Uludağ Üniversitesi
Eğitim Fakülteleri Dergisi
Cilt: IV, Sayı: 2, 1989

DEFORMATION OF CYLINDER SURFACE PRESERVED ON LENGTH

Aydın OKÇU*

SUMMARY

In this study, the cylinder was studied according to its deformation of its surface with respect to length due to an external source. In the deformation, the length of main lines of cylinder was held constant; and horizontal and vertical positions of second order curves, especially Euclidian helix were examined. We can summarize the conclusion as follows:

1. After the deformation, amount of transformation vs. α deformation angle was computed.

2. After the confident to the distances, the locus of the P_i points is a circle with diameter $\frac{r.p}{h}$, and horizontal projection C_1, touches the base circle at the begininng point of the motion.

3. Parameter circle of the motion found by the description of the Euclidian helix, is $p = d. \text{Cos}\ \theta$, while it is the loop of the orbit involute which touches the base circle at two points a horizontal projection.

4. The motions of description and involution are reversible.

5. Locus of points at height P on the main lines of the cylinder is a curve on the hyperboloid surface and its horizontal projection is concentric with the base circle of the cylinder.

ÖZET

Silindir Yüzeyinin Uzunluğa Sadık Deformasyonu

Bu çalışmada silindir yüzeyinin bir dış kuvvet etkisiyle uzunluğa sadık bir deformasyonu ele alınarak incelenmiştir. Deformasyonda silindir ana doğrularının uzunluğu sabit tutulmuş ve silindir yüzeyi üzerindeki ikinci dereceden eğrilerin özellikle de Öklit helisinin yatay ve düşey konumu incelenmiştir. Çalışmada elde edilen sonuçları şu şekilde özetliyebiliriz.

1. Deformasyon sonunda α deformasyon açısına bağlı olan ötelenme miktarı hesaplanmıştır.

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* *Doç. Dr.; U.Ü. Necatibey Eğitim Fakültesi Matematik Anabilim Dalı, Balıkesir.*

2. Uzunluğa sadık tasvir sonunda P_i noktalarının geometrik yerleri $\frac{r.p}{h}$ çaplı bir çemberdir ve yatay izdüşümü de C_1, taban dairesine hareketin başlangıç noktasından değmektedir.

3. Öklit helisinin tasviri ile elde edilen hareketin parametre dairesi $p = d \, Cos\theta$ olup yatay izdüşümde taban dairesini iki noktada kesen yörünge evoluventinin ilmiğidir.

4. Tasvir ile evoluvent hareketi tersinirdir.

5. Silindir ana doğruları üzerinde bulunan P yükseklikteki noktaların geometrik yeri, hiperboloid yüzeyi üzerinde ki bir eğridir ve yatay iz düşümü ise silindirin taban dairesi ile consantriktir.

1. INTRODUCTION:

In the most general form, the deformations of the surfaces can be defined as the change of the tensile forces acting on a surface by means of an external effect and the change of the surface in shape. At the deformation, the surface which changes in shape can be a liquid, solid or any elastic material.

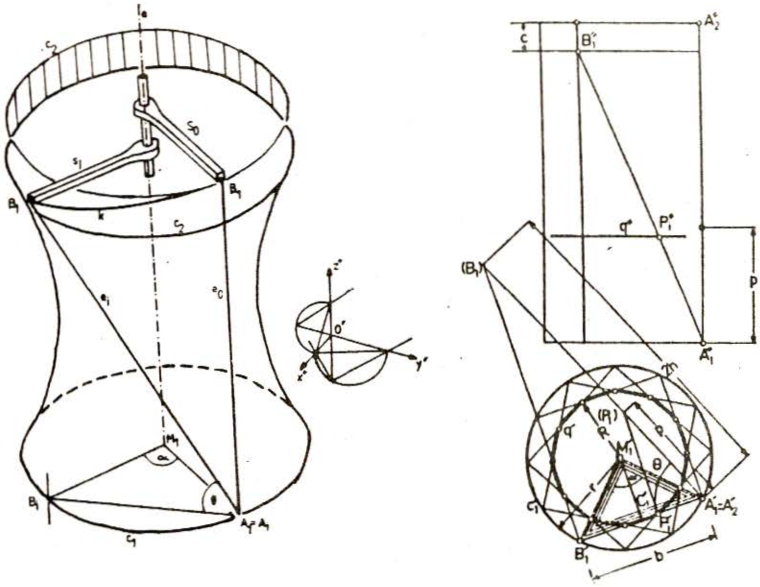
If we make a commentary geometrically, it is so clear that there will be a relation between the points of the main surface and the deformed surface. We can say that, after any deformation, some of geometrical features are kept, beside this, some of geometrical features are deformed. For example, if the Euclidean distance between the two point on the main surface does not change on the deformation surface, this distance i.e. the length is called as a confident deformation. When a deformation is concerned, the distances will be constant and no diversion will be expected on the angles. For example, if the angle between the points C and D from a referance point T is B, then it is expected that the same B angle will be between the points C' and D' from another reference point T'.

In this study, the status of the curves which are confident to the distances on a deformed surface as a result of the deformation of a second degree surface have been discussed. Firstly, let us emphasize the mechanical effect which is used at deformation.

Let us consider any intersectional area of a cylinder having a vertical axis which intersects with a vertical plane.

Let the main liner of the longitudinal area of the cylinder which we obtained have a length of 2h and their bottom and top circumferences be C_1 and C_2. Now the thing to do is to search the possibilities of projections of any surface on to another surface, by varying their slopes while keeping constant the points of the main lines of the cylinder with a length of 2h on the C_1 (See fig 1(a)).

Because, one of the points of the main line having a length of 2h is kept constant on the circumference C_1, the resultant displacement changes the slopes of those lines and they draw the surface of a sphere with a center C_1. If the slopes are changed ensuring the point on the main lines at on equal distance to the axle then, the motion of the B_i points is provided on the surface of the a cylinder. This motion is provided by rotating the main lines of the cylinder around their own axle with on angle of α. This axial rotation provides an approach of C_2 to C_1 in relation with the angle α.



(a) (b)

Fig 1

However, the angle α should be in the range $\theta \leqslant \alpha \leqslant 180$ the main line and the axis conform an oblique lines pair. In this way as result of the motion of the main line, a hiperboloidal surface is occured. If there is an angle of θ between the main lines of the hiperboloid and the horizontal, thus we can declare the following theorem.

Theorem 1:

The deformation surface of cylinder is a rotational hiperboloid surface. The family of main lines of the cylinder are formed of the main lines of hiperboloid. The family of the main lines of the hiperboloidal surface make an angle θ with the horizontal axis according to the deformation angle.

$$\mathrm{Cos}\theta = \frac{r\ \mathrm{Sin}\frac{\alpha}{2}}{h} \qquad (1)$$

As shown in (fig. 1 (a)) the end of main lines B_i of the cylinder is connected with a crank S_i the distance to its axis does not change; thus, the B_i end of the crank hand draws a space curve (h) in a distance r from the axis. This space curve is the trajectory of the points B_i as a result of axial translation of S_i and also rotation of them around the axle of the cylinder. Thus this is a curve such that, it has been generated as a result of a helical motion. If there is a parallel displacement θ slope angle having an α. We can express that,

$$C = 2h - 2h\ .\ \mathrm{Sin}\theta \qquad (2)$$

And we also express it as the rotation angle so;

$$C = 2h - 2h \sqrt{1 - \frac{\mathrm{Sin}^2 \frac{\alpha}{2}}{K^2}}$$

$$C = 2h \left(1 - \frac{1}{K} \sqrt{K^2 - \mathrm{Sin}^2 \frac{\alpha}{2}}\right) \qquad (3)$$

As considering the deformation angle in the range of $0 \leqslant \alpha \leqslant 180$, if we express the displacements of C_2 and C_1 circles with the equation (3), the following theorem will appear:

Theorem 2:

While keeping the base of a cylinder constant, the upper base of the same cylinder will be deformed angle α, the value of deformation is given by equation (3).

2. POINT TRAJECTORIES:

It is possible to define the point trajectories in some different ways. The transformation parameters are α and θ, Because these two transformation angles are related with each other with the equation (1), different positions of the transformation angle can be considered. For example, by setting the transformation angle to any fixed value, the situations in which the sampled points exist in a main line orthogonally and they make some certain motion can be considered.

a) Let us consider the main line passing through the point A_1' as the angle α is constant. Let the angle $\alpha = 90$ (fig. 1 (b)). The point P_1 which is on the hypotenuse divides the line in a ratio of p/2h. When the horizontal projection is concerned due the point P_1' divides the chord of $A'_1B'_1$ arc in the same ratio, these points are situated at equal distances from the centevs of the circle.

Thus; the equation for the distance R between the P_i points and the center M'_1 will be as follows:

$$R^2 = r^2 \,.\, \mathrm{Sin}^2 \frac{\alpha}{2} \quad \left(1 - \frac{p}{h}\right)^2 \quad + r^2 \,\mathrm{Cos}^2 \frac{\alpha}{2} \qquad (5)$$

Theorem 3:

The circle which is a locus of the points at a height of P over the main lines of cylinder is transformed into a circle on a hiperboloidal surface after a deformation. With a horizontal projection, this circle is coaxial with the bottom circle C'_1 and its radius is defined by the equation (5).

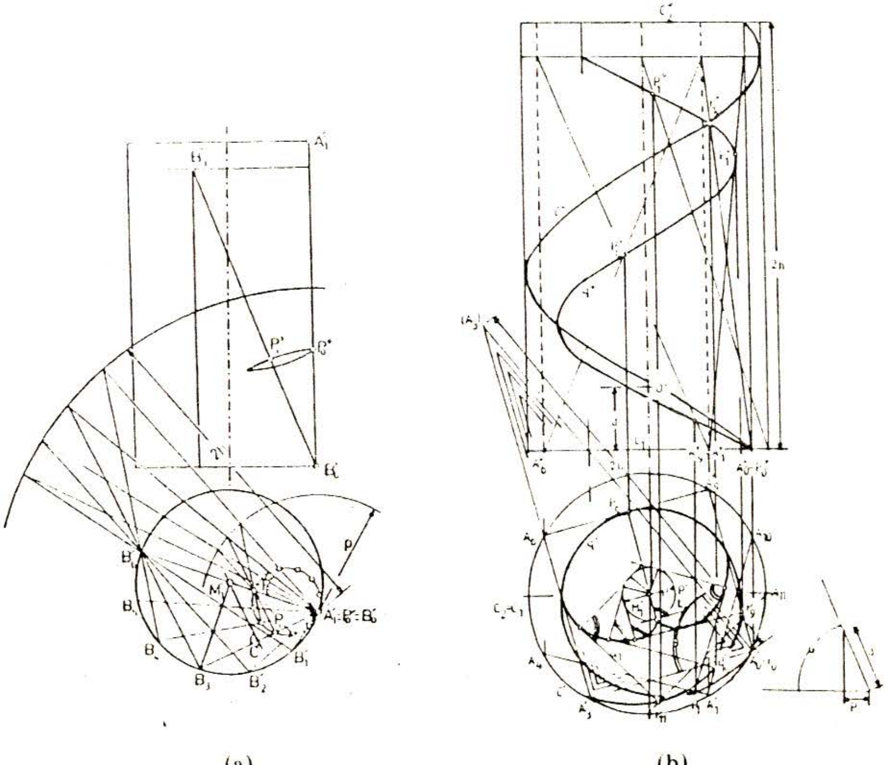
b) Then, assuming $\theta < \alpha < 180^\circ$ the points P_i on the main line of a cylinder divides the main lines with a ratio of p/2h. If we assume the angle $\alpha = 0$, then $A'_1 = P'_0$, and if $\alpha = 180^\circ$ then the point divides the radius in a ratio of p/2h. Due to the other main lines passing through the point P'_0 again, they divide the chords of the circles with the ratio p/2h. So the locus of these points which are taken with the ratio of p/2h on the chords, will be a semi-circle having a diameter of $P'_0P'_6$. By using the symmetrical characteristic of the deformation, the other half of the circle can easily be obtained.

Theorem 4:

When a rotational cylinder is projected confidently in length the locus of the points which are on the cylinder is a circle having a diameter of $\frac{r \cdot p}{h}$ and touching the lower base of the cylinder at the very moment of the motion (fig. 2 (a)).

a) Let us consider the surface deformation of any point P on the surface of a cylinder and its motion at that surface. Thus by doing that the deformation of a point at the surface of the cylinder will be revised (fig. 2 (b)). For example, as a result of the deformation of a C Euclidean helix on a cylindrical surface, a curve q is obtained on the hyperboloidal surface. As seen in figure, there is a relationship I between the parameter d of the helix and the deformation angle θ:

$$p = d \operatorname{Cos} \theta$$



(a) (b)

Fig 2

Assuming $\alpha = 90^\circ$, the point A_i which moves on the main lines of a cylinder, transforms into chord of an arc which it is seen from the center with an angle of 90° under the horizontal projection proportional to the angle θ. In the figure, it is seen from the triangle of $A'_0 = P'_0$ and $A'_3 = (A_3)$. If the point P_i arises to a height of 2.h then it scans the chord $A'_0 A'_3$. This motion with a constant velocity will provide the chord $A'_0 A'_3$ to be cut into two equal parts.

As it is described above, the heights of the points will be A'_3 (A_3) i.e just the modified length of 2h. Therefore, at every transformation of the main line of a cylinder, the points on the cylinder which belong to the Euclidean helix are projected on to the hyperboloid.

The curve which is the locus of these points is the locus of the points P_i connected to an involutional motion asumming the given length of p at equation (6) as the center of parameter. This connection is given by the involutional motion of the circle p. Thus, at every transformation of the main line of the cylinder, the points on the cylinder which belong the Euclidean helix are projected on to the hyperboloid.

The curve which is the locus of these points is the trajectory of the points P_i connected to an involutional motion assuming the given length of p at the equation (6) as the center of parameter. This connection is given by the involutional motion of the circle p. Thus, a new definition is obtained for the helical involutional motion. This is a new and third definition and is different from the H. Horninger's[1] and A. Günhan's[2] definitions on the involute helix.

Let us declare the relevant theorem:

Theorem 5:

A curve which as obtained from the deformation of the Euclidean helix is in relation with the motion of the $p = d \cos \theta$ parameter circle and is also a mode of the trajectory involute intercepting the lower base circle in two parts at the horizontal projection.

This property has been shown in Fig 5 axonometrically. That is, this property has been shown by shadowing the front part of the surface of the cylinder and the interval points of the Euclidean helix which is drawn by the points A_i on the surface of the cylinder. The involute helix having the trajectories of P_i points at the interval, has been drawn with a continuous curve on the main lines of the hyperboloidal deformation surface with a discreete curve $P_0, P_1, ..., P_{12}$. at the rear side.

Besides, the evoluvent ρ'_1 of the lower base P of the cylinder which is defined as a cylinder with a center M_1 inside and axle Z at the horizontal projection, has been shown with dotted line. The q being the space curve of this horizontal projection has been tied as the locus of ρ_i points.

3. RECIPROCITY OF THE TRANSFORMATION CURVE

As seen in fig 5, both the q evolvent-helix andthe one obtained by the deformation are two trajectorial curves attached to each other. So, the helical involutional motion of the circle q corresponds to P the involute helix, besides, the Archimedean Spiral which is on the horizontal plane drawn by P_i points is corresponded to

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1 H. Horninger; Uber eine Evolventen schraubung (Zylindrishe Schrotung einer Ebene); Revue de la Faculte' des Sciences de I' Universite d' İstanbul Serie A Tome XVI Fas. (1951).

2 A.V. Günhan; Helisel Evolvent hareketlerinin yörüngeleri hakkında Kurtuluş Matbaası (1953).

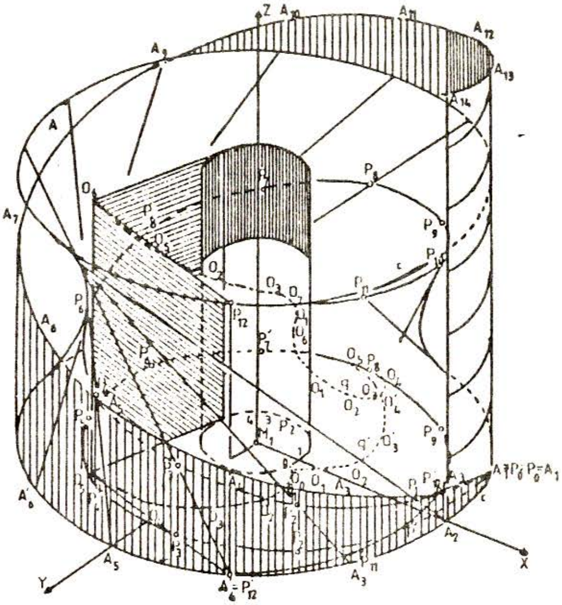

Fig 3

the involutional motion of the circle of parameter p in the horizontal plane. This curve is the horizontal projection of the involute helix P. This helix is a curve is situated at the main lines of the deformed cylinder. That is from reciprocal aspect, when the main lines of the hyperbolojid transformed to the main lines of the cylinder surface, the trajectory of the P_i points on the lines is barely an Euclidean helix. Thus we can declare the following theorem:

Theorem 6:

The deformation of a cylinder which is confident to length is reciprocal to the evolution motion. By this reciprocal motion, with the fact of transformation of each curve of the space involutional motion, i.e. the helical involutional motion, with a parameter p, the screwing motions in the pace can be obtained as the deformation curve, and vice versa.

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