

## AN APPLICATION OF THE AIRCRAFT IDENTIFICATION

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### ABSTRACT

*In this study, a solution of an aircraft identification problem is presented. Firstly, the essential equations of longitudinal motion are introduced and simplified. Subsequently, this motion is simulated with a suitable model as a real aircraft. Finally, the model system has been identified in real time for different environmental condition.*

### ÖZET

#### Bir Uzay Aracı Tanıma Uygulaması

*Bu çalışmada bir uzay aracı tanıma probleminin bir çözümü sunulmaktadır. Önce uzay aracının boyuna hareketinin gerekli eşitlikleri verilip sadeleştirilmektedir. Buna bağlı olarak bu hareketin gerçek bir uzay aracına benzer bir modelle simülasyonu yapılmaktadır. Son olarak da bu model farklı çevre koşullarında tanınmaktadır.*

### INTRODUCTION

The longitudinal dynamic motion of an aircraft can be divided into modes; the short time mode and phugoid mode<sup>1,2,3</sup>. The short time mode is important for

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the control because it may not be controlled by the pilot without autostabilisation. However the phugoid mode can be controlled by the pilot, even if it is divergent or unstable. To design an autostabilisation system, it is necessary to know the parameters of the longitudinal dynamic motion. The aircraft dynamic can change very rapidly and needs a more accurate model to control it. It is also desirable to avoid the use of the special inputs for the identification. A continuous model is assumed a more accurate model when it is compared with discrete model<sup>4</sup>. In this paper, the parameters of a remote controlled aircraft RAVEN 201 that belongs the Flight Refuelling Comp. in U.K. are identified in real time. The block diagram of the realization is given by Fig. 1.

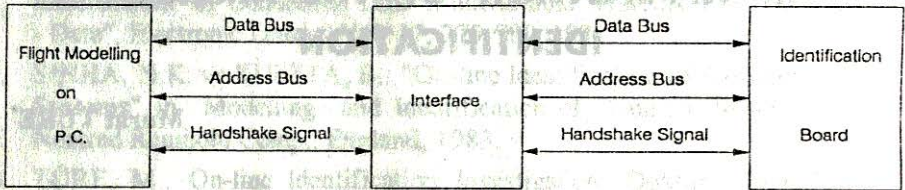


Fig. 1. Block diagram of the identification on real-time simulation

## FLIGHT MODELLING

The aircraft is assumed to be in a trimmed position with a constant speed. Therefore the angular displacements of elevator, ailerons and rudder are zero. It has been shown that the longitudinal motion only depends on the elevator<sup>1</sup>. On the other hand, the lateral motion is independent from the elevator movements. Thus the flight modelling during landing can be represented by the equation of the longitudinal motion. The equation of the longitudinal motion can be written as<sup>1</sup>:

$$\hat{D} + x_u) \hat{u} + (x_w + x_w) \hat{w} + x_q \hat{q} + \hat{g}_1 \theta + x_\eta \eta' - 0 \quad (1)$$

$$z_u \hat{u} + [(1 + z_w) \hat{D} + z_w] \hat{w} + (z_q - 1) \hat{q} + \hat{g}_2 \theta + z_\eta \eta' - 0 \quad (2)$$

$$m_u \hat{u} + (m_w + m_w) \hat{w} + (\hat{D} + m_q) \hat{q} + m_\eta \eta' - 0 \quad (3)$$

$\hat{u}$  is defined by  $\frac{u}{V_e}$  where  $u$  represents the difference between the disturbed flight velocity and the steady flight velocity along  $0_x$  and  $V_e$  is the aircraft speed in

steady flight. According the initial condition assumption  $\hat{u}$  can be omitted on the equations (1) to (2)<sup>1</sup>. The term  $\hat{g}_2\theta$  can also be neglected in comparison other term. In this case, Eqs (2) and (3) are rewritten as:

$$[(1 + z_w)\hat{D} + z_w]\hat{w} + (z_q - 1)\hat{q} + z_\eta\eta' = 0 \quad (4)$$

$$(m_w \hat{D} + m_w) \hat{w} + (\hat{D} + m_q) \hat{q} + m_\eta\eta' = 0 \quad (5)$$

where  $D$  is the differential operator  $(\hat{D} = \frac{d}{dt})$ ,  $\hat{w}$  is the attack angle, which is

equal to  $\frac{w}{V_e}$ ,  $q$  is the aircraft angular velocity in pitch,  $\eta'$  is the increment in

elevator angle from trimmed position. Other coefficients are aerodynamic derivatives of the aircraft that can change with flight condition. Therefore the aircraft dynamics change because of the change in coefficients. During the landing time, the parameters of the dynamic can be assumed to be constant because the landing time is small in comparison with the parameter varying time. The considered aircraft model is given as Eqs (4) and (5):

$$(ar_{11} \frac{d}{dt} + ar_{12}) \alpha - 0.9892 ar_{11}q - br_1\eta' \quad (6)$$

$$(ar_{21} \frac{d}{dt} + ar_{22}) \alpha + (ar_{23} \frac{d}{dt} + ar_{24}) q - br_2\eta' \quad (7)$$

where  $\alpha$  is equal to  $w$ . This system has been identified by direct continuous method using the Newton-Raphson algorithm. For this purpose, Eqs (6) and (7) were written the state space form as:

$$\frac{d}{dt} \begin{bmatrix} \alpha \\ q \end{bmatrix} = \begin{bmatrix} a_{11} & 0.9892 \\ a_{21} & a_{22} \end{bmatrix} \begin{bmatrix} \alpha \\ q \end{bmatrix} + \begin{bmatrix} b_1 \\ b_2 \end{bmatrix} \eta' \quad (8)$$

where  $\eta'$  includes a first order lag which is 0.1 sec. This model was simulated by using the fourth order Runge-Kutta integration method on a personal computer.

The step time is equal 40 msec which is determined according to the telemetry of the aircraft. The desired performance coefficients values which is given by

$$\frac{d}{dt} \begin{bmatrix} \alpha \\ q \end{bmatrix} = \begin{bmatrix} -0.0142 & 0.9892 \\ -1.244 & -1.924 \end{bmatrix} \begin{bmatrix} \alpha \\ q \end{bmatrix} + \begin{bmatrix} 1.17e-03 \\ -0.434 \end{bmatrix} \eta' \quad (9)$$

### THE DESIGN OF THE IDENTIFICATION ROUTINE

The Newton-Raphson method was used to identify the parameters. This method is given in the reference<sup>3</sup>, it is briefly given here again. The identified system has been considered as:

$$\frac{d}{dt} x(t) = Ax(t) + Bu(t) \quad (10)$$

$$\dot{x}(t) = f(x, u, a, b, t)$$

where A and B are the parameter matrices, whose element can be time varying, x is state variable matrix and u is control input matrix. The state observable values is defined by

$$s_i(t) = p_i(t) + \sum_{k=1}^n c_k h_{ki} \quad (11)$$

where s is the vector of the observable values of the process state variable, p is the vector of the state variables of the estimation model,  $h_k$  is the vector of the sum of the homogeneous variables and n is the number of the unknown parameters.  $c_k$  will be used to modify initial estimation of unknown parameters. Firstly, an estimation model and homogeneous models are described respectively. The estimation model was chosen to have exactly the same structure as the flight model but all coefficients were assumed to be equal to 1 as:

$$\frac{d}{dt} p(t) = \hat{A}p(t) + \hat{B}\eta' \quad (12)$$

$$\frac{d}{dt} \begin{bmatrix} p_1 \\ p_2 \end{bmatrix} = \begin{bmatrix} 1 & 0.9892 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} p_1 \\ p_2 \end{bmatrix} + \begin{bmatrix} 1 \\ 1 \end{bmatrix} \eta' \quad (13)$$

$\hat{A} \qquad \hat{B}$

The homogeneous systems are defined for each unknown parameter which are given as:

$$\frac{d}{dt} h_k(t) - \hat{A}h_k(t) + \frac{\partial f}{\partial a_k} \quad (14)$$

When  $\frac{\partial f}{\partial a_k}$  are calculated, the homogeneous systems are found as:

$$\frac{d}{dt} \begin{bmatrix} h_{11} \\ h_{12} \end{bmatrix} = \begin{bmatrix} 1 & 0.9892 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} h_{11} \\ h_{12} \end{bmatrix} + \begin{bmatrix} s_1 \\ 0 \end{bmatrix} \quad (15)$$

$$\frac{d}{dt} \begin{bmatrix} h_{21} \\ h_{22} \end{bmatrix} = \begin{bmatrix} 1 & 0.9892 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} h_{21} \\ h_{22} \end{bmatrix} + \begin{bmatrix} 0 \\ s_1 \end{bmatrix} \quad (16)$$

$$\frac{d}{dt} \begin{bmatrix} h_{31} \\ h_{32} \end{bmatrix} = \begin{bmatrix} 1 & 0.9892 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} h_{31} \\ h_{32} \end{bmatrix} + \begin{bmatrix} 0 \\ s_2 \end{bmatrix} \quad (17)$$

$$\frac{d}{dt} \begin{bmatrix} h_{41} \\ h_{42} \end{bmatrix} = \begin{bmatrix} 1 & 0.9892 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} h_{41} \\ h_{42} \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \end{bmatrix} \eta' \quad (18)$$

$$\frac{d}{dt} \begin{bmatrix} h_{51} \\ h_{52} \end{bmatrix} = \begin{bmatrix} 1 & 0.9892 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} h_{51} \\ h_{52} \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} \eta' \quad (19)$$

The control input vectors of  $h_4(t)$  and  $h_5(t)$  were found according the derivation of  $\frac{\partial f}{\partial b_i}$ . Eq (11) must be written in matrix form to solve the elements

of the  $c$  matrix. In this problem, the number of variables is 2, the number of the unknown parameters ( $c_k$ ) is 5, therefore we need to add enough observations in this case, Eq (11) can be rewritten in the matrix form as:

$$\begin{aligned}
 & \begin{bmatrix} s_1(t) \\ s_2(t) \\ s_1(t+1) \\ s_2(t+1) \\ s_2(t+2) \end{bmatrix} = \begin{bmatrix} p_1(t) \\ p_2(t) \\ p_1(t+1) \\ p_2(t+1) \\ p_2(t+2) \end{bmatrix} + \\
 & + \begin{bmatrix} h_{11}(t) & h_{21}(t) & h_{31}(t) & h_{41}(t) & h_{51}(t) \\ h_{21}(t) & h_{22}(t) & h_{32}(t) & h_{42}(t) & h_{52}(t) \\ h_{11}(t+1) & h_{21}(t+1) & h_{31}(t+1) & h_{41}(t+1) & h_{51}(t+1) \\ h_{21}(t+1) & h_{22}(t+1) & h_{32}(t+1) & h_{42}(t+1) & h_{52}(t+1) \\ h_{21}(t+2) & h_{22}(t+2) & h_{32}(t+2) & h_{42}(t+2) & h_{52}(t+2) \end{bmatrix} \begin{bmatrix} c_1 \\ c_2 \\ c_3 \\ c_4 \\ c_5 \end{bmatrix} \quad (20)
 \end{aligned}$$

The  $c$  matrix can be solved from Eq (20). The  $c$  matrix is corresponded with the unknown parameter as:

$$\begin{aligned}
 \hat{a}_{11}(k+1) - c_1 + \hat{a}_{11}(k), \quad \hat{a}_{21}(k+1) - c_2 + \hat{a}_{21}(k), \\
 \hat{a}_{22}(k+1) - c_3 + \hat{a}_{22}(k) \\
 \hat{b}_1(k+1) - c_4 + \hat{b}_1(k), \quad \hat{b}_2(k+1) - c_5 + \hat{b}_2(k) \quad (21)
 \end{aligned}$$

All of the identification operations were done with TMS320C30 Digital Signal Processor. The lag on the elevator response was also considered in the identification operation. The essential software and hardware can be seen in reference<sup>3</sup>.

## DESIGN OF THE INTERFACE

The identification board clock frequency was different from the clock frequency of the personal computer which was used for the flight modelling. Therefore a synchro - communication between PC and the identification board was not possible. Hence an interface was inserted between them in order to realize the a synchro - communication. This interface provided parallel communication.

## THE IDENTIFICATION RESULTS

The system representation is rewritten for the measurement with noise as;

$$\frac{d}{dt} x(t) - Ax(t) + Bu(t) \quad (22)$$

$$s(t) - Cx(t) + \zeta(t)$$

Where  $\zeta(t)$  is zero-mean random function, representing the noise. Eq (23) are used for the identification with noise.

$$[c]_{k,1} = \left( \sum_{i=1}^N [h(t_i)]^T [h(t_i)] \right)^{-1} \cdot \sum_{i=1}^N [h(t_i)]^T \{ [s(t_i)] - [x(t_i)] \} \quad (23)$$

It can be shown that the requirements of step numbers increase due to the amplitude of noise. The identifications were performance for different amplitudes of the random function. The identification results and control input are given by Fig. 2 to Fig. 6. For  $N = 30$ , the identification board was used to collect the data during the first 30 steps. Therefore the first estimation results are available after 30 samples time. The identification board can calculate the parameters with two iterations in one sample time. Hence, each step identification can be done recursively after 30 steps. This must be considered when choosing the initial estimation, because the particular system outputs may cause the processor to overflow in 30 samples time. However the initial estimation values were taken to be the same as the first estimation value when the control input was changed

completely. The system outputs, from which we tried to identify the parameters in this project, are not measured with a very good resolution. The maximum measurement error is around 0.007 which can be represented as a random signal amplitude. When  $N = 30$ , the identification is possible with a small error which is less than 20 percent. When the noise amplitude is bigger than 0.01, estimation error increases more than 50 percent. Therefore the number of  $N$  must be increased. The identification is possible until the noise amplitude is 0.1 with  $N = 60$ . The noise amplitude 0.1 means that the noise is bigger than 30 percent of the signal, which can be seen in Fig. 6.

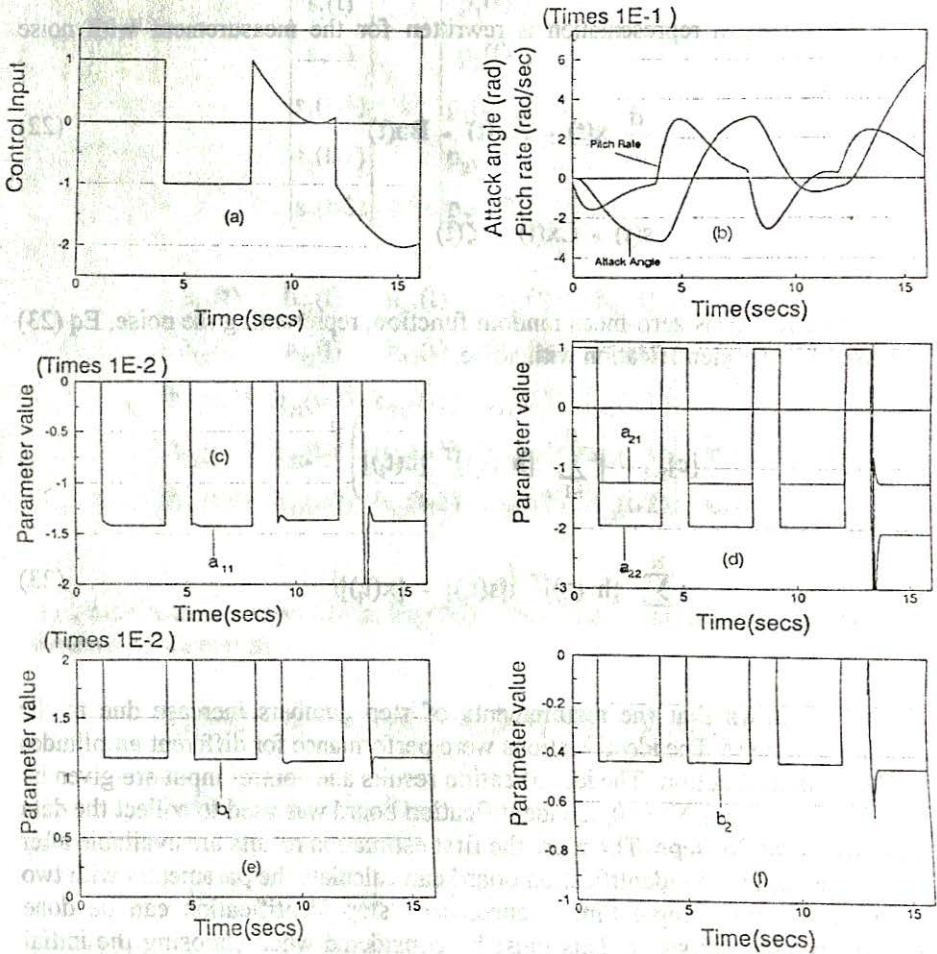


Fig. 2. Figures (a) to (f) show that are the control input, the output observations and the identified parameters for noise-free system, respectively



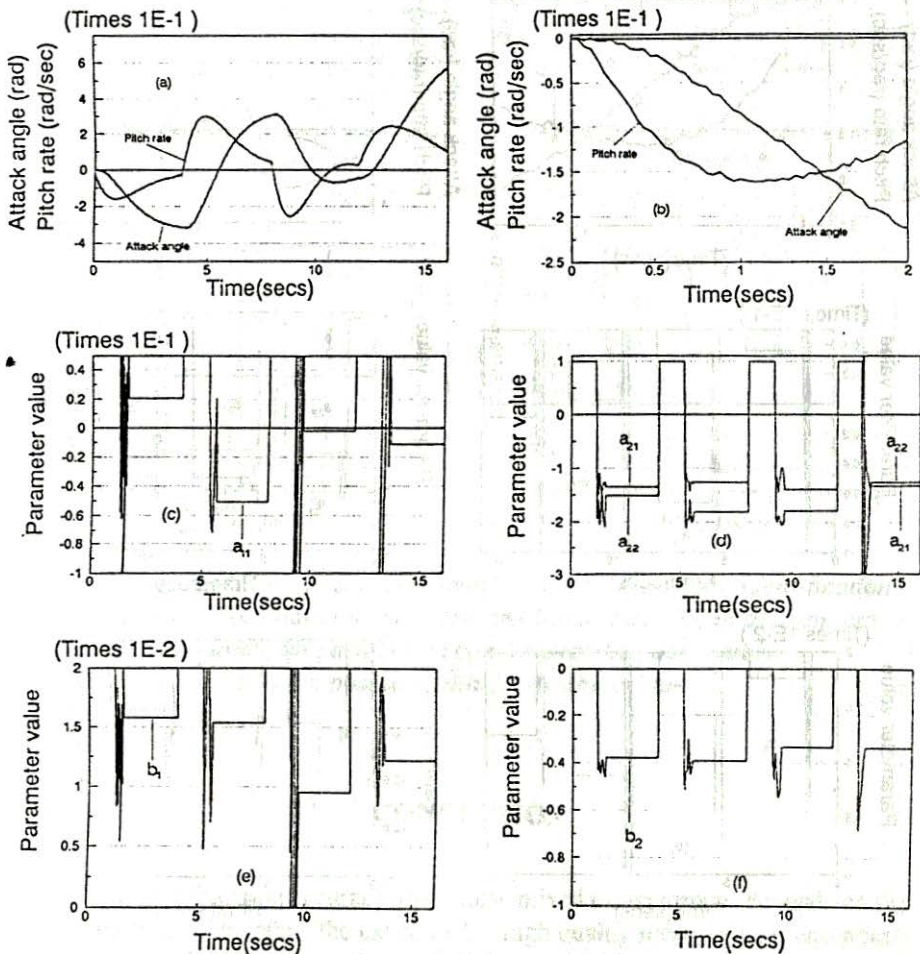


Fig. 3. Figure (a) shows that the output observations for the noise amplitude 0.007. Figure (b) is enlarging view of the figure (a). Figure (c) to (f) are the identified parameters with 30 step block data. Error increases for all parameters.

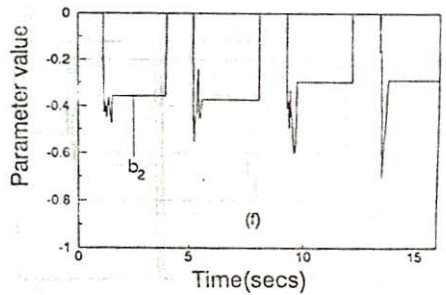
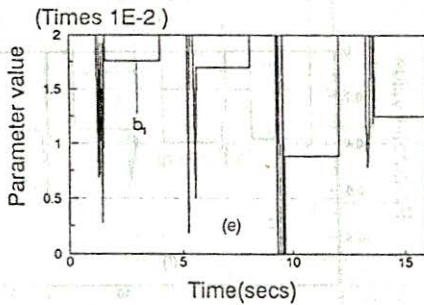
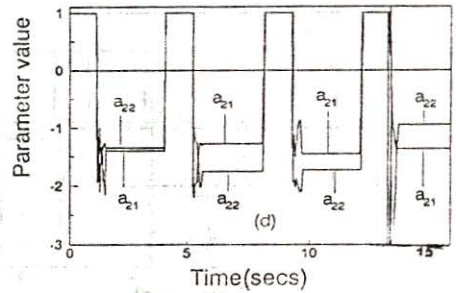
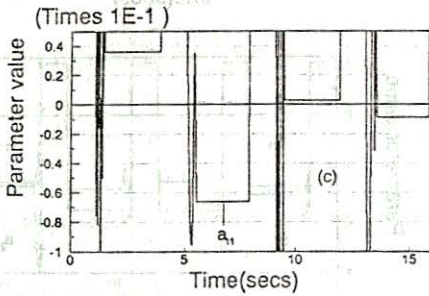
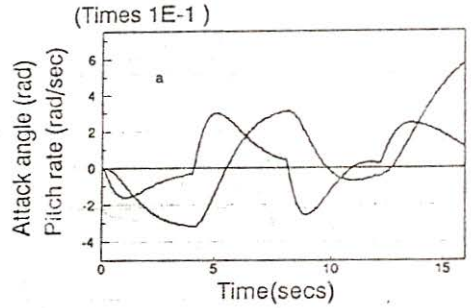
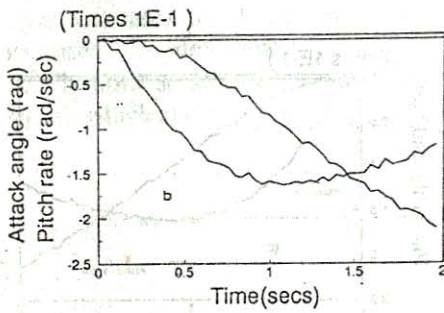
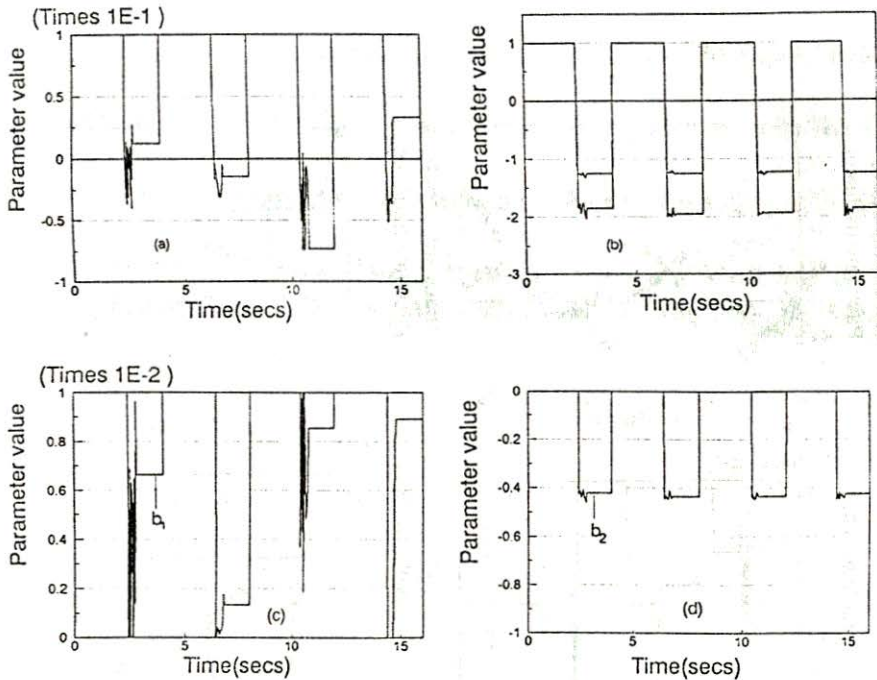


Fig. 4. Figure (a) shows that the output observations for the noise amplitude 0.001. Figure (b) is enlarging view of the figure (a). Figure (c) to (f) are the identified parameters with 30 step block data. Error increases up to 50% for system parameters but it is still less than 25% for the control parameters.



*Fig. 5. Figures (a) to (d) are the identification results of the identification with 60 step block data for the noise amplitude 0.01. It can be seen that the dominant parameters can be identified accurately. It is not possible with 30 step block data.*

## CONCLUSION

The identification accuracy with noise mixed measurement depends on the noise amplitude. Therefore, the usage of the high quality measurement equipment will increase the accuracy and also will decrease identification time. The main result of this study is that the identification board can identify the system within the transient response time without using a special input.

On the other hand, only the  $a_{11}$  value can not be identified from noise mixed observations. Because its effect on the observation value is smaller than the noise every time.

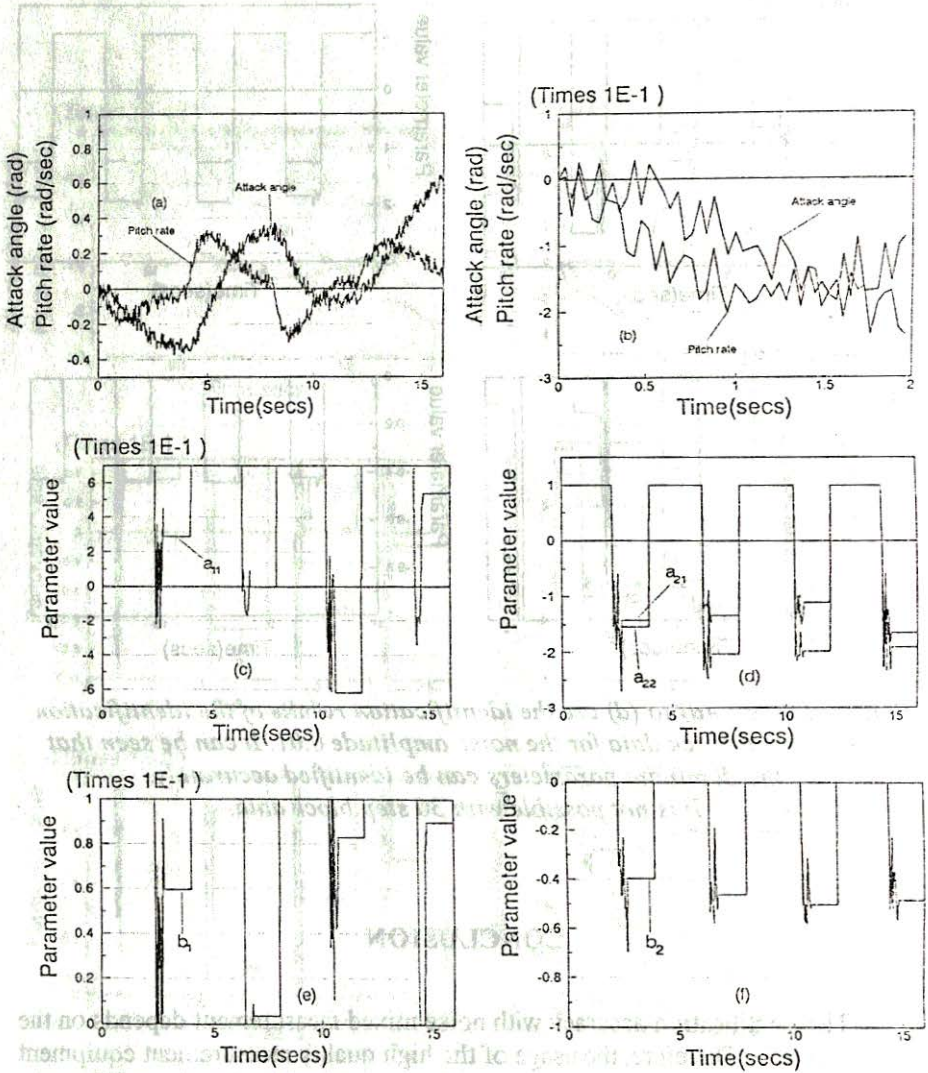


Fig. 6. Figure (a) shows that the output observations for the noise amplitude 0.1. Figure (b) is enlarging view of the figure (a). Figure (c) to (f) are the identified parameters with 60 step block data.

Error is still less than 20% for the dominant parameters. Figure (f) indicates that the error of  $b_2$  is still less than 10%.

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