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MULTI-PASS METAL-CUTTING OPTIMIZATION, PART 2: WITH DYNAMIC PROGRAMMING APPROACH

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SUMMARY

This paper describes an algorithm based on the dynamic programming approach for metal-cutting machining variables optimization. The multistage optimisation principles of dynamic programming are used to solve the cost function for multi-pass metal cutting operations. The proposed algorithm will be integrated into CNC tool path simulation program which is developed for TOFAŞ automotive factory.

ÖZET

Çok pasolu metal kesme işlemlerinin dinamik programlama yaklaşımı ile optimizasyonu

Bu makalede, metal kesme işlemlerinde makina değişkenlerinin optimizasyonunda dinamik programlama yaklaşımı anlatılmıştır. Dinamik programlamanın çoklu adım optimizasyon kuralları, çok pasolu metal kesme işlemlerinin maliyet esaslı optimizasyonunda kullanılmıştır. Önerilen algoritma, TOFAŞ otomobil fabrikası için geliştirilen CNC kesici yolu benzetimi programına entegre edilecektir.

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1. INTRODUCTION

In machining environment, the recommended values of machining are generally used but these are not necessarily the best or the most appropriate ones. There is a need to simulate and optimize the metal cutting process since the part production cost depends largely on machining time and NC part program preparation time. Reducing manufacturing lead time gives the manufacturers a competitive advantage in today's global market. To utilize the advantages of using NC machines, the machining parameters must be optimal ones and nonproductive time must be decreased by means of off-line tool path simulation.

The cost minimization criteria of multi-pass cutting consists of many sequential decisions that have a multistage mathematical structure which can be solved by the sequential optimisation principles of dynamic programming. Serial network type problems that occur in engineering, may contain several decisions that can not be easily handled. It is difficult to solve them because of the excessive time required. The reduction of highly complex systems into simple N-subproblems makes it possible to handle the systems in an easier way by carrying out the optimisation one at a time. This research is carried out for TOFAŞ automotive factory and it has two levels which are:

Level 1: The development of highly interactive tool path simulation CAD program.

Level 2: The optimization of machining variables of metal cutting.

This paper describes the proposed algorithm for level 2 which will be integrated into level 1. The algorithm presented here is intended to optimize multi-pass metal cutting case. A numerical example case study is given to show the applicability of the proposed method.

2. LITERATURE REVIEW

There is no standard mathematical formulation for the dynamic programming multistage solution algorithm. Each problem has its own formulations for the objective function and constraints on the decision levels. There has been a considerable number of researchers using various techniques to determine the optimum machining variables for metal cutting.(1-6) There is no one best solution technique that can be described as a universal one for metal cutting problem. Several techniques can be used but they must all cope with nonlinearities in the cutting equations and nonlinear constraints of machining. Some researchers used iterative techniques for the optimization of machining variables.(2,3) In these techniques, the initiation parameter of the solution procedure was estimated and the search was carried on using this parameter to satisfy the boundary limits of the constraints and to satisfy the machining requirements in order to determine the other parameters of the problem. These kinds of iterative procedures, which are intuitive, suffer problems as optimization techniques because the efficiency of convergence not guaranteed and it requires several trial attempts to reach the optimal solution. There is a considerable advantage in being able to transform a function. To convert the optimization problem to one with linear objective and constraints has advantages because one of the linear programming techniques can easily be used to solve the problem.(4,5) Because of most of the techniques have difficulties in transformation, they are not always preferred in practice. One of the widely used transformation method is the SUMT (sequential unconstrained minimization technique).(6) The effect of different starting values in the technique showed that it can lead to different results for machining variables, especially in milling. The major shortcoming of the approach is the determination of the penalty parameters.

Dynamic Programming is applied to the optimisation of multistage problems.(7-11) Even though each problem has its own alternative formulations the main principle is the same. The technique is first discussed by Bellman and it was called the principle of optimality. Another technique which is developed for different types of nonlinear problems is Geometric Programming which is implemented single-pass cutting in this research.(7)

Of the above methods, the best compatible technique for multi-pass metal cutting optimization is Dynamic Programming method since the multistage problems are well suited to the structure of program. Another advantage is due to division of the problem into N-sub problems and by optimizing each sub-problem to obtain the overall optimal solution. In this paper, dynamic programming approach is proposed to optimize multi-pass machining variables since it is suitable, satisfying most of the above mentioned points concerned with multistage structure of problem.

3. PROBLEM FORMULATION

The mathematical model of metal cutting cost in terms of the machining variables (speed, feed, depth of cut etc.) is as shown below:(12)

$$\text{Cost} = \sum C_i \quad i = 1, \ldots\ldots, n \tag{1}$$

where the cost components C_i can be expressed as:

$$C_i = C_i v^{a_{i1}} f^{a_{i1}} d^{a_{i3}} \qquad i = 1, \ldots\ldots, n \tag{2}$$

where

Ci=cost component coefficients,

v=machining speed,

f=machining feed, d=cutting depth of cut,

a_{i1}, a_{i2}, a_{i3}=machining variable exponents

and

$C_i > 0$

a_{ij} are arbitrary real numbers.

The objective function x is called a posynomial, which is a polynomial with positive term coefficients.

In practice the choice of variables for machining operations can vary considerably due to the many constraints that are applied such as maximum feed, speed, power or surface finish. The constraints can be expressed in polynomial form as shown below:

$$B_n = b_n v_n^{aml} f_n^{am2} d_n^{am3} \quad n = 1,.....,N \quad m = 1,.....,M \tag{3}$$

where
b_n= term coefficients of constraints,
M= number of terms in constraint,
N= total number of constraints
The most common form of expression is

$$\sum_{m=1}^{M} b_n v_n^{aml} f_n^{am2} d_n^{am3} \leq 1 \tag{4}$$

4. APPLICATION OF DYNAMIC PROGRAMMING TO MINIMUM COST ANALYSIS

The metal cutting cost function in terms of the machining variables (feed, speed, depth of cut etc.) can be expressed functionally by the polynomials shown below:

$$\text{Cost} = X(T_1 + T_2 + T_3 T_2/T) + y\, T_2/T \tag{5}$$

In the case of turning, the variables in Eqn. x are as follows:
x=operating cost of machining involves also the labor and overhead cost rates
T_1=non-productive time, min
T_2=machining time per part, min
T_3=tool changing time per part, min
T=tool life
y=tool cost of cutting edge
and the cutting time T_2 is given by:

$$T_2 = \pi DL/12vf \tag{6}$$

where
D=workpiece diameter,
L=length of cut,
v=cutting speed,
f=feed
The tool life equation is given by:

$$T = Kv^{-1/n} f^{-1/n}\, d^{-1/n2} \tag{7}$$

where
K=constant
n, n_1, , n_2 = exponents of machining variables of tool life, which depend on mate-

rial properties of tool-workpiece combination
Substituting Eqns. 6 and 7 into Eqn.5, the cost objective function per part is:

$$C=C_1 + C_2 v^{-1}f^{-1} + C_3 v^{-1/n}f^{-1/n1}d^{-1/n2} \quad (8)$$

where
$C_1=xT1$,
$C_2=x\pi DL/12$,
$C_3=\pi DL(xT_3+y)/12K$

The dynamic programming algorithm which is defined above section can be used to solve equations of the following form:

$$F_N(X_N)=\min_{D_N}(X_N,D_N) \quad n = 1 \quad (9)$$

and

$$F_N(X_N)=\min_{D_N}(R_N(X_N,D_N)+f_{n-1}(x_{n-1})) \quad n = 1$$

subject to

$$X_{n-1} = T_n (X_n ,D_n) \quad n=2,\ldots\ldots,N$$

where

X_n = state variables, D_n = decision variables

The first step in the solution procedure is determination of the optimum objective return value for the last stage, n=1 (the last stage is numbered 1, the second from last 2 etc. so that the numbering is in the opposite sense to that normally expected in mathematical series).

The objective return value is as shown below:

$$F_1^{opt} = f_1 (X_1^{opt},D_1^{opt}) \quad n = 1 \quad (10)$$

The objective return value F1 is a function of the input state variable X1 and the decision variable D1. The optimum decision policy D_1^{opt} must be determined to obtain the optimum input state variable X_1^{opt} and optimum return value F_1^{opt} for the last stage, n=1. The term X_1 is defined as the initial state which affects the other state outputs and inputs as shown by the transformation function below:

$$X_{n-1} = T_n (X_n ,D_n) \quad n=2,\ldots\ldots,N \quad (11)$$

The transformation in the case of a metal cutting process considering the finishing pass to be n=1 as shown:

$X_o = X_1 - D_1$,
$X_o = 0$ (for finishing pass) and
$X_1 = D_1$

The optimum decision policy D_1^{opt} of stage n=1 also affects the other stage decisions, see Egn. 10. The following solution steps are rather different from the first stage and backward (forward) sequential optimisation can be applied to the rest of the stages. For stage n=2 the equations are:

$$F_2^{opt} = \min_{D_2^{opt}} (R_2^{opt}(X_2^{opt}, D_2^{opt}) + F_1^{opt} \quad (12)$$

and similarly for the other equations which can all be optimized in a similar way as shown:

$$\begin{aligned} F_3^{opt} &= \min_{D_3^{opt}} (R_3^{opt}(X_3^{opt}, D_3^{opt}) + F_1^{opt} \\ &\cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \\ &\cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \\ F_N^{opt} &= \min_{D_N^{opt}} (R_N^{opt}(X_N^{opt}, D_N^{opt}) + F_{N-1}^{opt} \end{aligned} \quad (13)$$

Therefore the objective value, which is the minimum machining cost for metal cutting, F_N^{opt}, and the optimum decision policies D_N^{opt} for each stage, can be determined by the dynamic programming technique of multistage sequential decision making analysis. The optimisation procedure is summarized below:

1. Divide the problem into stages, n=1,.....,N
2. Define the states associated with each stage
3. Evaluate the recursive relationship, i.e. the return and transformation equations
4. Find the optimum value of the N-stage return in terms of optimal decision policies.

5. MULTI-PASS METAL CUTTING

In the multi-pass metal cutting case the depth of cut is not fixed. In this case the machining cost and constraint equations can be expressed as shown below:

$$C_i = C_i v_i^{a11} f_i^{a12} d_i^{a13} + b_i v_i^{a21} f_i^{a22} d_i^{23} \quad (14)$$

Subject to

$$C_j\, v_i^{aj1} f_i^{aj2} d_i^{aj3} \leq 1 \qquad j = 3,.....,n_i \qquad i = 1,.....,m$$

where
m= number of passes.

In solution algorithm of the multi-pass metal cutting case, multistage decision making analysis is used to assist with the determination of the optimum machining variables. The input to each pass consists of the remaining depth of cut after the previous pass. The outputs of each pass are the remaining depth of cut and the machining cost of the pass. The transformation function in this metal cutting case is depth of cut. At the nth pass it is expressed as follows:

$X_{n+1}= X_n-d_n$
where
X_{n+1}= the depth of cut before stage n+1
X_n= the depth of cut before stage n
d_n= the required depth of cut at stage n.

The metal cutting primal program for the N-multipass can be expressed as follows:

$$\text{Minimise } y_n(d_n,f_n,v_n) = \sum_{t=1}^{K_{on}} C_{ok}(d_m, f_n, v_n) \qquad n = 1,\ldots\ldots,N$$

subject to

$$X_{n+1} = X_n - d_n \tag{15}$$

$$\sum_{k=1}^{K_{mn}} C^n_{mk}(d_n, f_n, v_n) \le 1 \qquad n = 1,\ldots\ldots,N \qquad m = 1,\ldots\ldots,M$$

The backward (forward) optimisation concepts of dynamic programming are used to obtain the minimum metal cutting cost as shown in Fig. 1. The backward analysis solves the problem with using the end of the whole problem as the first stage and then proceeds to the beginning of the problem which is the final stage.

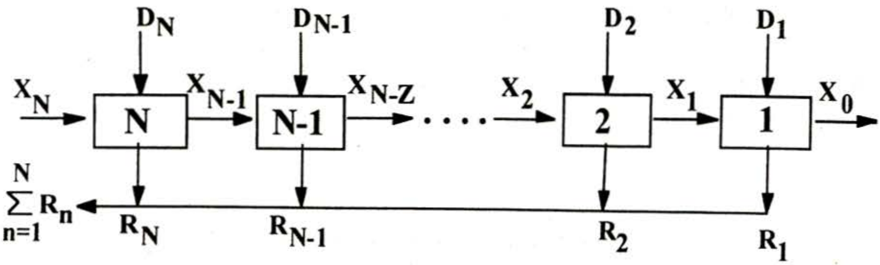

Figure 1. Structure of multi-stage decision system

The forward analysis is the other way around where the beginning of the problem is the first stage. The backward analysis is used in this research since the depth of cut at the end of machining can be selected as the amount required to complete the whole depth of cut of the machining as follows:

$$X_n = X_T - \sum_{n=0}^{N-1} X_n \qquad (16)$$

where

X_N=the final cut required, X_T=total depth of cut

In the case of multipass cutting, the division of the problem into small problems is inevitable to obtain an efficient computational approach to the solution. It makes it easier to handle a series of small problems rather than a large one. The stages are related to each other using the transfer functions of the depth of cut as shown in Eqn. 16. The constraints of Eqn. 15 are:

- the maximum cutting power available
- the machine-tool speed restrictions
- the machine-tool feed restrictions
- the surface finish requirements

The above constraints are the ones most generally used, however further restrictions on the machining can be added to the program if required without affecting the solution algorithm. The multipass cutting optimisation using geometric programming technique via multi-stage decision making is shown in Fig.2.

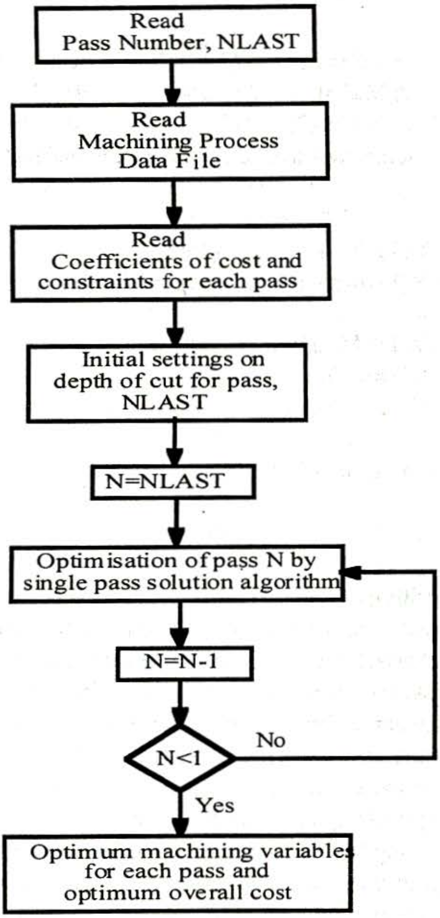

Figure 2. Multistage decision making technique of dynamic programming

6. MULTI-PASS METAL CUTTING EXAMPLE

In the multi pass situation, Eqn. 14 can be used for the sample problem of a work-piece of length L=203 mm, diameter D=152 mm which is constrained for the double pass case as follows (13):

$f \leq 2.54$ (feed mm / rev)
$0.015023v^{-1.52}f^{1.004}d^{0.25} \leq 25.4$ (surface finish µm)
$0.0499v^{0.95}f^{0.78}d^{0.75} \leq 20$ (pover h. p.)

Final pass costraints are the same as first pass except the surface finish constraint coefficient which is taken as 5.08 instead of 25.4. The exponents of the variables are:

A(1,1) = -1	A(1,2)= -1	A(2,1)= 3	A(2,2)= 0.16	A(2,3)= 1.14
A(3,1) = 0	A(3,2)=1	A(3,3)= 0	A(4,1)= -1.52	A(4,1)= -1.52
A(4,2) = 1.004	A(4,3)=0.25	A(5,1)= 0.95	A(5,2)= 0.78	A(5,3)= 0.75

The optimum machining variable results of the problem are computed as follows:

First pass:	V(1)=133 m/min	(speed)
	V(2)=2.54	(feed)
	V(3)=4.3 mm	(depth of cut)
Final pass:	V(1)=45 m/min	(speed)
	V(2)=2.54	(feed)
	V(3)=0.2 mm	(depth of cut)

The total depth of cut is 4.5 mm.

7. CONCLUSION

In solution algorithm of the multi-pass metal cutting case, multistage decision making analysis is used to assist with the determination of the optimum machining variables. The input to each pass consists of the remaining depth of cut after the previous pass. The outputs of each pass are the remaining depth of cut and the machining cost of the pass. Dynamic programming approach is successfully implemented into the multi-pass cutting case since the multistage problems are well suited to the structure of program. Another advantage is due to division of the problem into N-sub problems and by optimizing each sub-problem to obtain the overall optimal solution. The problem of solving for the cutting variables was converged to the optimum using the dynamic programming and geometric programming technique so that the optimum operations are determined. Dynamic programming solved the optimization problem with little difficulty. The analysis described in this paper is derived primarily for the turning process. The technique can also be applied to a wide range of processes: turning, milling, drilling, tapping, etc.

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