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The multiplicative Zagreb indices of graph operations

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Abstract

Recently, Todeschini *et al.* (Novel Molecular Structure Descriptors - Theory and Applications I, pp. 73-100, 2010), Todeschini and Consonni (MATCH Commun. Math. Comput. Chem. 64:359-372, 2010) have proposed the multiplicative variants of ordinary Zagreb indices, which are defined as follows:

$$\prod_1 = \prod_1(G) = \prod_{v\in V(G)} d_G(v)^2, \qquad \prod_2 = \prod_2(G) = \prod_{uv\in E(G)} d_G(u)d_G(v).$$

These two graph invariants are called *multiplicative Zagreb indices* by Gutman (Bull. Soc. Math. Banja Luka 18:17-23, 2011). In this paper the upper bounds on the multiplicative Zagreb indices of the join, Cartesian product, corona product, composition and disjunction of graphs are derived and the indices are evaluated for some well-known graphs.

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1 Introduction

Throughout this paper, we consider simple graphs which are finite, indirected graphs without loops and multiple edges. Suppose G is a graph with a vertex set $V(G)$ and an edge set $E(G)$. For a graph G, the degree of a vertex v is the number of edges incident to v and is denoted by $d_G(v)$. A topological index $\mathrm{Top}(G)$ of a graph G is a number with the property that for every graph H isomorphic to G, $\mathrm{Top}(H) = \mathrm{Top}(G)$. Recently, Todeschini *et al.* [1, 2] have proposed the multiplicative variants of ordinary Zagreb indices, which are defined as follows:

$$\prod_1 = \prod_1(G) = \prod_{v\in V(G)} d_G(v)^2, \qquad \prod_2 = \prod_2(G) = \prod_{uv\in E(G)} d_G(u)d_G(v).$$

Mathematical properties and applications of multiplicative Zagreb indices are reported in [1–6]. Mathematical properties and applications of multiplicative sum Zagreb indices are reported in [7]. For other undefined notations and terminology from graph theory, the readers are referred to [8].

In [9, 10], Khalifeh *et al.* computed some exact formulae for the hyper-Wiener index and Zagreb indices of the join, Cartesian product, composition, disjunction and symmetric difference of graphs. Some more properties and applications of graph products can be seen in the classical book [11].

In this paper, we give some upper bounds for the multiplicative Zagreb index of various graph operations such as join, corona product, Cartesian product, composition, disjunction, *etc.* Moreover, computations are done for some well-known graphs.

2 Multiplicative Zagreb index of graph operations

We begin this section with two standard inequalities as follows.

Lemma 1 (AM-GM inequality) *Let $x_1, x_2, \dots, x_n$ be nonnegative numbers. Then*

$$\frac{x_1 + x_2 + \cdots + x_n}{n} \geq \sqrt[n]{x_1 x_2 \cdots x_n} \tag{1}$$

holds with equality if and only if all the x_k's are equal.

Lemma 2 (Weighted AM-GM inequality) *Let $x_1, x_2, \dots, x_n$ be nonnegative numbers and also let $w_1, w_2, \dots, w_n$ be nonnegative weights. Set $w = w_1 + w_2 + \cdots + w_n$. If $w > 0$, then the inequality*

$$\frac{w_1 x_1 + w_2 x_2 + \cdots + w_n x_n}{w} \geq \sqrt[w]{x_1^{w_1} x_2^{w_2} \cdots x_n^{w_n}} \tag{2}$$

holds with equality if and only if all the x_k with $w_k > 0$ are equal.

Let G_1 and G_2 be two graphs with n_1 and n_2 vertices and m_1 and m_2 edges, respectively. The join $G_1 \vee G_2$ of graphs G_1 and G_2 with disjoint vertex sets $V(G_1)$ and $V(G_2)$ and edge sets $E(G_1)$ and $E(G_2)$ is the graph union $G_1 \cup G_2$ together with all the edges joining $V(G_1)$ and $V(G_2)$. Thus, for example, $\overline{K}_p \vee \overline{K}_q = K_{p,q}$, the complete bipartite graph. We have $|V(G_1 \vee G_2)| = n_1 + n_2$ and $|E(G_1 \vee G_2)| = m_1 + m_2 + n_1 n_2$.

Theorem 1 *Let G_1 and G_2 be two graphs. Then*

$$\prod_1 (G_1 \vee G_2) \leq \left[\frac{M_1(G_1) + 4m_1 n_2 + n_1 n_2^2}{n_1}\right]^{n_1} \times \left[\frac{M_1(G_2) + 4n_1 m_2 + n_2 n_1^2}{n_2}\right]^{n_2} \tag{3}$$

and

$$\prod_2 (G_1 \vee G_2) \leq \left[\frac{M_2(G_1) + n_2 M_1(G_1) + m_1 n_2^2}{m_1}\right]^{m_1} \times \left[\frac{M_2(G_2) + n_1 M_1(G_2) + m_2 n_1^2}{m_2}\right]^{m_2} \times \left[\frac{4m_1 m_2 + 2n_1 n_2 (m_1 + m_2) + (n_1 n_2)^2}{n_1 n_2}\right]^{n_1 n_2}, \tag{4}$$

where n_1 and n_2 are the numbers of vertices of G_1 and G_2, and m_1, m_2 are the numbers of edges of G_1 and G_2, respectively. Moreover, the equality holds in (3) *if and only if both G_1 and G_2 are regular graphs, that is, $G_1 \vee G_2$ is a regular graph and the equality holds in* (4) *if and only if both G_1 and G_2 are regular graphs, that is, $G_1 \vee G_2$ is a regular graph.*

Proof Now,

$$\begin{aligned}\prod_1(G_1 \vee G_2) &= \prod_{(u_i,v_j)\in V(G_1\vee G_2)} d_{G_1\vee G_2}(u_i,v_j)^2 \\ &= \prod_{u_i\in V(G_1)} \big(d_{G_1}(u_i)+n_2\big)^2 \prod_{v_j\in V(G_2)} \big(d_{G_2}(v_j)+n_1\big)^2 \\ &= \prod_{u_i\in V(G_1)} \big(d_{G_1}(u_i)^2+2n_2d_{G_1}(u_i)+n_2^2\big) \prod_{v_j\in V(G_2)} \big(d_{G_2}(v_j)^2+2n_1d_{G_2}(v_j)+n_1^2\big)\end{aligned}$$

and by (1) this above equality is actually less than or equal to

$$\begin{aligned}&\le \left[\frac{\sum_{u_i\in V(G_1)}(d_{G_1}(u_i)^2+2n_2d_{G_1}(u_i)+n_2^2)}{n_1}\right]^{n_1} \\ &\quad\times \left[\sum_{v_j\in V(G_2)} \frac{(d_{G_2}(v_j)^2+2n_1d_{G_2}(v_j)+n_1^2)}{n_2}\right]^{n_2} \\ &= \left[\frac{M_1(G_1)+4m_1n_2+n_1n_2^2}{n_1}\right]^{n_1} \times \left[\frac{M_1(G_2)+4n_1m_2+n_2n_1^2}{n_2}\right]^{n_2}.\end{aligned}$$

Moreover, the above equality holds if and only if

$$d_{G_1}(u_i)^2+2n_2d_{G_1}(u_i)+n_2^2 = d_{G_1}(u_k)^2+2n_2d_{G_1}(u_k)+n_2^2 \quad \big(u_i,u_k\in V(G_1)\big)$$

and

$$d_{G_2}(v_j)^2+2n_1d_{G_2}(v_j)+n_1^2 = d_{G_2}(v_\ell)^2+2n_1d_{G_2}(v_\ell)+n_1^2 \quad \big(v_j,v_\ell\in V(G_2)\big)$$

(by Lemma 1), that is, for $u_i,u_k\in V(G_1)$ and $v_j,v_\ell\in V(G_2)$,

$$\big(d_{G_1}(u_i)-d_{G_1}(u_k)\big)\big(d_{G_1}(u_i)+d_{G_1}(u_k)+2n_2\big)$$

and

$$\big(d_{G_2}(v_j)-d_{G_2}(v_\ell)\big)\big(d_{G_2}(v_j)+d_{G_2}(v_\ell)+2n_1\big).$$

That is, for $u_i,u_k\in V(G_1)$ and $v_j,v_\ell\in V(G_2)$, we get $d_{G_1}(u_i)=d_{G_1}(u_k)$ and $d_{G_2}(v_j)=d_{G_2}(v_\ell)$. Hence the equality holds in (3) if and only if both G_1 and G_2 are regular graphs, that is, $G_1\vee G_2$ is a regular graph.

Now, since

$$\prod_2(G_1\vee G_2) = \prod_{(u_i,v_j)(u_k,v_\ell)\in E(G_1\vee G_2)} d_{G_1\vee G_2}(u_i,v_j)d_{G_1\vee G_2}(u_k,v_\ell),$$

we then obtain

$$\begin{aligned}&= \prod_{u_iu_k\in E(G_1)} \big(d_{G_1}(u_i)+n_2\big)\big(d_{G_1}(u_k)+n_2\big) \prod_{v_jv_\ell\in E(G_2)} \big(d_{G_2}(v_j)+n_1\big)\big(d_{G_2}(v_\ell)+n_1\big) \\ &\quad\times \prod_{u_i\in V(G_1),v_j\in V(G_2)} \big(d_{G_1}(u_i)+n_2\big)\big(d_{G_2}(v_j)+n_1\big)\end{aligned}$$

and by (1)

$$\leq \left[\frac{\sum_{u_i u_k \in E(G_1)} (d_{G_1}(u_i) d_{G_1}(u_k) + n_2(d_{G_1}(u_i) + d_{G_1}(u_k)) + n_2^2)}{m_1} \right]^{m_1}$$
$$\times \left[\frac{\sum_{v_j v_\ell \in E(G_2)} (d_{G_2}(v_j) d_{G_2}(v_\ell) + n_1(d_{G_2}(v_j) + d_{G_2}(v_\ell)) + n_1^2)}{m_2} \right]^{m_2}$$
$$\times \left[\frac{\sum_{u_i \in V(G_1), v_j \in V(G_2)} (d_{G_1}(u_i) d_{G_2}(v_j) + n_2 d_{G_2}(v_j) + n_1 d_{G_1}(u_i) + n_1 n_2)}{n_1 n_2} \right]^{n_1 n_2}. \tag{5}$$

However, from the last inequality, we get

$$= \left[\frac{M_2(G_1) + n_2 M_1(G_1) + m_1 n_2^2}{m_1} \right]^{m_1} \times \left[\frac{M_2(G_2) + n_1 M_1(G_2) + m_2 n_1^2}{m_2} \right]^{m_2}$$
$$\times \left[\frac{\sum_{u_i \in V(G_1)} d_i \sum_{v_j \in V(G_2)} d_j^* + n_1 n_2 \sum_{v_i \in V(G_1)} d_i + n_1 n_2 \sum_{v_j \in V(G_2)} d_j^* + n_1^2 n_2^2}{n_1 n_2} \right]^{n_1 n_2}$$
$$= \left[\frac{M_2(G_1) + n_2 M_1(G_1) + m_1 n_2^2}{m_1} \right]^{m_1} \times \left[\frac{M_2(G_2) + n_1 M_1(G_2) + m_2 n_1^2}{m_2} \right]^{m_2}$$
$$\times \left[\frac{4 m_1 m_2 + 2 n_1 n_2 (m_1 + m_2) + (n_1 n_2)^2}{n_1 n_2} \right]^{n_1 n_2}.$$

Furthermore, for both connected graphs G_1 and G_2, the equality holds in (5) iff

$$d_{G_1}(u_i) d_{G_1}(u_r) + n_2 \big(d_{G_1}(u_i) + d_{G_1}(u_r) \big) + n_2^2 = d_{G_1}(u_i) d_{G_1}(u_k) + n_2 \big(d_{G_1}(u_i) + d_{G_1}(u_k) \big) + n_2^2$$

for any $u_i u_r, u_i u_k \in E(G_1)$; and

$$d_{G_2}(v_j) d_{G_2}(v_r) + n_1 \big(d_{G_2}(v_j) + d_{G_2}(v_r) \big) + n_1^2 = d_{G_2}(v_j) d_{G_2}(v_\ell) + n_1 \big(d_{G_2}(v_j) + d_{G_2}(v_\ell) \big) + n_1^2$$

for any $v_j v_r, v_j v_\ell \in E(G_2)$ as well as

$$d_{G_1}(u_i) d_{G_2}(v_j) + n_2 d_{G_2}(v_j) + n_1 d_{G_1}(u_i) + n_1 n_2$$
$$= d_{G_1}(u_i) d_{G_2}(v_\ell) + n_2 d_{G_2}(v_\ell) + n_1 d_{G_1}(u_i) + n_1 n_2$$

for any $u_i \in V(G_1)$, $v_j, v_\ell \in V(G_2)$; and

$$d_{G_1}(u_i) d_{G_2}(v_j) + n_2 d_{G_2}(v_j) + n_1 d_{G_1}(u_i) + n_1 n_2$$
$$= d_{G_1}(u_k) d_{G_2}(v_j) + n_2 d_{G_2}(v_j) + n_1 d_{G_1}(u_k) + n_1 n_2$$

for any $v_j \in V(G_2)$, $u_i, u_k \in V(G_1)$ by Lemma 1. Thus one can easily see that the equality holds in (5) if and only if for $u_i, u_k \in V(G_1)$ and $v_j, v_\ell \in V(G_2)$,

$$d_{G_1}(u_i) = d_{G_1}(u_k) \quad \text{and} \quad d_{G_2}(v_j) = d_{G_2}(v_\ell).$$

Hence the equality holds in (4) if and only if both G_1 and G_2 are regular graphs, that is, $G_1 \vee G_2$ is a regular graph. □

Example 1 Consider two cycle graphs C_p and C_q. We thus have

$$\prod_1 (C_p \vee C_q) = (p+2)^{2q}(q+2)^{2p} \quad \text{and} \quad \prod_2 (C_p \vee C_q) = (p+2)^{(p+2)q}(q+2)^{(q+2)p}.$$

The *Cartesian product* $G_1 \boxtimes G_2$ of graphs G_1 and G_2 has the vertex set $V(G_1 \times G_2) = V(G_1) \times V(G_2)$ and $(u_i, v_j)(u_k, v_\ell)$ is an edge of $G_1 \boxtimes G_2$ if

either $u_i = u_k$ and $v_j v_\ell \in E(G_2)$,

or $u_i u_k \in E(G_1)$ and $v_j = v_\ell$.

Theorem 2 *Let G_1 and G_2 be two connected graphs. Then*

(i)

$$\prod_1 (G_1 \boxtimes G_2) \le \left[\frac{n_2 M_1(G_1) + n_1 M_1(G_2) + 8m_1 m_2}{n_1 n_2}\right]^{n_1 n_2}. \tag{6}$$

The equality holds in (6) *if and only if $G_1 \boxtimes G_2$ is a regular graph.*

(ii)

$$\prod_2 (G_1 \boxtimes G_2) \le \frac{1}{(2n_1 m_2)^{2n_1 m_2}} \big(n_1 M_1(G_2) + 4m_1 m_2\big)^{2n_1 m_2} \times \frac{1}{(2n_2 m_1)^{2n_2 m_1}} \big(n_2 M_1(G_1) + 4m_1 m_2\big)^{2n_2 m_1}. \tag{7}$$

Moreover, the equality holds in (7) *if and only if $G_1 \boxtimes G_2$ is a regular graph.*

Proof By the definition of the first multiplicative Zagreb index, we have

$$\begin{aligned} \prod_1 (G_1 \boxtimes G_2) &= \prod_{(u_i,v_j) \in V(G_1 \boxtimes G_2)} \big(d_{G_1}(u_i) + d_{G_2}(v_j)\big)^2 \\ &= \prod_{u_i \in V(G_1)} \prod_{v_j \in V(G_2)} \big(d_{G_1}(u_i) + d_{G_2}(v_j)\big)^2. \end{aligned}$$

On the other hand, by (1)

$$\le \left[\frac{\sum_{u_i \in V(G_1)} \sum_{v_j \in V(G_2)} (d_{G_1}(u_i)^2 + d_{G_2}(v_j)^2 + 2d_{G_1}(u_i) d_{G_2}(v_j))}{n_1 n_2}\right]^{n_1 n_2}. \tag{8}$$

But as $\sum_{u_i \in V(G_1)} d_{G_1}(u_i)^2 = M_1(G_1)$ and $\sum_{v_j \in V(G_2)} d_{G_2}(v_j)^2 = M_1(G_2)$, the last statement in (8) is less than or equal to

$$\le \left[\frac{\sum_{u_i \in V(G_1)} (d_{G_1}(u_i)^2 \sum_{v_j \in V(G_2)} 1 + \sum_{v_j \in V(G_2)} d_{G_2}(v_j)^2 + 2d_{G_1}(u_i) \sum_{v_j \in V(G_2)} d_{G_2}(v_j))}{n_1 n_2}\right]^{n_1 n_2}$$

which equals to

$$\left[\frac{n_2 M_1(G_1) + n_1 M_1(G_2) + 8m_1 m_2}{n_1 n_2}\right]^{n_1 n_2}.$$

Moreover, the equality holds in (8) if and only if $d_{G_1}(u_i) + d_{G_2}(v_j) = d_{G_1}(u_k) + d_{G_2}(v_\ell)$ for any $(u_i, v_j), (u_k, v_\ell) \in V(G_1 \boxtimes G_2)$ by Lemma 1. Since both G_1 and G_2 are connected graphs, one can easily see that the equality holds in (8) if and only if $d_{G_1}(u_i) = d_{G_1}(u_k)$, $u_i, u_k \in V(G_1)$ and $d_{G_2}(v_j) = d_{G_2}(v_\ell)$, $v_j, v_\ell \in V(G_2)$. Hence the equality holds in (6) if and only if both G_1 and G_2 are regular graphs, that is, $G_1 \boxtimes G_2$ is a regular graph. This completes the first part of the proof.

By the definition of the second multiplicative Zagreb index, we have

$$\prod_2(G_1 \boxtimes G_2) = \prod_{(u_i,v_j)(u_k,v_\ell)\in E(G_1\boxtimes G_2)} \big(d_{G_1}(u_i) + d_{G_2}(v_j)\big)\big(d_{G_1}(u_k) + d_{G_2}(v_\ell)\big).$$

This actually can be written as

$$\begin{aligned}\prod_2(G_1 \boxtimes G_2) = {} & \prod_{u_i\in V(G_1)} \prod_{v_jv_\ell\in E(G_2)} \big(d_{G_1}(u_i) + d_{G_2}(v_j)\big)\big(d_{G_1}(u_i) + d_{G_2}(v_\ell)\big) \\ & \times \prod_{v_j\in V(G_2)} \prod_{u_iu_k\in E(G_1)} \big(d_{G_1}(u_i) + d_{G_2}(v_j)\big)\big(d_{G_1}(u_k) + d_{G_2}(v_j)\big)\end{aligned}$$

or, equivalently,

$$\begin{aligned}\prod_2(G_1 \boxtimes G_2) = {} & \prod_{u_i\in V(G_1)} \prod_{v_j\in V(G_2)} \big(d_{G_1}(u_i) + d_{G_2}(v_j)\big)^{d_{G_2}(v_j)} \\ & \times \prod_{v_j\in V(G_2)} \prod_{u_i\in V(G_1)} \big(d_{G_1}(u_i) + d_{G_2}(v_j)\big)^{d_{G_1}(u_i)}.\end{aligned}$$

After that, by (2) we get

$$\begin{aligned}\prod_2(G_1 \boxtimes G_2) \le {} & \prod_{u_i\in V(G_1)} \left[\frac{\sum_{v_j\in V(G_2)} d_{G_2}(v_j)(d_{G_1}(u_i) + d_{G_2}(v_j))}{2m_2}\right]^{2m_2} \\ & \times \prod_{v_j\in V(G_2)} \left[\frac{\sum_{u_i\in V(G_1)} d_{G_1}(u_i)(d_{G_1}(u_i) + d_{G_2}(v_j))}{2m_1}\right]^{2m_1}.\end{aligned} \tag{9}$$

Moreover, since

$$\sum_{u_i\in V(G_1)} d_{G_1}(u_i) = 2m_1, \qquad \sum_{v_j\in V(G_2)} d_{G_2}(v_j) = 2m_2 \quad \text{and}$$
$$\sum_{u_i\in V(G_1)} d_{G_1}(u_i)^2 = M_1(G_1), \qquad \sum_{v_j\in V(G_2)} d_{G_2}(v_j)^2 = M_1(G_2).$$

By (1) the final statement in (9) becomes

$$\begin{aligned} & = \prod_{u_i\in V(G_1)} \left[\frac{M_1(G_2) + 2m_2 d_{G_1}(u_i)}{2m_2}\right]^{2m_2} \times \prod_{v_j\in V(G_2)} \left[\frac{M_1(G_1) + 2m_1 d_{G_2}(v_j)}{2m_1}\right]^{2m_1} \\ & \le \frac{1}{(2m_2)^{2n_1m_2}} \left[\frac{\sum_{u_i\in V(G_1)}(M_1(G_2) + 2m_2 d_{G_1}(u_i))}{n_1}\right]^{2n_1m_2}\end{aligned}$$

$$\times \frac{1}{(2m_1)^{2n_2m_1}}\left[\frac{\sum_{v_j\in V(G_2)}(M_1(G_1)+2m_1 d_{G_2}(v_j))}{n_2}\right]^{2n_2m_1} \tag{10}$$

$$= \frac{1}{(2n_1m_2)^{2n_1m_2}}\left(n_1M_1(G_2)+4m_1m_2\right)^{2n_1m_2}$$

$$\times \frac{1}{(2n_2m_1)^{2n_2m_1}}\left(n_2M_1(G_1)+4m_1m_2\right)^{2n_2m_1}.$$

Hence the second part of the proof is over.

The equality holds in (9) and (10) if and only if $d_{G_2}(v_j) = d_{G_2}(v_\ell)$ for any $v_j, v_\ell \in V(G_2)$ and $d_{G_1}(u_i) = d_{G_1}(u_k)$ for any $u_i, u_k \in V(G_1)$ by Lemmas 1 and 2. Hence the equality holds in (7) if and only if both G_1 and G_2 are regular graphs, that is, $G_1 \boxtimes G_2$ is a regular graph. This completes the proof. □

Example 2 Consider a cycle graph C_p and a complete graph K_q. We thus have

$$\prod_1 (C_p \boxtimes K_q) = (q+1)^{2pq} \quad \text{and} \quad \prod_2 (C_p \boxtimes K_q) = (q+1)^{(q+1)pq}.$$

The *corona product* $G_1 \circ G_2$ of two graphs G_1 and G_2 is defined to be the graph Γ obtained by taking one copy of G_1 (which has n_1 vertices) and n_1 copies of G_2, and then joining the ith vertex of G_1 to every vertex in the ith copy of G_2, $i = 1, 2, \ldots, n_1$.

Let $G_1 = (V, E)$ and $G_2 = (V, E)$ be two graphs such that $V(G_1) = \{u_1, u_2, \ldots, u_{n_1}\}$, $|E(G_1)| = m_1$ and $V(G_2) = \{v_1, v_2, \ldots, v_{n_2}\}$, $|E(G_2)| = m_2$. Then it follows from the definition of the corona product that $G_1 \circ G_2$ has $n_1(1+n_2)$ vertices and $m_1 + n_1m_2 + n_1n_2$ edges, where $V(G_1 \circ G_2) = \{(u_i, v_j), i = 1, 2, \ldots, n_1; j = 0, 1, 2, \ldots, n_2\}$ and $E(G_1 \circ G_2) = \{((u_i, v_0), (u_k, v_0)), (u_i, u_k) \in E(G_1)\} \cup \{((u_i, v_j), (u_i, v_\ell)), (v_j, v_\ell) \in E(G_2), i = 1, 2, \ldots, n_1\} \cup \{((u_i, v_0), (u_i, v_\ell)), \ell = 1, 2, \ldots, n_2, i = 1, 2, \ldots, n_1\}$. It is clear that if G_1 is connected, then $G_1 \circ G_2$ is connected, and in general $G_1 \circ G_2$ is not isomorphic to $G_2 \circ G_1$.

Theorem 3 *The first and second multiplicative Zagreb indices of the corona product are computed as follows*:

(i)

$$\prod_1 (G_1 \circ G_2) \le \frac{1}{n_1^{n_1} n_2^{n_1n_2}} M_1(G_1)^{n_1}\left(M_1(G_2)+4m_2+n_2\right)^{n_1n_2}, \tag{11}$$

(ii)

$$\prod_2 (G_1 \circ G_2) \le \left[\frac{M_2(G_1)+n_2M_1(G_1)+n_2^2}{m_1}\right]^{m_1}\left[\frac{M_2(G_2)+M_1(G_2)+1}{m_2}\right]^{n_1m_2}$$

$$\times \left[\frac{4m_1m_2+n_1n_2^2+2m_1n_2+2m_2n_1n_2}{n_1n_2}\right]^{n_1n_2}, \tag{12}$$

where $M_1(G_i)$ and $M_2(G_i)$ are the first and second Zagreb indices of G_i, where $i = 1, 2$, respectively. Moreover, both equalities in (11) *and* (12) *hold if and only if $G_1 \circ G_2$ is a regular graph.*

Proof By the definition of the first multiplicative Zagreb index, we have

$$\begin{aligned}
\prod_1(G_1\circ G_2) &= \prod_{(u_i,v_j)\in V(G_1\circ G_2)} d_{G_1\circ G_2}(u_i,v_j)^2 \\
&= \prod_{u_i\in V(G_1)} \big(d_{G_1}(u_i)+n_2\big)^2 \prod_{u_i\in V(G_1)}\prod_{v_j\in V(G_2)} \big(d_{G_2}(v_j)+1\big)^2 \\
&= \prod_{u_i\in V(G_1)} \big(d_{G_1}(u_i)^2+2n_2 d_{G_1}(u_i)+n_2^2\big) \\
&\quad\times \Big[\prod_{v_j\in V(G_2)} \big(d_{G_2}(v_j)^2+2d_{G_2}(v_j)+1\big)^2\Big]^{n_1} \\
&\le \Big[\frac{\sum_{u_i\in V(G_1)}(d_{G_1}(u_i)^2+2n_2 d_{G_1}(u_i)+n_2^2)}{n_1}\Big]^{n_1} \\
&\quad\times \Big[\frac{\sum_{v_j\in V(G_2)}(d_{G_2}(v_j)^2+2d_{G_2}(v_j)+1)}{n_2}\Big]^{n_1 n_2} \quad \text{by (1)} \qquad (13) \\
&= \frac{1}{n_1^{n_1} n_2^{n_1 n_2}}\big(M_1(G_1)+4n_2 m_1+n_1 n_2^2\big)^{n_1}\big(M_1(G_2)+4m_2+n_2\big)^{n_1 n_2}.
\end{aligned}$$

The equality holds in (13) if and only if $d_{G_1}(u_i)=d_{G_1}(u_k)$, $u_i,u_k\in V(G_1)$ and $d_{G_2}(v_j)=d_{G_2}(v_\ell)$, $v_j,v_\ell\in V(G_2)$, that is, both G_1 and G_2 are regular graphs, that is, $G_1\circ G_2$ is a regular graph.

By the definition of the second multiplicative Zagreb index, we have

$$\begin{aligned}
\prod_2(G_1\circ G_2) &= \prod_{(u_i,v_j)(u_k,v_\ell)\in E(G_1\circ G_2)} d_{G_1\circ G_2}(u_i,v_j)d_{G_1\circ G_2}(u_k,v_\ell) \\
&= \prod_{u_iu_k\in E(G_1)} \big(d_{G_1}(u_i)+n_2\big)\big(d_{G_1}(u_k)+n_2\big) \\
&\quad\times \prod_{u_i\in V(G_1)}\prod_{v_j\in V(G_2)} \big(d_{G_1}(u_i)+n_2\big)\big(d_{G_2}(v_j)+1\big) \\
&\quad\times \prod_{u_i\in V(G_1)}\prod_{v_jv_\ell\in E(G_2)} \big(d_{G_2}(v_j)+1\big)\big(d_{G_2}(v_\ell)+1\big) \\
&= \prod_{u_iu_k\in E(G_1)} \big(d_{G_1}(u_i)d_{G_1}(u_k)+n_2\big(d_{G_1}(u_i)+d_{G_1}(u_k)\big)+n_2^2\big) \\
&\quad\times \Big[\prod_{u_i\in V(G_1)} \big(d_{G_1}(u_i)+n_2\big)\Big]^{n_2}\Big[\prod_{v_j\in V(G_2)} \big(d_{G_1}(v_j)+1\big)\Big]^{n_1} \\
&\quad\times \Big[\prod_{v_jv_\ell\in E(G_2)} \big(d_{G_2}(v_j)d_{G_2}(v_\ell)+\big(d_{G_2}(v_j)+d_{G_2}(v_\ell)\big)+1\big)\Big]^{n_1} \\
&\le \Big[\frac{M_2(G_1)+n_2M_1(G_1)+n_2^2m_1}{m_1}\Big]^{m_1}\times\Big[\frac{2m_1+n_1n_2}{n_1}\Big]^{n_1n_2} \\
&\quad\times \Big[\frac{2m_2+n_2}{n_2}\Big]^{n_1n_2}\times\Big[\frac{M_2(G_2)+M_1(G_2)+m_2}{m_2}\Big]^{n_1m_2} \quad \text{by (1)}.
\end{aligned}$$

The above equality holds if and only if $d_{G_1}(u_i) = d_{G_1}(u_k)$ for any $u_i, u_k \in V(G_1)$ and $d_{G_2}(v_j) = d_{G_2}(v_\ell)$ for any $v_j, v_\ell \in V(G_2)$, that is, both G_1 and G_2 are regular graphs, which implies that $G_1 \circ G_2$ is a regular graph. This completes the proof. □

Example 3 $\prod_1(C_p \circ K_q) = q^{2pq}(q+2)^{2p}$ and $\prod_2(C_p \circ K_q) = q^{pq^2}(q+2)^{p(q+2)}$.

The *composition* (also called *lexicographic product* [12]) $G = G_1[G_2]$ of graphs G_1 and G_2 with disjoint vertex sets $V(G_1)$ and $V(G_2)$ and edge sets $E(G_1)$ and $E(G_2)$ is the graph with a vertex set $V(G_1) \times V(G_2)$ and (u_i, v_j) is adjacent to (u_k, v_ℓ) whenever

either u_i is adjacent to u_k,

or $u_i = u_k$ and v_j is adjacent to v_ℓ.

Theorem 4 *The first and second multiplicative Zagreb indices of the composition* $G_1[G_2]$ *of graphs* G_1 *and* G_2 *are bounded above as follows*:

(i)

$$\prod_1 \bigl(G_1[G_2]\bigr) \le \frac{1}{(n_1n_2)^{n_1n_2}} \bigl[n_2^3 M_1(G_1) + 8n_2m_1m_2 + n_1M_1(G_2)\bigr]^{n_1n_2}, \tag{14}$$

(ii)

$$\prod_2 \bigl(G_1[G_2]\bigr) \le \frac{1}{(n_1m_2)^{n_1m_2}} \bigl[m_2n_2^2M_1(G_1) + 2n_2m_1M_1(G_2) + n_1M_2(G_2)\bigr]^{n_1m_2} \times \frac{1}{(n_2m_1)^{m_1n_2^2}} \bigl[n_2^3M_2(G_1) + m_1M_1(G_2) + 2m_2n_2M_1(G_1)\bigr]^{n_2^2m_1}, \tag{15}$$

where $M_1(G_i)$ *and* $M_2(G_i)$ *are the first and second Zagreb indices of* G_i, *where* $i = 1, 2$. *Moreover, the equalities in* (14) *and* (15) *hold if and only if* $G_1 \circ G_2$ *is a regular graph.*

Proof By the definition of the first multiplicative Zagreb index, we have

$$\begin{aligned}
&\prod_1 \bigl(G_1[G_2]\bigr) \\
&\quad = \prod_{(u_i,v_j) \in V(G_1[G_2])} d_{G_1[G_2]}(u_i, v_j)^2 \\
&\quad = \prod_{u_i \in V(G_1)} \prod_{v_j \in V(G_2)} \bigl(d_{G_1}(u_i)n_2 + d_{G_2}(v_j)\bigr)^2 \\
&\quad \le \biggl[\frac{\sum_{u_i \in V(G_1)} \sum_{v_j \in V(G_2)} (n_2^2 d_{G_1}(u_i)^2 + 2n_2 d_{G_1}(u_i) d_{G_2}(v_j) + d_{G_2}(v_j)^2)}{n_1n_2}\biggr]^{n_1n_2} \qquad (16) \\
&\quad = \frac{1}{(n_1n_2)^{n_1n_2}} \bigl[n_2^3 M_1(G_1) + 8n_2m_1m_2 + n_1M_1(G_2)\bigr]^{n_1n_2}.
\end{aligned}$$

The equality holds in (16) if and only if $d_{G_1}(u_i) = d_{G_1}(u_k)$, $u_i, u_k \in V(G_1)$ and $d_{G_2}(v_j) = d_{G_2}(v_\ell)$, $v_j, v_\ell \in V(G_2)$ (by Lemma 1), that is, both G_1 and G_2 are regular graphs, that is, $G_1 \circ G_2$ is a regular graph.

By the definition of the second multiplicative Zagreb index, we have

$$\begin{aligned}
&\prod_2(G_1[G_2]) \\
&\quad= \prod_{(u_i,v_j)(u_k,v_\ell)\in E(G_1[G_2])} d_{G_1\circ G_2}(u_i,v_j)d_{G_1[G_2]}(u_k,v_\ell) \\
&\quad= \prod_{u_i\in V(G_1)}\prod_{v_jv_\ell\in E(G_2)} \big(d_{G_1}(u_i)n_2+d_{G_2}(v_j)\big)\big(d_{G_1}(u_i)n_2+d_{G_2}(v_\ell)\big) \\
&\qquad\times \prod_{u_iu_k\in E(G_1)}\prod_{v_j\in V(G_2)} \big[\big(d_{G_1}(u_i)n_2+d_{G_2}(v_j)\big)\big(d_{G_1}(u_k)n_2+d_{G_2}(v_j)\big)\big]^{n_2} \\
&\quad\le \prod_{u_i\in V(G_1)}\left[\frac{m_2n_2^2d_{G_1}(u_i)^2+n_2d_{G_1}(u_i)M_1(G_2)+M_2(G_2)}{m_2}\right]^{m_2} \\
&\qquad\times \prod_{u_iu_k\in E(G_1)}\left[\frac{n_2^3d_{G_1}(u_i)d_{G_1}(u_k)+M_1(G_2)+2m_2n_2(d_{G_1}(u_i)+d_{G_1}(u_k))}{n_2}\right]^{n_2^2} \qquad (17)\\
&\quad\le \frac{1}{m_2^{n_1m_2}}\left[\frac{m_2n_2^2M_1(G_1)+2n_2m_1M_1(G_2)+n_1M_2(G_2)}{n_1}\right]^{n_1m_2} \\
&\qquad\times \frac{1}{(n_2)^{m_1n_2^2}}\left[\frac{n_2^3M_2(G_1)+m_1M_1(G_2)+2m_2n_2M_1(G_1)}{m_1}\right]^{n_2^2m_1}, \qquad (18)
\end{aligned}$$

which gives the required result in (15).

The equality holds in (17) and (18) if and only if $d_{G_1}(u_i)=d_{G_1}(u_k)$, $u_i,u_k\in V(G_1)$ and $d_{G_2}(v_j)=d_{G_2}(v_\ell)$, $v_j,v_\ell\in V(G_2)$ (by Lemma 1), that is, both G_1 and G_2 are regular graphs, that is, $G_1\circ G_2$ is a regular graph. □

Example 4 $\prod_1(C_p[C_q])=2^{2pq}(q+1)^{2pq}$ and $\prod_2(C_p[C_q])=2^{2pq(q+1)}(q+1)^{2pq(q+1)}$.

The *disjunction* $G_1\otimes G_2$ of graphs G_1 and G_2 is the graph with a vertex set $V(G_1)\times V(G_2)$ and (u_i,v_j) is adjacent to (u_k,v_ℓ) whenever $u_iu_k\in E(G_1)$ or $v_jv_\ell\in E(G_2)$.

Theorem 5 *The first and second multiplicative Zagreb indices of the disjunction are computed as follows*:

(i)

$$\begin{aligned}
\prod_1(G_1\otimes G_2) \le \frac{1}{(n_1n_2)^{n_1n_2}}\big[&n_2^3M_1(G_1)+n_1^3M_1(G_2)+M_1(G_1)M_1(G_2) \\
&+8n_1n_2m_1m_2-4n_1m_1M_1(G_2)-4n_2m_2M_1(G_1)\big]^{n_1n_2}, \qquad (19)
\end{aligned}$$

(ii)

$$\begin{aligned}
&\prod_2(G_1\otimes G_2) \\
&\quad\le \left[\frac{M_1(G_1)(n_2^3+M_1(G_2)-4n_2m_2)+M_1(G_2)(n_1^2-4n_1m_1)+8n_1n_2m_1m_2}{Q}\right]^Q, \qquad (20)
\end{aligned}$$

where $Q = \sum_{u_i \in V(G_1)} \sum_{v_j \in V(G_2)} P = 2(n_2^2 m_1 + n_1^2 m_2 - 2m_1 m_2)$ *and* $M_1(G_i)$ *is the first Zagreb index of* G_i, $i = 1, 2$. *Moreover, the equalities in* (19) *and* (20) *hold if and only if* $G_1 \circ G_2$ *is a regular graph.*

Proof We have $d_{G_1 \otimes G_2}(u_i, v_j) = n_2 d_{G_1}(u_i) + n_1 d_{G_2}(v_j) - d_{G_1}(u_i) d_{G_2}(v_j)$. By the definition of the first multiplicative Zagreb index, we have

$$\begin{aligned} &\prod_1 (G_1 \otimes G_2) \\ &\quad = \prod_{(u_i, v_j) \in V(G_1 \otimes G_2)} d_{G_1 \otimes G_2}(u_i, v_j)^2 \\ &\quad = \prod_{u_i \in V(G_1)} \prod_{v_j \in V(G_2)} \bigl(n_2 d_{G_1}(u_i) + n_1 d_{G_2}(v_j) - d_{G_1}(u_i) d_{G_2}(v_j)\bigr)^2 \\ &\quad \le \left[\frac{\sum_{u_i \in V(G_1)} \sum_{v_j \in V(G_2)} (n_2 d_{G_1}(u_i) + n_1 d_{G_2}(v_j) - d_{G_1}(u_i) d_{G_2}(v_j))^2}{n_1 n_2} \right]^{n_1 n_2} \qquad (21) \\ &\quad = \frac{1}{(n_1 n_2)^{n_1 n_2}} \bigl[n_2^3 M_1(G_1) + n_1^3 M_1(G_2) + M_1(G_1) M_1(G_2) \\ &\qquad + 8 n_1 n_2 m_1 m_2 - 4 n_1 m_1 M_1(G_2) - 4 n_2 m_2 M_1(G_1) \bigr]^{n_1 n_2}. \end{aligned}$$

The equality holds in (21) if and only if $d_{G_1}(u_i) = d_{G_1}(u_k)$, $u_i, u_k \in V(G_1)$ and $d_{G_2}(v_j) = d_{G_2}(v_\ell)$, $v_j, v_\ell \in V(G_2)$ (by Lemma 1), that is, both G_1 and G_2 are regular graphs, that is, $G_1 \circ G_2$ is a regular graph.

By the definition of the second multiplicative Zagreb index, we have

$$\begin{aligned} \prod_2 (G_1 \otimes G_2) &= \prod_{(u_i, v_j)(u_k, v_\ell) \in E(G_1 \otimes G_2)} d_{G_1 \otimes G_2}(u_i, v_j) d_{G_1 \otimes G_2}(u_k, v_\ell) \\ &= \prod_{u_i \in V(G_1)} \prod_{v_j \in V(G_2)} P^P, \end{aligned}$$

where

$$P = n_2 d_{G_1}(u_i) + n_1 d_{G_2}(v_j) - d_{G_1}(u_i) d_{G_2}(v_j).$$

Using the weighted arithmetic-geometric mean inequality in (2), $\prod_2 (G_1 \otimes G_2)$ is less than or equal to

$$\begin{aligned} &\le \left[\frac{\sum_{u_i \in V(G_1)} \sum_{v_j \in V(G_2)} (n_2 d_{G_1}(u_i) + n_1 d_{G_2}(v_j) - d_{G_1}(u_i) d_{G_2}(v_j))^2}{\sum_{u_i \in V(G_1)} \sum_{v_j \in V(G_2)} P} \right]^{\sum_{u_i \in V(G_1)} \sum_{v_j \in V(G_2)} P} \qquad (22) \\ &= \left[\frac{M_1(G_1)(n_2^3 + M_1(G_2) - 4 n_2 m_2) + M_1(G_2)(n_1^2 - 4 n_1 m_1) + 8 n_1 n_2 m_1 m_2}{Q} \right]^Q, \end{aligned}$$

where

$$Q = \sum_{u_i \in V(G_1)} \sum_{v_j \in V(G_2)} P = 2\bigl(n_2^2 m_1 + n_1^2 m_2 - 2 m_1 m_2\bigr).$$

Hence the first part of the proof is over.

The equality holds in (22) if and only if $d_{G_1}(u_i) = d_{G_1}(u_k)$, where $u_i, u_k \in V(G_1)$ and $d_{G_2}(v_j) = d_{G_2}(v_\ell)$, where $v_j, v_\ell \in V(G_2)$ (by Lemma 1), that is, both G_1 and G_2 are regular graphs, and so the graph $G_1 \circ G_2$ is regular. □

Example 5 $\prod_1(K_p \otimes C_q) = (pq - q + 2)^{2pq}$ and $\prod_2(K_p \otimes C_q) = (pq - q + 2)^{pq(pq-q+2)}$.

The *symmetric difference* $G_1 \oplus G_2$ of two graphs G_1 and G_2 is the graph with a vertex set $V(G_1) \times V(G_2)$ in which (u_i, v_j) is adjacent to (u_k, v_ℓ) whenever u_i is adjacent to u_k in G_1 or v_i is adjacent to v_ℓ in G_2, but not both. The degree of a vertex (u_i, v_j) of $G_1 \oplus G_2$ is given by

$$d_{G_1 \oplus G_2}(u_i, v_j) = n_2 d_{G_1}(u_i) + n_1 d_{G_2}(v_j) - 2 d_{G_1}(u_i) d_{G_2}(v_j),$$

while the number of edges in $G_1 \oplus G_2$ is $n_1^2 m_2 + n_2^2 m_1 - 4 m_1 m_2$.

Theorem 6 *The first and second multiplicative Zagreb indices of the symmetric difference $G_1 \oplus G_2$ of two graphs G_1 and G_2 are bounded above as follows*:

(i)

$$\prod_1(G_1 \oplus G_2) \le \frac{1}{(n_1 n_2)^{n_1 n_2}} \bigl[n_2^3 M_1(G_1) + n_1^3 M_1(G_2) + 4 M_1(G_1) M_1(G_2) + 8 n_1 n_2 m_1 m_2 - 8 n_1 m_1 M_1(G_2) - 8 n_2 m_2 M_1(G_1)\bigr]^{n_1 n_2}, \tag{23}$$

(ii)

$$\prod_2(G_1 \oplus G_2) \le \left[\frac{M_1(G_1)(n_2^3 + 4 M_1(G_2) - 8 n_2 m_2) + M_1(G_2)(n_1^3 - 8 n_1 m_1) + 8 n_1 n_2 m_1 m_2}{Q}\right]^Q, \tag{24}$$

where $Q = \sum_{u_i \in V(G_1)} \sum_{v_j \in V(G_2)} P = 2(n_2^2 m_1 + n_1^2 m_2 - 4 m_1 m_2)$ and $M_1(G_i)$ is the first Zagreb index of G_i, for $i = 1, 2$. Moreover, the equalities in (23) *and* (24) *hold if and only if $G_1 \circ G_2$ is a regular graph.*

Proof We have

$$d_{G_1 \oplus G_2}(u_i, v_j) = n_2 d_{G_1}(u_i) + n_1 d_{G_2}(v_j) - 2 d_{G_1}(u_i) d_{G_2}(v_j).$$

By the definition of the first multiplicative Zagreb index, we have

$$\begin{aligned}\prod_1(G_1 \oplus G_2) &= \prod_{(u_i, v_j) \in V(G_1 \oplus G_2)} d_{G_1 \otimes G_2}(u_i, v_j)^2 \\ &= \prod_{u_i \in V(G_1)} \prod_{v_j \in V(G_2)} \bigl(n_2 d_{G_1}(u_i) + n_1 d_{G_2}(v_j) - 2 d_{G_1}(u_i) d_{G_2}(v_j)\bigr)^2\end{aligned}$$

$$\leq \left[\frac{\sum_{u_i\in V(G_1)}\sum_{v_j\in V(G_2)}(n_2 d_{G_1}(u_i)+n_1 d_{G_2}(v_j)-2d_{G_1}(u_i)d_{G_2}(v_j))^2}{n_1 n_2}\right]^{n_1 n_2} \tag{25}$$

$$= \frac{1}{(n_1 n_2)^{n_1 n_2}}\left[n_2^3 M_1(G_1)+n_1^3 M_1(G_2)+4M_1(G_1)M_1(G_2)+8n_1 n_2 m_1 m_2 - 8n_1 m_1 M_1(G_2)-8n_2 m_2 M_1(G_1)\right]^{n_1 n_2}.$$

The equality holds in (25) if and only if $d_{G_1}(u_i) = d_{G_1}(u_k)$, $u_i, u_k \in V(G_1)$ and $d_{G_2}(v_j) = d_{G_2}(v_\ell)$, $v_j, v_\ell \in V(G_2)$ (by Lemma 1), that is, both G_1 and G_2 are regular graphs, which implies that $G_1 \circ G_2$ is a regular graph.

By the definition of the second multiplicative Zagreb index, we have

$$\prod_2(G_1\oplus G_2) = \prod_{(u_i,v_j)(u_k,v_\ell)\in E(G_1\otimes G_2)} d_{G_1\otimes G_2}(u_i,v_j)d_{G_1\otimes G_2}(u_k,v_\ell) = \prod_{u_i\in V(G_1)}\prod_{v_j\in V(G_2)} P^P,$$

where $P = n_2 d_{G_1}(u_i) + n_1 d_{G_2}(v_j) - 2d_{G_1}(u_i)d_{G_2}(v_j)$.

Using the weighted arithmetic-geometric mean inequality in (2), we get

$$\prod_{u_i\in V(G_1)}\prod_{v_j\in V(G_2)} P^P \leq \left[\frac{\sum_{u_i\in V(G_1)}\sum_{v_j\in V(G_2)}(n_2 d_{G_1}(u_i)+n_1 d_{G_2}(v_j)-2d_{G_1}(u_i)d_{G_2}(v_j))^2}{\sum_{u_i\in V(G_1)}\sum_{v_j\in V(G_2)} P}\right]^{\sum_{u_i\in V(G_1)}\sum_{v_j\in V(G_2)} P} \tag{26}$$

$$= \left[\frac{M_1(G_1)(n_2^3+4M_1(G_2)-8n_2 m_2)+M_1(G_2)(n_1^3-8n_1 m_1)+8n_1 n_2 m_1 m_2}{Q}\right]^Q,$$

where $Q = \sum_{u_i\in V(G_1)}\sum_{v_j\in V(G_2)} P = 2(n_2^2 m_1 + n_1^2 m_2 - 4m_1 m_2)$. First part of the proof is over.

The equality holds in (26) if and only if $d_{G_1}(u_i) = d_{G_1}(u_k)$, $u_i, u_k \in V(G_1)$ and $d_{G_2}(v_j) = d_{G_2}(v_\ell)$, $v_j, v_\ell \in V(G_2)$ (by Lemma 1), that is, both G_1 and G_2 are regular graphs, which implies that $G_1 \circ G_2$ is a regular graph. □

Example 6 $\prod_1(G_1 \oplus G_2) = (p+q-2)^{2pq}$ and $\prod_2(G_1 \oplus G_2) = (p+q-2)^{pq(p+q-2)}$.

Competing interests
The authors declare that they have no competing interests.

Authors' contributions
All authors completed the paper together. All authors read and approved the final manuscript.

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