

A Geometric Modeling of Dog Intestine

Hüseyin YILDIZ^{1,*}, Kadri ARSLAN², İhsaniye COŞKUN³, Bahri YILDIZ¹

¹Department of Anatomy, Faculty of Veterinary Medicine, Uludağ University, 16059 Görükle, Bursa - TURKEY

²Department of Mathematics, Faculty of Arts and Science, Uludağ University, 16059 Görükle, Bursa - TURKEY

³Department of Anatomy, Faculty of Medicine, Uludağ University, 16059 Görükle, Bursa - TURKEY

Received: 09.11.2005

Abstract: In computer-assisted surgical applications, reconstruction of 3-D morphological peculiarities are required. Herein, we present a geometric modeling of the separate intestinal sections of the canine. The intestine of the dog was considered a tubular shape along a special curve. Seven male Turkish shepherd dogs were used for the modeling study. The length (cm) and diameter (mm) of the intestines were measured with digital calipers and formulated. These models were then compared to their original photographs. It was concluded that the geometric modeling and experimental work were consistent. These kinds of organ modeling techniques will enable medical lecturers to show 3-D figures to their students.

Key Words: Canine, intestine, surface, tube, parameterized curve

Köpek Bağırsağının Geometrik Modellenmesi

Özet: Bilgisayar destekli cerrahi uygulamalarda morfolojik özelliklerin 3 boyutlu görünümü zorunludur. Bu nedenle, çalışmada köpeğin bağırsak bölümlerinin geometrik modellenmesi gerçekleştirildi. Köpek bağırsağı, özel kıvrımlara sahip bir tüp şeklinde düşünüldü ve modelleme için 7 erkek Türk Çoban Köpeği kullanıldı. Bağırsak uzunluk (cm) ve çapları (mm) dijital kumpas ile ölçülerek formülleri yapıldı. Oluşturulan modeller, orjinal fotoğrafları ile karşılaştırıldı. Sonuç olarak geometrik modelleme ile deneysel çalışma değerlerinin birbirlerine uyumlu olduğu tespit edildi. Bu tür organ modelleme teknikleri tıp öğrencilerine 3 boyutlu görünümünün sunulması bakımından faydalı olacaktır.

Anahtar Sözcükler: Köpek, bağırsak, yüzey, tüp, parametrize kıvrım

Introduction

Computerized modeling of anatomical morphologies has become very useful for visualizing complex 3-D forms. In computer-assisted surgical applications, efficient volume reconstructions of 3-D anatomical features are needed. Computer models not only provide a means to visualize complex morphology derived from 2-D tissue outlines, they also permit mathematical modeling of growth or functional attributes. Numerical simulation and prediction can be used to test functional or morphogenetic hypotheses. Problems can arise due to the very complex shapes of many structures, such as inaccurate modeling and the loss of small features. To quantify the quality of the reconstruction, geometrical measurements of the obtained models are made and compared with the original object (1,2).

The most anatomically detailed simulations made are those of the human face. Skin is generally modeled as a surface mesh whose points must move as expression changes (3).

Biological modeling of cells, organs, and systems has reached a very significant stage of development. At the cellular level, in particular, there has been a long period of iteration between simulation and experiment (4,5). Considerable effort was spent during the 1980s to develop algorithms capable of modeling surfaces. Two general approaches were developed: surface modeling and solid modeling. Surface modeling reconstructs the object as a shell. Geometric mapping algorithms are used to map the coordinates of each contour to the subsequent contours on subsequent sections, providing a wire-mesh model (6).

* E-mail: yildiz@uludag.edu.tr

The aim of this study was to investigate the consistency between the geometrical and experimental work of the canine intestines.

Materials and Methods

Experimental work

The study included 7 male Turkish shepherd dogs (Karabash) 17-18 months of age, 28.30 ± 1.14 kg, and 81.20 ± 2.57 cm in body length. The canines were primarily used as cadavers for student education.

An important question for mathematical modelers is how much detail to include in a model. If added detail simply means including more free parameters, the answer must be as little as possible (7). The length (cm) and diameter (mm) of the intestines were measured with digital calipers (Mitutoyo Corporation, Kawasaki, Japan) and photographed with a digital camera (Nikon D100, Nikon Corporation, Japan).

Geometrical work

In the sequel, a tubular shape along a certain regular curve was defined. First a regular curve was defined.

Definition 1. Let $\alpha: (a, b) \rightarrow \mathbb{R}^n$ be a function where (a, b) is an open interval in \mathbb{R} . We write $\alpha(t) = (\alpha_1(t), \alpha_2(t), \dots, \alpha_n(t))$, where each α_j is an ordinary real-valued function of a real variable. We say that α is differentiable if and only if each α_j is differentiable for $j = 1, \dots, n$. Similarly, α is piecewise differentiable if and only if α_j is piecewise differentiable.

A (parameterized) curve in \mathbb{R}^n can be considered a piecewise differentiable function, $\alpha: (a, b) \rightarrow \mathbb{R}^n$, where (a, b) is an open interval in \mathbb{R} . The velocity of α is given by $\alpha'(t) = (\alpha'_1(t), \alpha'_2(t), \dots, \alpha'_n(t))$. If $\alpha'(t) \neq 0$, then α is said to be regular (8).

Definition 2. Let $\alpha: (a, b) \rightarrow \mathbb{R}^3$ be a regular curve in E^3 . For the normal N and binormal B vectors, which are perpendicular to α' , the circle $\theta \rightarrow \cos\theta N(t) + \sin\theta B(t)$ is perpendicular to α' , at $\alpha(t)$. As this circle moves along α it traces out a surface called the tube along α , which is defined by $X(t, \theta) = \alpha(t) + r(\cos t N(t) + \sin t B(t))$, where r is the radius of the circle (8).

We considered that the dog intestine is a tubular shape about a space curve $\alpha(t) = (\alpha_1(t), \alpha_2(t), \alpha_3(t))$. To draw these tubular shapes we typed into Maple IX the command: `tubeplot([\alpha_1(t), \alpha_2(t), \alpha_3(t)], t = a..b, radius = c, numpoints = d)`.

Results

Experimental work

The length and diameter of the intestines are presented in Table 1.

Table 1. Length and diameter (arithmetic mean \pm standard deviation) of the intestines.

Parts of intestine	Diameter (mm)	Length (cm)
Duodenum	20.65 ± 1.87	32.50 ± 3.41
Jejunum	13.42 ± 1.05	348.42 ± 16.13
Ileum	15.58 ± 0.97	11.24 ± 0.21
Cecum	18.56 ± 1.54	15.53 ± 0.84
Colon	28.39 ± 0.76	33.32 ± 2.16
Rectum	37.39 ± 2.04	8.51 ± 0.22

Geometrical work

In this section, a geometric modeling of dog intestines (duodenum, jejunum, ileum, cecum, colon and rectum) is presented. These models were compared to their original photographs.

Duodenum: The graph of duodenum was plotted (Figure 1) with the Maple IX plotting command:

```
tubeplot([cos(t/3), t^2, (1/7)*t*sin(t/2)
t = -10..16, radius = 0.4, style = patch]);
```

Jejunum: The graph of dog jejunum was plotted (Figure 2) with the Maple IX plotting command:

```
tubeplot([sin(5*t), 2*t, exp(2*t), t = 1..1, radius =
0.2, style = patch]);
```

Ileum: The ileum of the canine (Figure 3) can be considered a tubular shape over the space curve $a(t) = (\sin t, t, \exp(t))$. To draw this tube in Maple IX, one can use the plotting command:

```
tubeplot[sin(t), t, exp(t)]; t = -1..1, radius = 0.3,
style = patch];
```

Cecum: The graph of dog cecum was plotted (Figure 4) using the Maple IX plotting command:

```
r := array([exp(u)*cos(u), ln(u)*u*sin(u), 0.1*exp(2*u)]);
R := 0.3*u; u0 := 0; u1 := 2;
dr := map(diff, r, u); ddr := map(diff, r, u$2);
```

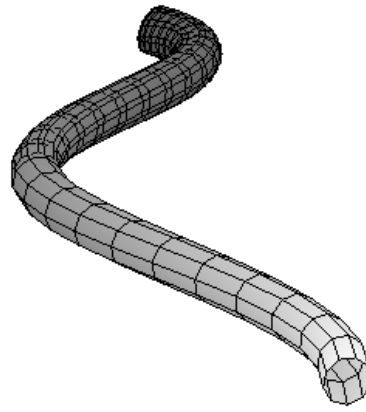
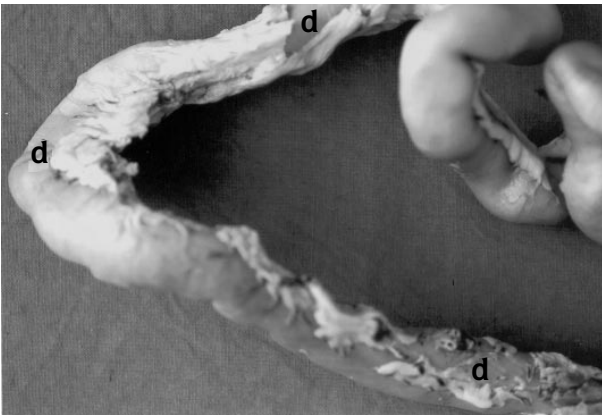


Figure 1. Photograph and the geometric modeling of dog duodenum (d).

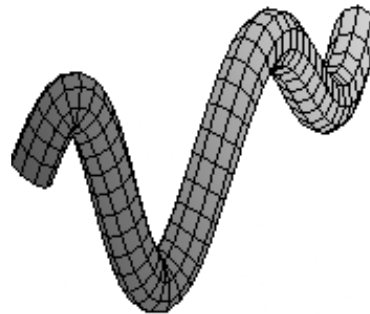
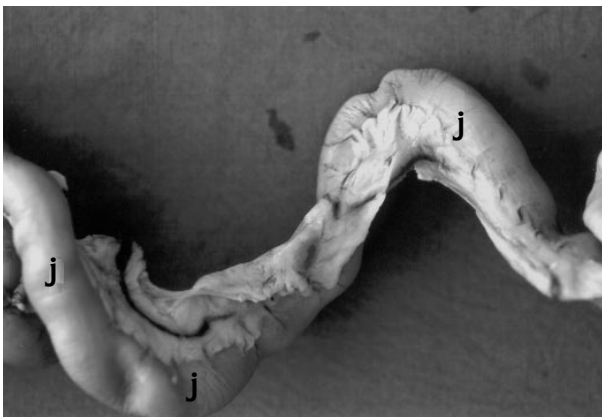


Figure 2. Photograph and the geometric modeling of dog jejunum (j).

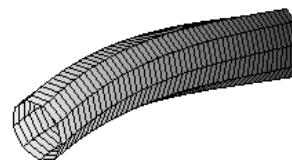
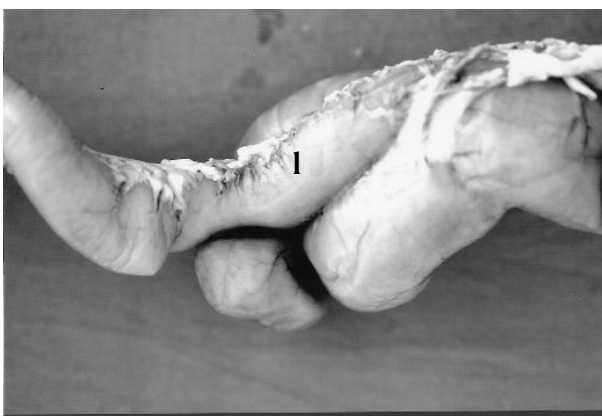


Figure 3. Photograph and geometric modeling of dog ileum (l).

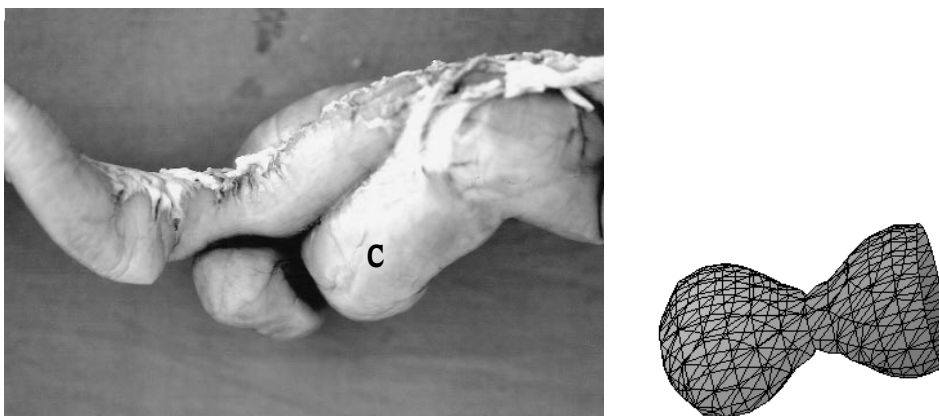


Figure 4. Photograph and geometric modeling of dog cecum (c).

```

tau: = scalarmul(dr, 1/norm(dr,2)):
b: = crossprod(dr, ddr):
beta: = scalarmul(b, 1/simplify(norm(b,2))):
nu: = crossprod(beta, tau):
rr: = evalm(r+scalarmul(nu, R*cos(v))+scalarmul
(beta, R*sin(v))): plot3d(rr, u = 0..2, v = 0..2*Pi):
implicitplot3d((4+x^2+y^2+z^2/2)^2 =
15*x^2+17, x = -2..3, y = -2..2, z = -2..2);
    
```

Colon: The dog colon (Figure 5) is short, and its 3 segments, the ascending, transverse, and descending colons, are arranged as their names indicate. However, the colon of the canine can be considered a tubular shape

over the plane curve $a(t) = (t, t^4)$. To draw these tubes in Maple IX we used the following plotting command:

```

tubeplot([24*t, t^6, sin(3*t), t = -1.3..1.3, radius =
0.6, style = patch]);
    
```

Rectum: The graph of canine rectum was plotted (Figure 6) with the Maple IX plotting command:

```

tubeplot([sin(t), t, exp(t), t = -1.8..1.3, radius =
0.8, style = patch]);
    
```

The geometrical values of the diameters of the middle of each intestinal section were measured and are presented in Table 2.

The ratios of experimental values (a) to geometric values (b) are given in Table 3.

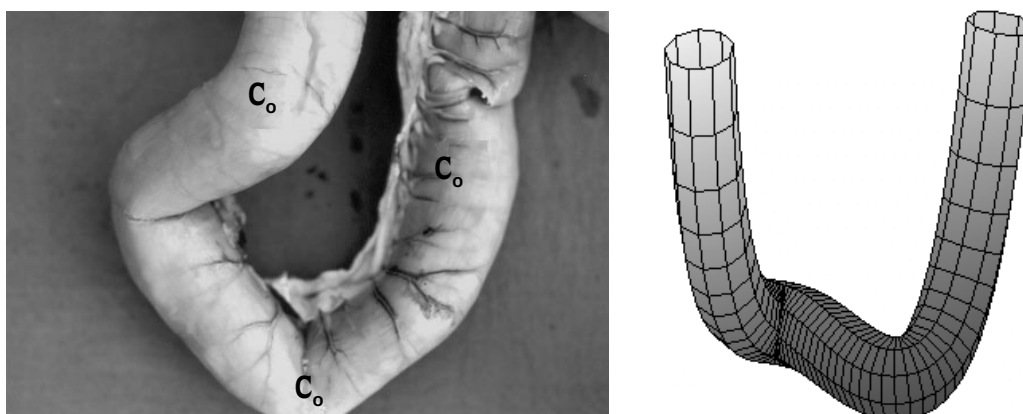


Figure 5. Photograph and geometric modeling of dog colon (Co).

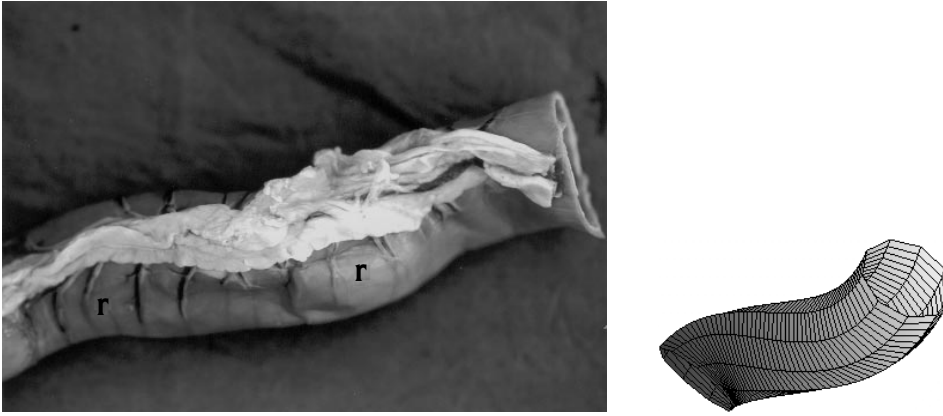


Figure 6. Photograph and geometric modeling of dog rectum (r).

Table 2. Diameter of each section of dog intestines.

Parts of intestine	Diameter (unit)
Duodenum	0.80
Jejunum	0.40
Ileum	0.60
Cecum	1.00
Colon	1.20
Rectum	1.60

Table 3. The ratios of geometric values (b) to experimental values (a).

Parts of intestine	Ratio (b/a)
Duodenum	0.038
Jejunum	0.029
Ileum	0.038
Cecum	0.053
Colon	0.042
Rectum	0.042

Discussion

There are some studies (9-13) about organ modeling with different methods in animals and humans. The present study presents a modeling technique for the tubular (surface) shapes of dog intestines.

Koch et al. (9) described a system for simulating facial surgery using finite element models. Different from these studies, we constructed a modeling technique for the tubular (surface) shapes of the intestinal organs. Shen (10) and Scheepers et al. (11) also implemented muscle models using spheroids. LEMAN, developed by Turner (12), uses implicit surfaces (includes sphere and cylinder) to model muscles. The deformation is implemented by changing the global parameters of implicit surfaces, which is not intuitional or interactive. Lung lobes, the rectus femoris muscle, and the lower limb bones are used as examples to illustrate finite element geometries (13). In the present experimental study, the measured values of the intestines and the tubular models formed according to the geometrical formulas were consistent, and the accuracy of the 3-D graphics was confirmed by the real-life photographs.

Our previous works (14,15) were related to anatomical organ modeling. In the first study, a geometric description of the ascending colons of some domestic animals, such as pig, ruminants (only the ansa spiralis coli), and dog were presented. In the second study, we presented a geometric model of the ascending colon of the horse. We showed that these cross-sections correspond to the closed algebraic curves known as epitrochoids. In the present study, a geometric modeling of the intestines (duodenum, jejunum, ileum, cecum, colon, and rectum) of the dog is presented. We consider these parts of the intestines to be tubular surfaces and plotted their graphs with the plotting command of Maple IX in different forms. It has been concluded that the geometric modeling and experimental work were consistent. For this reason, we think that such geometric modeling work will contribute to anatomical modeling studies in the future. Moreover, these kinds of organ modeling techniques will enable medical lecturers to show 3-D figures to their students.

The intestines extend from the pylorus of the stomach to the anus. Their proximal part, because of its relatively small lumen, is called the small intestine; the wider part is known as the large intestine. While the small intestine is subdivided into the duodenum, jejunum, and ileum, the large intestine includes the cecum, colon, and rectum. The duodenum is the first and most fixed part of the small intestine. It begins from the pylorus and ends at the duodenocolic fold. It is U-shaped and 25 cm in length. The jejunum begins at the duodenal flexure at the cranial end of the duodenocolic fold. It ends with the ileocecal ligament. The ileum is the terminal portion of the small intestine. It begins with the ileocecal ligament and ends by opening into the initial portion of the ascending colon. The cecum is usually described as the first part of the large intestine. It has an irregular blunt apex and ends with the ileocolic sphincter. It is about 5 cm long and 2 cm in diameter. The colon begins at the ileocolic sphincter. It terminates at a transverse plane through the pelvic inlet. It has 3 segments and is approximately 25 cm in length and 2 cm in diameter. The rectum begins at the pelvic inlet, where it is cranially continuous with the descending colon and ends at the anus. It is about 5 cm in length and 3 cm in diameter

(16-19). The diameter and the length measures of the intestinal segments of the dogs obtained in our experimental work were close to the values given both by the geometric work and the literature.

The ratio $\frac{a}{b}$ of the geometric value (b) to the experimental value (a) of the small and large intestine diameters was close; therefore, the dog intestines were modeled with a tubular shape along a certain regular curve.

This paper described a new and improved modeling and animation approach for animals and humans that is based on actual 3-D representations of individual body components, such as bones and muscles. We think this is the most natural approach for creating realistic animal and human models.

Acknowledgments

This research was supported by the Research Fund of the University of Uludağ, project number 2002/40.

References

1. Gaunt, P.N., Gaunt, W.A.: Solid reconstruction. In: Gaunt, P.N., Gaunt, W.A., Eds. *Three Dimensional Reconstruction in Biology*. Pitman Medical Press, Kent, 1978; 6373.
2. Zou, L., Li, J.T., Wang, Z-Q.: Anatomical human musculature modelling for real-time deformation. *JWSCG*, 2003; 11: 3-7.
3. Yuencheng, L., Terzepoulos, D., Waters, K.: Realistic modeling for facial animation. In *SIGGRAPH 95 Conference Proceedings*, Wesley. 1995; 55-62.
4. Avis, N.J., Briggs, N.M., Kleinermann, F., Hose, D.R., Brown, B.H., Edwards, M.H.: Anatomical and physiological models for surgical simulation. *Stud. Health Technol. Inform.*, 1999; 62: 23-29.
5. Noble, D.: Modelling the heart. *Physiology*, 2004; 19: 191-197.
6. Lozanoff, S.: Accuracy and precision of computerized models of the anterior cranial base in young mice. *Anat. Rec.*, 1992; 234: 618-624.
7. Rice, J.J., Jafri, M.S.: Modelling calcium handling in cardiac cells. *Phil. Trans. R. Soc. A.*, 2001; 359: 1143-1157.
8. Gray, A.: *Modern Differential Geometry of Curves and Surfaces*. CRS Press, Tokyo. 1993.
9. Koch, R.M., Gross, M.H., Carls, F.R., Buerin, D.F., Frankhauser, G., Parish, Y.I.H.: Simulating Facial Surgery Using Finite Element Models. In *SIGGRAPH 96 Conference Proceedings*, Wesley. 1996; 421-428.
10. Shen, J.: Human body modelling and deformations. PhD Thesis, Swiss Federal Institute of Technology, Lausanne, Switzerland, 1996.
11. Scheepers, F., Parent, R.E., Carlson, W.E., May, S.F.: Anatomy-based modelling of the human musculature. *ACM Siggraph Proceedings*, 1997; 163-172.
12. Turner, R.: Leman a system for construction and animating layered elastic characters. In: Earnshaw, R.A., Vince, J.A., Eds. *Computer Graphics*. Academic Press Ltd, England, 1995; 185-203.
13. Fernandez, J.W., Mithraratne, P., Thrupp, S.F., Tawhai, M.H., Hunter, P.J.: Anatomically based geometric modelling of the musculo-skeletal system and other organs. *Biomech. Model. Mechanobiol.*, 2004; 2: 139-155.
14. Yıldız, B., Arslan, K., Yıldız, H., Özgür, C.: A geometric description of the ascending colon of some domestic animals. *Ann. Anat.*, 2001; 183: 555-557.
15. Yıldız, H., Yıldız, B., Arslan, K., Özgür, C.: A mathematical model of the ascending colon of the horse. *Ann. Anat.*, 2001; 183: 551-554.
16. Getty, R.: *The Anatomy of the Domestic Animals*. 5th ed., WB Saunders Company, Philadelphia. 1975.
17. Nickel, R., Schummer, A., Seiferle, E.: *The Viscera of the Domestic Mammals*. 2nd ed., Verlag Paul Parey, Berlin. 1979.
18. Evans, H.E.: *Miller's Anatomy of the Dog*. 3rd ed., WB Saunders Company, Philadelphia. 1993.
19. Budras, K.D., Fricke, W., Richter, R., Mcarty, P.H.: *Anatomy of the Dog*. 4th ed., Schlütersche Verlag und Druckerei, Hannover. 2002.