $g-2$) [11], has a discrepancy with the experimental results [10]:

$$\Delta a_\mu \equiv a_\mu(\text{exp}) - a_\mu(\text{SM}) = (28.6 \pm 8.0) \times 10^{-10} \ . \tag{2}$$

If supersymmetry is to offer a solution to this discrepancy, the smuon and gaugino (bino or wino) SSB masses should be $\mathcal{O}(100)$ GeV. Thus, it is hard to simultaneously explain the observed Higgs boson mass and resolve the muon $g-2$ anomaly if universality of all sfermion soft masses is imposed at the grand unified theory (GUT) scale, as in cMSSM.

Recently there have been several attempts to reconcile this (presumed) tension between muon $g-2$ and Higgs boson mass within the MSSM framework by assuming non-universal SSB mass terms for the gauginos [12] or the sfermions [13] at the GUT scale. A simultaneous explanation of m_h and muon $g-2$ is possible [14] even with $t-b-\tau$ Yukawa coupling unification condition [15]. It has been known for some time [16] that constraints from FCNC processes are very mild and easily satisfied for the case in which the third generation sfermion masses are split from those of the first two generations. However, when the muon $g-2$ anomaly and the Higgs boson mass are simultaneously explained with non-universal gaugino and/or sfermion masses, the correct relic abundance of neutralino dark matter is typically not obtained [13]. Consistency with the observed dark matter abundance would further constrain the SUSY parameter space.

In this paper we develop further the framework of flavor symmetry–based minimal supersymmetric standard model, sMSSM, suggested recently [17]. It will be shown that in this framework, which consists of seven phenomenological parameters that describe supersymmetry breaking, it is possible to explain the muon $g-2$ anomaly and the Higgs boson mass simultaneously, along with the observed dark matter abundance. While the parameter set of sMSSM (seven) is larger than that of cMSSM (four), it is still rather restrictive. In comparison, the phenomenological MSSM (pMSSM) [18] describes SUSY breaking in terms of nineteen parameters. In the sMSSM framework SUSY breaking is dictated by symmetry considerations alone. It is realized by combining a grand unified symmetry such as $SO(10)$ with a flavor symmetry acting on the three families which could be a gauge symmetry based on $SU(2)$ or $SO(3)$ or a discrete non-Abelian symmetry such as S_3 or A_4. These models admit either a **2+1** or a **3** family assignment. Both assignments would lead to the same low energy phenomenology, since a large top quark mass effectively breaks the **3** assignment down to a **2+1** assignment. The soft SUSY breaking Lagrangian is the most general one consistent with these symmetries. FCNC processes mediated by SUSY particles are adequately suppressed by the flavor symmetry, while the grand unified symmetry further reduces the parameter set. As a consequence of these symmetries, the soft masses of the first two families are equal, which differs from that of the third family. This additional freedom helps explain the muon $g-2$, m_h and dark matter abundance simultaneously. The framework is still rather restrictive, leading to the result that the sfermions of the first two families, as well as the gluino, should have masses below about 2 TeV, which will be tested in the near future at the LHC14.

The outline for the rest of the paper is as follows. In Section 2 we summarize the salient features of flavor symmetry–based MSSM. In Section 3 we briefly describe the dominant contributions to the muon anomalous magnetic moment arising from low scale supersymmetry. In Section 4 we summarize the scanning procedure and the experimental constraints applied in our analysis. Here we also present the parameter space that we scan over. Our results are presented in Section 5. Section 6 has our conclusions.

2 Flavor symmetry–based minimal supersymmetric standard model: sMSSM

In this Section we provide a brief description of the sMSSM setup and its motivations [17]. We also describe at the end of this section a complete model based on $SU(2)$ flavor symmetry that leads to sMSSM phenomenology at energies below the GUT scale. We refer the reader to Ref. [17] for a more detailed discussion including additional models that generate the sMSSM spectrum.

In supergravity models, it is generally assumed that supersymmetry breaks dynamically in a hidden sector, which is communicated to the visible sector via gravity. With no further restrictions imposed, this setup would lead to over a hundred parameters in the soft SUSY breaking Lagrangian of MSSM, assuming that R–parity remains unbroken. These parameters are phenomenologically restricted by stringent constraints from flavor changing neutral currents that the SUSY particles mediate. To satisfy such constraints, it is often assumed that the sfermions of all three generations have a universal mass at the GUT scale. In the constrained MSSM, for example, SUSY breaking is described by a set of four parameters, traditionally chosen to be $\{m_0,\ M_{1/2},\ A_0,\ \tan\beta\}$, along with a discrete parameter $\mathrm{sgn}(\mu)$. Such a choice, however, is not dictated by any symmetry argument, and modifications of this scheme have been widely discussed. An example is the phenomenological MSSM (pMSSM), which describes the soft SUSY breaking Lagrangian in terms of nineteen parameters, chosen such that SUSY mediated flavor violation is sufficiently suppressed. As in the case of cMSSM this setup is also not dictated by an underlying symmetry. The flavor symmetry–based minimal supersymmetric standard model (sMSSM) suggested in Ref. [17] is a framework for controlled SUSY breaking based on symmetry principles. As the framework is based on gauge symmetries, the Lagrangian is guaranteed to be protected against quantum gravitational corrections.

In the sMSSM framework the soft SUSY breaking Lagrangian is the most general one consistent with two symmetries. First, it is compatible with a grand unified symmetry such as $SO(10)$. Second, a non-Abelian flavor symmetry of gauge origin acts on the three families with either a **2+1** or a **3** family assignment. This symmetry suppresses SUSY mediated flavor violation. The grand unified symmetry, which is well motivated, and supported by the merging of the three gauge couplings at a GUT scale of $\approx 2 \times 10^{16}$ GeV within MSSM, reduces the soft SUSY breaking parameters considerably. For example, gaugino mass unification is implied by GUT, which reduces the gaugino soft parameters of MSSM from three down to one. Similarly, all members of a family would have a common soft mass, as they are unified into a **16**-plet of $SO(10)$. Combined with the non-Abelian flavor symmetry, the 15 soft squared mass parameters of the 15 chiral fermions of the MSSM are reduced to just two in sMSSM. The SUSY phenomenology of sMSSM is described by seven parameters, chosen to be

$$\{m_{1,2},\ m_3,\ M_{1/2},\ A_0,\ \tan\beta,\ \mu,\ m_A\}\ . \tag{3}$$

Here $m_{1,2}$ is the common mass of the first two family sfermions, while m_3 is the soft mass of the third family sfermions. $M_{1/2}$ is the unified gaugino mass and m_A is the mass of the pseudoscalar Higgs boson. We shall now describe how the symmetries of sMSSM lead to the parameter set of Eq. (3).

A non-Abelian flavor symmetry, denoted as H, acts on the three families in sMSSM. Ideally any symmetry should be a gauge symmetry, which suggests $SU(2)$, $SO(3)$ and $SU(3)$ as possible candidates for H as these groups contain **2** and **3**–dimensional irreducible representations. Among these, $SU(2)$ and $SO(3)$ can yield simple and realistic models of SUSY breaking and simultaneously generate realistic fermion masses [17], while this is not easily achieved in the case with $SU(3)$. Note that the representations of $SU(2)$ and $SO(3)$ are (pseudo)real, and gauge theories based on these groups are automatically free of triangle anomalies, which is not the case with $SU(3)$. Gauging a flavor symmetry, however, is generally problematic in SUSY models, as it induces new and potentially dangerous flavor violation via the D-terms [19]. In Ref. [17] an interchange symmetry was suggested acting on a pair of doublets that break $SU(2)$ which would set the D-terms to zero. Similarly, a simple solution for the D-term problem was found in the case of $SO(3)_H$ as well [20, 17]. In this case, although the soft masses of all members of the $SO(3)$ triplets would be degenerate at the GUT scale, there is significant mixing between the third family and ceratin vector-like families of GUT scale mass that generates a large top quark mass. As a result, the effective low energy SUSY breaking Lagrangian would have a common mass for the first two family sfermions that is different from that of the third family. Thus, both $SU(2)$ and $SO(3)$ would lead to the parameter set of Eq. (3) for low energy phenomenology.

The spectrum of sMSSM can also follow from a non-Abelian discrete flavor symmetry such as S_3 and A_4 [17]. We envision such symmetries to have a fundamental gauge origin and note that in string theory constructions such non-Abelian symmetries often emerge. In this case there is no issue with the D-term, since discrete symmetries do not have associated D-terms. S_3, the permutation group of three letters, which is the simplest non-Abelian symmetry, admits a **2+1** family assignment. A_4, the symmetry group of a regular tetrahedron, which is the simplest group with a triplet representation, admits a **3** assignment of families. Realistic fermion mass generation and symmetry breaking mechanism with these symmetry groups have been analyzed in Ref. [17], which all yield the spectrum of sMSSM. The case of S_3 symmetry is similar to the $SU(2)_H$ model, while the case of A_4 symmetry resembles the $SO(3)_H$ model.

sMSSM is a systematized approach which addresses and solves the D-term problem [19] that generally exists in gauged family symmetry models [21] by auxiliary symmetries. Non-Abelian discrete family symmetries have been used in the literature to address the SUSY flavor violation problem [22, 23], but typically the low energy theory is not the MSSM. In the sMSSM, on the other hand, the low energy theory is the MSSM with the parameter set relevant for SUSY breaking given as in Eq. (3).

We conclude this section with a brief description of one model based on $SU(2)_H$ flavor symmetry that yields sMSSM at low energies [17]. The three families are assigned under $SO(10) \times SU(2)_H$ as $(\mathbf{16},\, \mathbf{2}) + (\mathbf{16},\, \mathbf{1})$, with the $(\mathbf{16},\, \mathbf{1})$ identified as the third family. We use the notation of $SO(10)$, but it is not required that the model be grand unified; the only requirement is compatibility with a GUT symmetry such as $SO(10)$. $SU(2)_H$ symmetry breaking is achieved by introducing a pair of $(\mathbf{1},\, \mathbf{2})$ Higgs fields, denoted as ϕ and $\overline{\phi}$, which acquire vacuum expectation values (VEVs) of order the GUT scale, through a superpotential given by

$$W_{\rm sym} = \mu_\phi \, \phi \, \overline{\phi} + \kappa \, (\phi \, \overline{\phi})^2 \; . \tag{4}$$

Here κ is a parameter with inverse dimensions of mass, obtained by integrating a gauge singlet field, or arising from quantum gravity effects. Including the $SU(2)_H$ D-terms, and soft SUSY breaking terms, one obtains from the minimization of the potential a condition

$$|u|^2 - |\overline{u}|^2 = \frac{2(m_{\overline{\phi}}^2 - m_\phi^2)}{g_H^2}, \tag{5}$$

where $\langle \phi \rangle = (0,\, u)^T$ and $\langle \overline{\phi} \rangle = (\overline{u},\, 0)^T$, and where m_ϕ^2 and $m_{\overline{\phi}}^2$ are the soft squared masses of the ϕ and $\overline{\phi}$ fields respectively. This condition yields a nonzero D-term, which would split the masses of the up and down–type members of all $SU(2)_H$ doublet sfermions, and thus induce flavor violation (once CKM mixing is included) in meson–antimeson mixing, for example. In Ref. [17] it was noted that this D-term problem can be avoided simply by imposing an interchange symmetry $\phi \leftrightarrow \overline{\phi}$, which would set $m_\phi^2 = m_{\overline{\phi}}^2$, and thus $|u|^2 = |\overline{u}|^2$. Such an interchange symmetry is a subgroup of an anomaly free $SU(2)$ global symmetry which exists in the model with two doublets.

Realistic fermion masses are induced in the model through the Yukawa superpotential

$$W_{\rm Yuk} = \mathbf{16}_3\mathbf{16}_3\mathbf{10}_H + \mathbf{16}_i\mathbf{16}_3\mathbf{10}_H\left(\frac{\phi_j+\overline{\phi}_j}{M_*}\right)\epsilon^{ij} + \mathbf{16}_i\mathbf{16}_j\epsilon^{ij}\mathbf{10}_H\left(\frac{\mathbf{45}_H}{M_*}\right) + \ldots \tag{6}$$

Here ellipsis stands for higher order terms suppressed by more powers of M_*, which is presumable the Planck scale, much larger than $|u|$ and $\langle\mathbf{45}_H\rangle \sim M_{\rm GUT}$. The coupling $\mathbf{16}_i\mathbf{16}_j\epsilon^{ij}\mathbf{10}_H$ will not be allowed if the full $SO(10)$ symmetry is utilized, however the term shown with an additional $\mathbf{45}_H$, used for GUT symmetry breaking, would be allowed because of its antisymmetric property. We see that only the third generation acquires a mass at the renormalizable level, while the lighter family masses are suppressed by inverse powers such as $|u|/M_*$. After some rotations, the fermion mass matrices resulting from Eq. (6) can be written in the form

$$M_f = \begin{pmatrix} 0 & c & 0 \\ -c & 0 & b \\ 0 & b' & a \end{pmatrix}_f \tag{7}$$

for $f = u,\, d,\, \ell, \nu^D$, which fits the observed masses and mixings of quarks and leptons quite well [23]. CP violation can have a spontaneous origin in this context, which would make all SUSY breaking parameters real, and thus solve the SUSY CP problem arising from limits on the electric dipole moments of the electron and the neutron. The CKM phase can be still be of order one, if some of the fields, such as the $\mathbf{45}_H$ of Eq. (6), acquire complex VEVs [17].

Owing to the $SU(2)_H$ flavor symmetry, the soft masses of the scalars in the $(\mathbf{16},\,\mathbf{2})$ multiplet are all the same (denoted as $m_{1,2}$), while members of the $(\mathbf{16},\,\mathbf{1})$ would have a common mass that is different (denoted as m_3). The gaugino masses are unified because of the $SO(10)$ symmetry. There is no reason for the soft masses of the MSSM Higgs doublets H_u and H_d to be equal to $m_{1,2}$ or m_3, as these fields belong to different representations of $SO(10)$ such as **10** and **16**. These two Higgs soft masses have been traded in Eq. (3) with μ and m_A. Finally, in the sMSSM framework it is not required that the trilinear A-terms be proportional to the respective Yukawa couplings. Nevertheless, these A-terms would exhibit the same hierarchy as the Yukawa couplings, and non-proportionality does not result in excessive SUSY induced flavor violation. For low energy collider phenomenology, only the third family A-terms are relevant, which we denote as A_0 at the GUT scale. In a more general setting this A_0 can break into A_0^t, A_0^b and A_0^τ, which need not be all the same. Such a difference will be relevant only for the case of large $\tan\beta$. In our analysis we define $A_0 = A_t^0 = A_b^0 = A_\tau^0$, which is realized in at least some versions of sMSSM.

3 The Muon Anomalous Magnetic Moment in sMSSM

The leading contribution from low scale supersymmetry to the muon anomalous magnetic moment, applicable to sMSSM, is given by [24, 25]:

$$\begin{aligned} \Delta a_\mu &= \frac{\alpha\, m_\mu^2\, \mu\, M_2 \tan\beta}{4\pi \sin^2\theta_W\, m_{\tilde{\mu}_L}^2}\left[\frac{f_\chi(M_2^2/m_{\tilde{\mu}_L}^2) - f_\chi(\mu^2/m_{\tilde{\mu}_L}^2)}{M_2^2-\mu^2}\right] \\ &+ \frac{\alpha\, m_\mu^2\, \mu\, M_1 \tan\beta}{4\pi \cos^2\theta_W\, (m_{\tilde{\mu}_R}^2 - m_{\tilde{\mu}_L}^2)}\left[\frac{f_N(M_1^2/m_{\tilde{\mu}_R}^2)}{m_{\tilde{\mu}_R}^2} - \frac{f_N(M_1^2/m_{\tilde{\mu}_L}^2)}{m_{\tilde{\mu}_L}^2}\right], \end{aligned} \tag{8}$$

where α is the fine-structure constant, m_μ is the muon mass, μ denotes the bilinear Higgs mixing term, and $\tan\beta$ is the ratio of the vacuum expectation values (VEVs) of MSSM Higgs doublets. M_1 and M_2 denote the $U(1)_Y$ and $SU(2)$ gaugino masses respectively, θ_W is the weak mixing angle, and $m_{\tilde{\mu}_L}$, $m_{\tilde{\mu}_R}$ are left and right handed smuon masses. The loop functions are defined as follows:

$$f_\chi(x) = \frac{x^2 - 4x + 3 + 2\ln x}{(1-x)^3}\,, \qquad f_\chi(1) = -2/3, \tag{9}$$

$$f_N(x) = \frac{x^2 - 1 - 2x\ln x}{(1-x)^3}\,, \qquad f_N(1) = -1/3\,. \tag{10}$$

The first term in Eq. (8) stands for the dominant contribution arising from the one loop diagram with Higgsino-wino exchange, while the second term describes contributions from the bino-smuon loop. As the Higgsino mass parameter μ increases, the first term decreases in Eq. (8) and the second term becomes dominant. On the other hand, the smuon need to be light, $O(100~\text{GeV})$, in both cases in order to make a sizeable contribution to muon $g-2$. Note that due to decoupling, the formulae will eventually fail to be accurate for large values of $\mu\tan\beta$. The Eq. (8) does not contain the trilinear SSB term A_μ, since it is assumed that $A_\mu < \mu\tan\beta$. From Eq. (8), the parameter set

$$\{M_1,\, M_2,\, \mu,\, \tan\beta, m_{\tilde{\mu}_L},\, m_{\tilde{\mu}_R}\}, \tag{11}$$

is relevant at low energies for the muon $g-2$ calculation. Since the gaugino masses are universal at the GUT scale in sMSSM, and the sfermions of the first two families have a common mass, we have $M_2 \approx 2M_1$ at low scale due to renormalization group equation (RGE) running. On the other hand, in order to have sizeable contribution to muon $g-2$ from supersymmetry, the gauginos should be sufficiently light. Because of relatively small values of bino and wino masses we can assume that $m_{\tilde{\mu}_L} \approx m_{\tilde{\mu}_R}$. With these constraints the number of independent parameters for the $g-2$ calculation can be reduced to four:

$$\{M_1,\, \mu,\, \tan\beta,\, m_{\tilde{\mu}_R}\}. \tag{12}$$

We pay special attention to these parameters, which are functions of the seven fundamental parameters shown in Eq. (3) in sMSSM.

4 Scanning Procedure, Parameter Space and Experimental Constraints

We employ the ISAJET 7.84 package [26] to perform random scans over the fundamental parameter space of sMSSM as shown in Eq. (3). In this package, the weak scale values of gauge and third generation Yukawa couplings are evolved to M_{GUT} via the MSSM RGEs in the $\overline{DR}$ regularization scheme. We do not strictly enforce the unification condition $g_3 = g_1 = g_2$ at M_{GUT}, since a few percent deviation from unification can be assigned to unknown GUT-scale threshold corrections [27]. The deviation between $g_1 = g_2$ and g_3 at M_{GUT} is no worse than $3-4\%$. For simplicity, we do not include the Dirac neutrino Yukawa coupling in the RGEs, whose contribution is expected to be small.

The various boundary conditions are imposed at M_{GUT} and all the SSB parameters, along with the gauge and Yukawa couplings, are evolved back to the weak scale M_{Z}. In the evolution of Yukawa couplings the SUSY threshold corrections [28] are taken into account at the common scale $M_{\rm SUSY} = \sqrt{m_{\tilde{t}_L} m_{\tilde{t}_R}}$, where $m_{\tilde{t}_L}$ and $m_{\tilde{t}_R}$ denote the masses of the third generation left and right-handed stop quarks. The entire parameter set is iteratively run between $M_{\rm Z}$ and $M_{\rm GUT}$ using the full 2-loop RGEs until a stable solution is obtained. To better account for leading-log corrections, one-loop step-beta functions are adopted for the gauge and Yukawa couplings, and the SSB parameters m_i are extracted from RGEs at multiple scales $m_i = m_i(m_i)$. The RGE-improved 1-loop effective potential is minimized at $M_{\rm SUSY}$, which effectively accounts for the leading 2-loop corrections. Full 1-loop radiative corrections are incorporated for all sparticle masses.

An approximate error of around 2 GeV [29] in the estimate of the Higgs boson mass largely arises from theoretical uncertainties in the calculation of the minimum of the scalar potential, and to a lesser extent from experimental uncertainties in the values for m_t and α_s.

An important constraint on the parameter space arises from limits on the cosmological abundance of stable charged particles [30]. This excludes regions in the parameter space where a charged SUSY particle becomes the lightest supersymmetric particle (LSP). We accept only those solutions for which one of the neutralinos is the LSP and saturates the WMAP bound on relic dark matter abundance.

We have performed random scans for the following parameter range:

$$\begin{aligned}
0 \leq m_{1,2} &\leq 3\,\mathrm{TeV} \\
0 \leq m_3 &\leq 3\,\mathrm{TeV} \\
0 \leq M_{1/2} &\leq 3\,\mathrm{TeV} \\
-5\,\mathrm{TeV} \leq \mathrm{A}_0 &\leq 5\,\mathrm{TeV} \\
-3 \leq A_0/m_3 &\leq 3 \\
2 \leq \tan\beta &\leq 60 \\
0 \leq \mu &\leq 3\,\mathrm{TeV} \\
0 \leq m_A &\leq 3\,\mathrm{TeV} \\
\mu &> 0.
\end{aligned} \tag{13}$$

Here $m_{1,2}$ is the SSB scalar mass parameters for the first two generations, while m_3 is for the third generation. $M_{1/2}$ is the SSB gaugino mass, and A_0 is the SSB trilinear scalar interaction coupling. The parameters μ and m_A are bilinear Higgs mixing term and mass of the CP-odd Higgs boson respectively. In contrast to the other parameters, the values for μ and m_A are set at low scale. We make $m_t = 173.3\,\mathrm{GeV}$ [31], and we show that our results are not too sensitive to one or two sigma variations from this central value [32]. Note that $m_b(m_Z) = 2.83$ GeV, which is hard-coded into ISAJET. The choice of the sgn(μ) to be positive is dictated by the desire to explain the muon $g-2$ anomaly. All SUSY breaking parameters are restricted to lie below 3 TeV (except for A_0 which is allowed to be somewhat larger), which would make the fine tuning in the Higgs mass relatively mild. Since most of the SUSY particles have masse below about 4 TeV, essentially all particles are within reach of the LHC.

In scanning the parameter space, we employ the Metropolis-Hastings algorithm as described in [33]. The data points collected all satisfy the requirement of radiative electroweak symmetry breaking (REWSB), with the neutralino being the LSP in each case. After collecting the data, we impose the mass bounds on the particles [30] and use the IsaTools

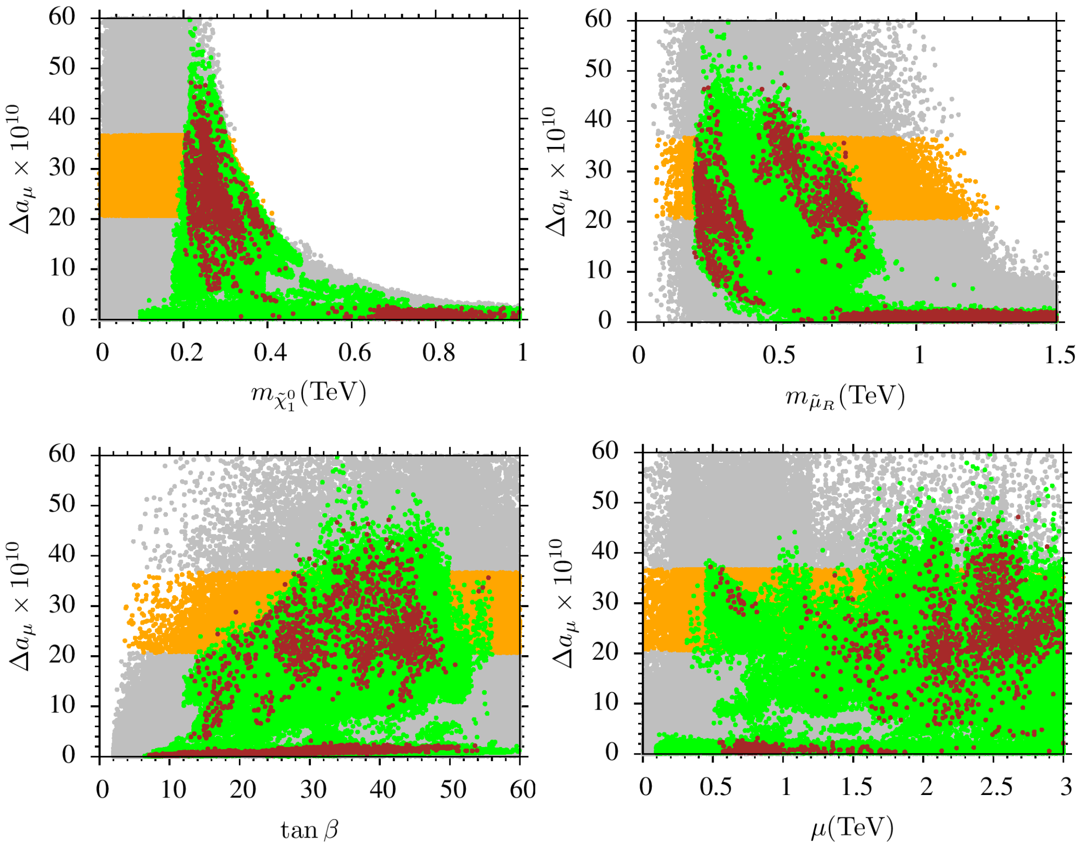

Figure 1: Plots in the $\Delta a_\mu - m_{\tilde{\chi}_1^0}$, $\Delta a_\mu - m_{\tilde{\mu}_R}$, $\Delta a_\mu - \tan\beta$, $\Delta a_\mu - \mu$ planes. Gray points are consistent with REWSB and neutralino LSP. Yellow points represent values of Δa_μ that would bring theory and experiment to within 1σ. Green points form a subset of gray points and satisfy sparticle mass and B-physics constraints described on Table 1. In addition these points satisfy the lightest CP-even Higgs mass range $123\,\mathrm{GeV} \leq m_h \leq 127$ GeV. Brown points belong to a subset of green points and satisfy the WMAP bound (5σ) on neutralino dark matter abundance.

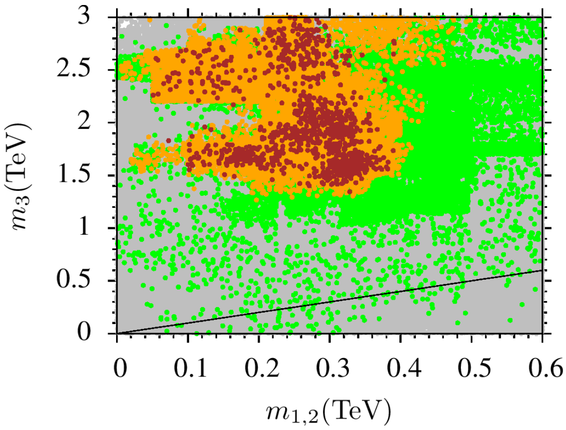

Figure 2: Plots in the $m_3 - m_{1,2}$ plane. The color coding is the same as Figure 1, but in this case yellow points are a subset of green points and brown points belong to a subset of yellow. The unit slope line is to guide the eye.

package [34] to implement the various phenomenological constraints. We successively apply the experimental constraints presented in Table 1 on the data that we acquire from ISAJET:

$123\,\text{GeV} \leq m_h \leq 127\ \text{GeV}$	[1, 2]
$0.8 \times 10^{-9} \leq \text{BR}(B_s \to \mu^+\mu^-) \leq 6.2 \times 10^{-9}\ (2\sigma)$	[35]
$2.99 \times 10^{-4} \leq \text{BR}(b \to s\gamma) \leq 3.87 \times 10^{-4}\ (2\sigma)$	[36]
$0.15 \leq \frac{\text{BR}(B_u \to \tau\nu_\tau)_{\text{MSSM}}}{\text{BR}(B_u \to \tau\nu_\tau)_{\text{SM}}} \leq 2.41\ (3\sigma)$	[37]
$0.0913 \leq \Omega_{\text{CDM}}h^2 \leq 0.1363\ (5\sigma)$	[38]

Table 1: Phenomenological constraints implemented in our study.

5 Results

We next present the results of the scan over the parameter space listed in Eq. (13). In Figure 1 we show the results in the $\Delta a_\mu - m_{\tilde{\chi}_1^0}$, $\Delta a_\mu - m_{\tilde{\mu}_R}$, $\Delta a_\mu - \tan\beta$, $\Delta a_\mu - \mu$ planes. Gray points are consistent with REWSB and neutralino LSP. Yellow points represent Δa_μ values which would bring theory and experiment within 1σ. Green points form a subset of gray points and satisfy the sparticle mass bounds and B-physics constraints described in Table 1. In addition, the lightest CP-even Higgs mass range $123\,\text{GeV} \leq m_h \leq 127\ \text{GeV}$ is applied. Brown points belong to a subset of green points and satisfy the WMAP bound (5σ) on the neutralino dark matter abundance.

In the $\Delta a_\mu - m_{\tilde{\chi}_1^0}$ plane of Figure 1, we show that muon $g-2$ prefers relatively light gauginos in the SUSY spectrum for Δa_μ to be large enough to explain the discrepancy between theory and experiment. The brown points belong to a subset of green points and satisfy the WMAP bound (5σ) on neutralino dark matter abundance. We will consider later on how to obtain the correct relic abundance of neutralino dark matter in this model. The lower bound on the neutralino mass arises mostly from the current gluino mass bound, and there is a sharp upper bound on the former, of about 2 TeV, if we want to have Δa_μ within a 1σ deviation.

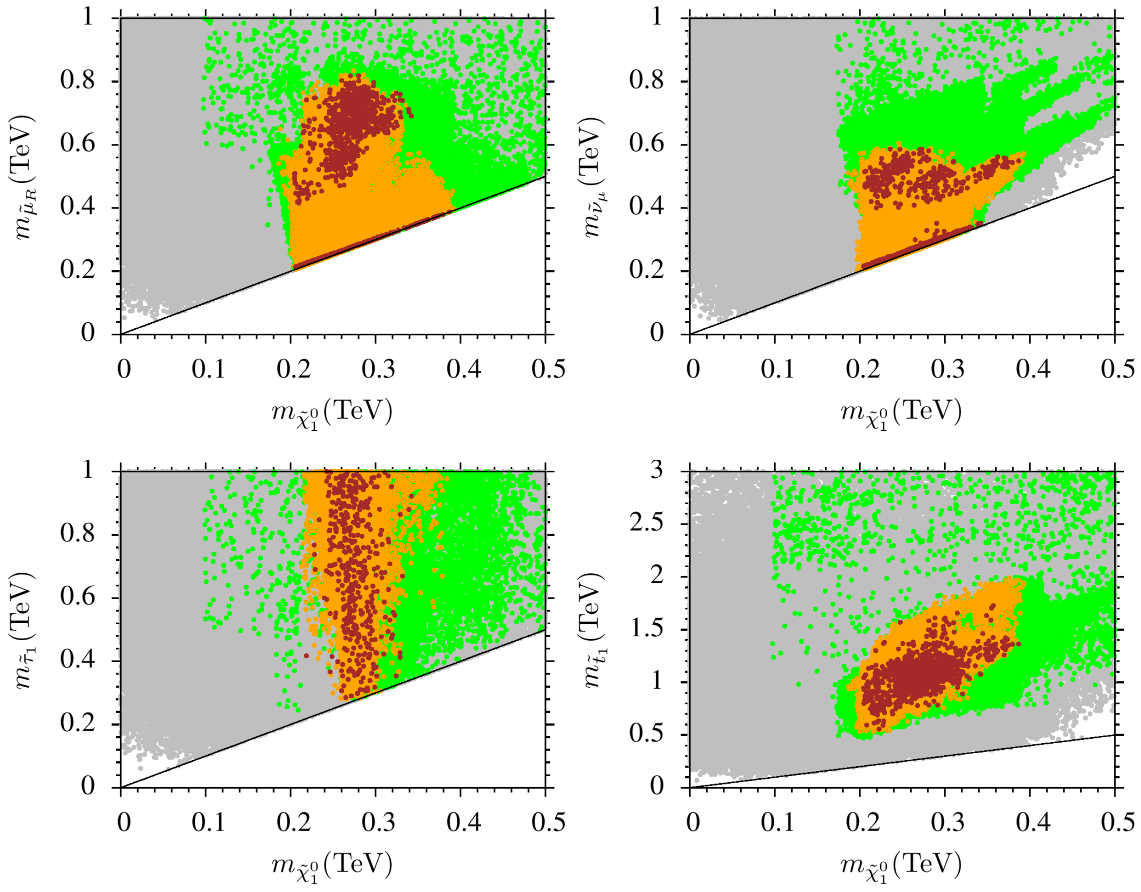

Figure 3: Plots in the $m_{\tilde{\mu}_R}-m_{\tilde{\chi}_1^0}$, $m_{\tilde{\nu}_{1,2}}-m_{\tilde{\chi}_1^0}$, $m_{\tilde{\tau}_1}-m_{\tilde{\chi}_1^0}$, $m_{\tilde{t}}-m_{\tilde{\chi}_1^0}$ planes. The color coding is the same as Figure 2 except that the mass bound on stop is not applied in $m_{\tilde{t}}-m_{\tilde{\chi}_1^0}$.

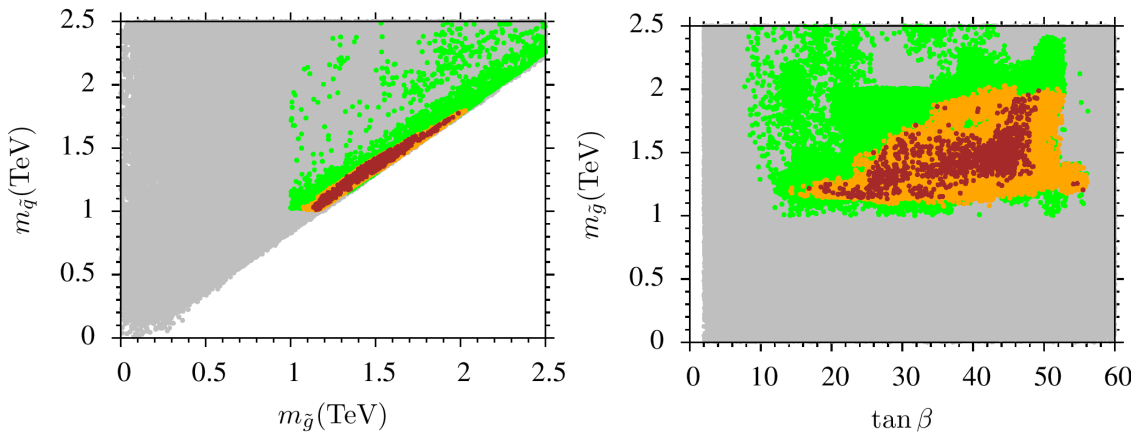

Figure 4: Plots in the $m_{\tilde{q}}-m_{\tilde{g}}$ and $m_{\tilde{g}}-\tan\beta$ planes. Color coding is the same as Figure 2.

	Point 1	Point 2	Point 3	Point 4
$m_{1,2}$	222	302	282	244
m_3	2862	1760	1678	2671
$M_{1/2}$	545.6	494	692	754
$\tan\beta$	35.4	20.9	44.4	46.1
A_0/m_3	-1.54	-2.24	-2.65	-2.18
μ	503.1	2179	2676	2895
m_A	2891	1648	2749	2972
m_t	173.3	173.3	173.3	173.3
Δa_μ	$\mathbf{31.8 \times 10^{-10}}$	$\mathbf{24.3 \times 10^{-10}}$	$\mathbf{22.5 \times 10^{-10}}$	$\mathbf{23.1 \times 10^{-10}}$
m_h	**123.2**	**124.1**	**124.6**	**125.2**
m_H	2910	1658	2767	2991
m_A	2891	1648	2749	2972
$m_{H^\pm}$	2911	1661	2768	2993
$m_{\tilde{\chi}^0_{1,2}}$	232,420.7	211, 410	299, 573	330, 631
$m_{\tilde{\chi}^0_{3,4}}$	514.2, 548	2164, 2164	2658, 2658	2874, 2875
$m_{\tilde{\chi}^\pm_{1,2}}$	423.5, 546.5	411, 2169	574, 2659	633, 2877
$m_{\tilde{g}}$	1290	1171	1579	1724
$m_{\tilde{u}_{L,R}}$	1137, 1041	1465, 1298	1399, 1218	1561, 1401
$m_{\tilde{t}_{1,2}}$	1066, 1960	896, 1553	1019, 1597	1267, 2030
$m_{\tilde{d}_{L,R}}$	1140, 1117.5	1069, 1022	1468, 1431	1563, 1521
$m_{\tilde{b}_{1,2}}$	1976, 2466	1532, 1892	1545, 1755	2014, 2354
$m_{\tilde{\nu}_{1,2}}$	244	473	326	340
$m_{\tilde{\nu}_3}$	2541	1724	1146	2021
$m_{\tilde{e}_{L,R}}$	319, 474	491, 218	355, 706	387, 687
$m_{\tilde{\tau}_{1,2}}$	2195, 2546	1581, 1731	318, 1159	1109, 2025
σ_{SI}(pb)	0.35×10^{-9}	0.53×10^{-11}	0.36×10^{-11}	0.13×10^{-11}
σ_{SD}(pb)	0.19×10^{-5}	0.44×10^{-7}	0.43×10^{-8}	0.32×10^{-8}
$\Omega_{CDM}h^2$	0.11	0.11	0.12	0.11

Table 2: Four benchmark points satisfying all phenomenological constraints including muon $g-2$ in sMSSM. All the masses are in units of GeV. All this points are chosen to satisfy the constraints described in Section 3. The points 1-4 respectively correspond to muon sneutrino, smuon, stau and muon sneutrino coannihilation channels.

From the $\Delta a_\mu - m_{\tilde{\mu}_R}$ panel, we see that in order to stay within a 1σ range of muon $g-2$ and comply with all the constraints listed in Section 4, the smuon mass should lie in the range $200\,\text{GeV} \lesssim m_{\tilde{\mu}_R} \lesssim 800\,\text{GeV}$.

The results in the $\Delta a_\mu - \tan\beta$ plane show that it is hard to have substantial contribution to muon $g-2$ if $\tan\beta \lesssim 14$. The interval $30 \lesssim \tan\beta \lesssim 50$ is preferred from the muon $g-2$ point of view, which is also a desirable range for $t-b-\tau$ Yukawa coupling unification as well [15, 39].

It is interesting to see from the $\Delta a_\mu - \mu$ plane that there exist large μ solutions, which means that in this case we have significant contribution from the bino-smuon loop. It has bees shown [40] that if bino and smuons are dominant contributors to the muon $g-2$, the corresponding parameter space for sleptons can be tested at the LHC and ILC. The $\Delta a_\mu - \mu$ plane also shows the possibility of smaller μ values consistent with desirable values for muon $g-2$. Small values of μ-term may make "the little hierarchy" problem less severe.

It is interesting to show the amount of mass splitting necessary between the third and first two-family sfermion SSB masses in order to satisfy all current phenomenological constraints including muon $g-2$. We present our results in the $m_3 - m_{1,2}$ plane in Figure 5. The color coding is the same as Figure 1 but in this case the yellow points are a subset of the green points, and the brown points belong to a subset of yellow. The unit slope line is to guide the eye. As we see, the yellow points are sufficiently above the unit line, and we need to have $m_3/m_{1,2} > 4$. The splitting becomes larger $(m_3/m_{1,2} > 10)$ as $\tan\beta$ decreases.

In Figure 3 we show the relic density channels consistent with muon $(g-2)$ in the $m_{\tilde{\mu}_R} - m_{\tilde{\chi}_1^0}$, $m_{\tilde{\nu}_\mu} - m_{\tilde{\chi}_1^0}$, $m_{\tilde{\tau}_1} - m_{\tilde{\chi}_1^0}$, $m_{\tilde{t}} - m_{\tilde{\chi}_1^0}$ planes. We see that a variety of coannihilation scenarios are compatible with muon $g-2$ and neutralino dark matter. In the $m_{\tilde{\mu}_R} - m_{\tilde{\chi}_1^0}$ plane in Figure 3, we draw the unit slope line which indicates the presence of smuon-neutralino coannihilation scenario. From the $m_{\tilde{\nu}_\mu} - m_{\tilde{\chi}_1^0}$ and $m_{\tilde{\tau}_1} - m_{\tilde{\chi}_1^0}$ planes we see that is is also possible to realize stau and muon sneutrino coannihilation scenarios.

The results in the $m_{\tilde{\tau}_1} - m_{\tilde{\chi}_1^0}$ plane show that it is hard to realize stop coannihilation scenario in this framework. The stop in this scenario can be as light as 500 GeV and cannot be heavier than 2 TeV. We expect that the A-funnel scenario is also consistent with muon $g-2$, although we have not found it, perhaps due to lack of statistics.

Figure 4 shows plots in the $m_{\tilde{q}} - m_{\tilde{g}}$, $m_{\tilde{q}} - \tan\beta$, $m_{\tilde{g}} - \tan\beta$ and $m_{\tilde{\mu}_R} - \tan\beta$ planes, with color coding the same as in Figure 2. The $m_{\tilde{q}} - m_{\tilde{g}}$ plane shows that imposing 1σ deviation from the measured muon $g-2$ requires the first and second generation squark masses to be less than 2 TeV, which can be tested in the on upcoming LHC second run. If the bound $m_{\tilde{g}} \gtrsim 1.4$ TeV (for $m_{\tilde{g}} \sim m_{\tilde{q}}$), observed from an analysis based on the cMSSM parameter space, is confirmed for the case of general low scale SUSY, then $\tan\beta \lesssim 30$ will be excluded in this scenario.

Finally, in Table 2 we present four characteristic benchmark points which summarize the salient features of this model. For these points the $g-2$ constraints as well as sparticle mass and B-physics constraints described in Section 4 are satisfied. The points 1-4 respectively correspond to muon sneutrino, smuon, stau and muon sneutrino coannihilation channels. Point 1 depicts a solution with a relatively low value of μ and accordingly it has relatively large neutralino-nucleon spin-independent and spin dependent cross section, which can be tested at the upcoming SuperCDMS, XENON 1T and IceCube DeepCore experiments. Point 4 displays a solution with heavy gluino and squarks of the first two families.

6 Conclusion

We have explored the sparticle and Higgs phenomenology of the flavor symmetry–based MSSM framework, referred to here as sMSSM. Such models are motivated by a grand unified symmetry such as $SO(10)$ along with a non-Abelian flavor symmetry that suppresses SUSY flavor violation. The SUSY breaking Lagrangian in sMSSM is the most general consistent with these two symmetries. Explicit ultra-violet complete models that generate sMSSM spectrum at low energies have been presented. These include models based on

$SU(2)$ and $SO(3)$ gauged flavor symmetries, as well as those based on non-Abelian discrete symmetries such as S_3 and A_4. The SUSY phenomenology of these models is described by seven parameters listed in Eq. (3). sMSSM contains three additional parameters compared to cMSSM. Specifically, the (common) soft mass of the first two family sfermions is different from that of the third family. This freedom helps us explain the muon $g-2$ anomaly, along with the Higgs boson mass and the correct relic abundance of neutralino dark matter. The parameter space is still rather restrictive, and we have shown that the simultaneous explanation of these observables requires the mass of the gluino to be less than about 2 TeV, and the mass of the first two family sleptons to be less than about 800 GeV. The parameter $\tan\beta$ is preferred to be relatively large, $\tan\beta > 15$.

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